Examiners’ Report

Principal Examiner Feedback

Summer 2017

Pearson Edexcel International A-Level
in Further Pure Mathematics (WFM02/01)

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General Introduction

This paper proved to be a good test of student knowledge and understanding. There were many accessible marks available to students who were confident with topics such as complex numbers, inequalities, the method of differences, differential equations, series expansions and polar coordinates.

Reports on Individual Questions

Question 1

This question required the fifth roots of 32 to be determined and was a good source of marks for most students, with many scoring the full five marks very confidently. However, some students could not use de Moivre's theorem correctly, leading to errors such as $z = 2(\cos 2k\pi + \mathrm{i}\sin 2k\pi)$. Occasionally the incorrect $\arg(32) = \frac{\pi}{2}$ was used. A more common error was to provide the correct five solutions but with arguments of $-\frac{2\pi}{5}$ and $-\frac{4\pi}{5}$ used instead of $\frac{8\pi}{5}$ and $\frac{6\pi}{5}$ respectively. Some solutions were seen in the form $r(\cos\theta - i\sin\theta)$. The $k = 0$ solution of $2(\cos 0 + \mathrm{i}\sin 0)$ or just

z = 2 was occasionally missing. A very small minority of candidates gave answers in degrees.

Question 2

This question on solving a fractional inequality saw many students obtaining eight or nine of the available nine marks. Approaches taken were equally split between collecting the fractions on one side or multiplying both sides by a suitable positive expression. In the latter approach, the multiplier was often needlessly complicated such as $x^2(x+3)^4$. A few attempts considered the inequality within different regions of x values and were largely correct. Graphical attempts were very rare.

Students who had identified the critical values of 0 and -3 usually had little difficulty in obtaining all four and the correct inequalities were then commonly seen. A notable error was with the strictness of the inequality signs since many students did not remember to exclude -3 and 0 from their solution set.

Question 3

This question required use of the method of differences and summation algebra to prove the standard result for the sum of the squares. A wide range of mark profiles were seen here. Almost all students were able to prove the identity in part (a) without error, but part (b) proved rather more demanding.

Some students did not use the result in part (a) and instead used the standard result for the sum of the cubes or proof by induction. The method of differences was usually correctly applied and with sufficient terms included. Errors were seen in handling the three term summation although the mark for replacing $\sum 1$ with n was commonly awarded. Those who had obtained the correct algebraic expression for $\sum r^2$ tended to reach the printed answer convincingly.

Question 4

Varied responses were seen to this second order differential equation question and some attempts were abandoned early on. Part (a) required the determination of a constant in a non-standard particular integral. A significant number of students began by writing the auxiliary equation and this often led to subsequent confusion between complementary functions, particular integrals, general and particular solutions. The differentiation required proved demanding, although students who simplified their expressions as they proceeded were more likely to be successful. Four of the five marks in part (a) were for method and were commonly scored. Some students introduced their own incorrect particular integral. Those who chose to use $\lambda \mathrm{e}^{-x}\cos x + \mu \mathrm{e}^{-x}\sin x$ could access all the marks but gave themselves additional simultaneous equations to solve.

Most produced the correct auxiliary equation in part (b) although occasional incorrect solutions were seen. The correct form of complementary function usually followed although the e^{x} or constants were sometimes missing. Those who chose the alternative exponential form often ran into difficulties differentiating in part (c). The subsequent follow through mark for combining their complementary function and particular integral was widely scored.

Three of the four marks in part (c) were for method and were fairly accessible. The final mark for a fully correct solution was only scored by the most confident and organised students.

Question 5

A Maclaurin series expansion of $e^{\cos^2 x}$ was required here and many fully correct responses were seen. In part (a), most students were able to obtain the correct $\frac{dy}{dx}$. Those that then replaced the $2\sin x\cos x$ with $\sin 2x$ usually proceeded correctly. Use of $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$ prior to differentiation helped some to follow this route. A few students took natural logarithms of both sides before differentiating.

The method to produce the series expansion in part (b) was not always known. Attempts were seen which tried to use the series expansions of e^x and $\cos x$. Those who used the correct formula were usually able to obtain full marks although $f'(0)$ was occasionally evaluated as e. A few students failed to calculate their trigonometric expressions or gave their answer in a decimal form.

Question 6

This first order differential equation question saw good scoring by the vast majority. It was rare to see any student fail to divide through by $\cos x$ before forming the integrating factor. A small number produced $e^{\sec^2 x}$ instead of $e^{\ln \sec x}$ or $e^{-\ln \cos x}$ after integrating $\tan x$. The method of multiplying both sides by the integrating factor was well known, although not all could obtain a right hand side of $\ln x$ or $\int \ln x$, usually the result of copying errors. The last two marks were more discriminating. $\int \ln x$ was widely given as $\frac{1}{x}$. Those who applied integration by parts were almost always successful although a few forgot to include the constant of integration in their final answer or had it incorrectly placed.

Question 7

This question on polar coordinates proved to be quite demanding and fully correct solutions to all parts were not widely seen. In part (a), most knew the initial step of using $r\sin\theta$ and correct differentiation usually followed. Solving $\frac{\mathrm{d}y}{\mathrm{d}\theta} = 0$ proved challenging to many. Those who had replaced $\cos 2\theta$ with $1 - 2\sin^2\theta$ before differentiation tended to have more success. A variety of approaches were seen to the trigonometric equation but obtaining one of the correct values for θ was elusive. It was unfortunate to see $2\pi - \theta$ rather than $\pi - \theta$ used for the second value. A common error was to neglect to find the corresponding value for r.

Part (b) was a reasonable source of marks for most students and almost all knew that integration of $k\int(4\cos 2\theta)^2$ was required. Almost all were able to write the integrand in terms of $\cos 4\theta$ and integrate correctly. The last method mark was more difficult and inappropriate limits and/or wrong multipliers were widely seen, resulting in various incorrect multiples of π. Some students thought that the values of θ from part (a) were required as limits. Students who explicitly showed their method to obtain the area bounded by two loops of the graph were more successful. Many were unaware of how the use of limits outside the range of θ for which the graph was defined risked incorporating extra loops into their calculation.

Part (c) continued to challenge, although the mark scheme was designed to reward all students who used an appropriate method. A few variations were possible as in part (b) although finding the area of the entire rectangle *PQRS* directly was the usual route. Some students needlessly embarked upon solving $\frac{\mathrm{d}}{\mathrm{d}x}(r\cos\theta) = 0$. Many were able to write down the length of the rectangle but the width was often incorrect, with values of $2r$ rather than $2r\sin\theta$ a common misconception. Those who obtained a value for the length and width invariably scored the two method marks for an acceptable attempt at the shaded area, but a correct final answer was not common.

Question 8

The final question on using de Moivre's theorem for trigonometric identities provided most students with marks in parts (a) and (b), but the last two question parts proved very discriminating. In part (a), the method of expanding $(\cos\theta + \mathrm{i}\sin\theta)^5$ was well known and often fully correct. Some students were clearly rushing their working here and left themselves more vulnerable to sign errors and other slips. Some expressions for $\sin 5\theta$ were offered without "i" being removed. Use of $\left(z + \frac{1}{z}\right)^5$ was not common and most attempts via this route became bogged down in awkward algebra.

The first mark in part (b) was widely scored but many were unable to achieve the printed result. Those who identified that the numerator and denominator had to be divided by $\cos^5\theta$ usually produced the answer with little effort. Others attempted to use various identities and other manipulations and found reaching the given answer elusive.

Many students offered no response to the final two parts. The key in part (c) was to take note of the "Hence". Solutions from multiplying out $\left(x - \tan^2\frac{\pi}{5}\right)(x - \tan^2\frac{2\pi}{5})$ could receive no credit. Those that attempted to use the result from part (b) almost always arrived at the correct quartic equation from which they could usually derive the required quadratic.

Part (d) was a deduction and it was essential that use of the product of the roots from the quadratic was clearly evident. Attempts that purely consisted of $\tan^2\frac{\pi}{5}\tan^2\frac{2\pi}{5} = 5$ followed by the given answer could not score here. Acceptable evidence included an explanation or sight of $x_1x_2 = 5$, $\alpha\beta = 5$ or use of $\frac{c}{a}$. An alternative by calculating the exact roots as surds and then multiplying them together was also acceptable and a successful route for some students.

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