Write your name here

Surname	Other names

Pearson
Edexcel GCE

Centre Number

Candidate Number

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Friday 19 May 2017 – Morning
Time: 1 hour 30 minutes

Paper Reference
6667/01

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Pearson

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1.

$$f(x) = \frac{1}{3}x^2 + \frac{4}{x^2} - 2x - 1, \quad x > 0$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval $[6, 7]$

(2)

(b) Taking 6 as a first approximation to α, apply the Newton-Raphson process once to $f(x)$ to obtain a second approximation to α. Give your answer to 2 decimal places.

(5)

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Question 1 continued

(Total 7 marks)

Q1

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2.

$$\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}, \quad \mathbf{P} = \begin{pmatrix} 3 & 6 \\ 11 & -8 \end{pmatrix}$$

(a) Find $\mathbf{A}^{-1}$

(2)

The transformation represented by the matrix **B** followed by the transformation represented by the matrix **A** is equivalent to the transformation represented by the matrix **P**.

(b) Find **B**, giving your answer in its simplest form.

(3)

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Question 2 continued

(Total 5 marks)

Q2

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3. The rectangular hyperbola H has parametric equations

$$x = 4t, \quad y = \frac{4}{t} \qquad t \neq 0$$

The points P and Q on this hyperbola have parameters $t = \frac{1}{4}$ and $t = 2$ respectively.

The line l passes through the origin O and is perpendicular to the line PQ.

(a) Find an equation for l.

(3)

(b) Find a cartesian equation for H.

(1)

(c) Find the exact coordinates of the two points where l intersects H.
Give your answers in their simplest form.

(3)

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Question 3 continued

(Total 7 marks)

Q3

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4. (i) The complex number w is given by

$$w = \frac{p - 4\text{i}}{2 - 3\text{i}}$$

where p is a real constant.

(a) Express w in the form $a + b\text{i}$, where a and b are real constants.
Give your answer in its simplest form in terms of p.

(3)

Given that $\arg w = \frac{\pi}{4}$

(b) find the value of p.

(2)

(ii) The complex number z is given by

$$z = (1 - \lambda\text{i})(4 + 3\text{i})$$

where λ is a real constant.

Given that

$$|z| = 45$$

find the possible values of λ.
Give your answers as exact values in their simplest form.

(3)

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Question 4 continued

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Question 4 continued

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Question 4 continued

Q4

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5. (i)

$$\mathbf{A} = \begin{pmatrix} p & 2 \\ 3 & p \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} -5 & 4 \\ 6 & -5 \end{pmatrix}$$

where p is a constant.

(a) Find, in terms of p, the matrix **AB**

(2)

Given that

$$\mathbf{AB} + 2\mathbf{A} = k\mathbf{I}$$

where k is a constant and **I** is the 2 × 2 identity matrix,

(b) find the value of p and the value of k.

(4)

(ii)

$$\mathbf{M} = \begin{pmatrix} a & -9 \\ 1 & 2 \end{pmatrix}, \text{ where } a \text{ is a real constant}$$

Triangle T has an area of 15 square units.

Triangle T is transformed to the triangle T' by the transformation represented by the matrix **M**.

Given that the area of triangle T' is 270 square units, find the possible values of a.

(5)

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Question 5 continued

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Question 5 continued

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Question 5 continued

Q5

(Total 11 marks)

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6. Given that 4 and $2i - 3$ are roots of the equation

$$x^3 + ax^2 + bx - 52 = 0$$

where a and b are real constants,

(a) write down the third root of the equation,

(1)

(b) find the value of a and the value of b.

(5)

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Question 6 continued

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Question 6 continued

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Question 6 continued

Q6

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7. The parabola C has equation $y^2 = 4ax$, where a is a constant and $a > 0$
The point $Q(aq^2, 2aq)$, $q > 0$, lies on the parabola C.

(a) Show that an equation of the tangent to C at Q is

$$qy = x + aq^2$$

(4)

The tangent to C at the point Q meets the x-axis at the point $X\left(-\frac{1}{4}a, 0\right)$ and meets the directrix of C at the point D.

(b) Find, in terms of a, the coordinates of D.

(4)

Given that the point F is the focus of the parabola C,

(c) find the area, in terms of a, of the triangle FXD, giving your answer in its simplest form.

(2)

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Question 7 continued

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Question 7 continued

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Question 7 continued

Q7

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8. (a) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n}(3r^2 + 8r + 3) = \frac{1}{2}n(2n+5)(n+3)$$

for all positive integers n.

(5)

Given that

$$\sum_{r=1}^{12}\left(3r^2 + 8r + 3 + k(2^{r-1})\right) = 3520$$

(b) find the exact value of the constant k.

(4)

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Question 8 continued

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Question 8 continued

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Question 8 continued

Q8

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9. (i) A sequence of numbers is defined by

$$u_1 = 6, \qquad u_2 = 27$$

$$u_{n+2} = 6u_{n+1} - 9u_n \qquad n \geqslant 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$

$$u_n = 3^n(n + 1)$$

(6)

(ii) Prove by induction that, for $n \in \mathbb{Z}^+$

$$\mathrm{f}(n) = 3^{3n-2} + 2^{3n+1} \text{ is divisible by } 19$$

(6)

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Question 9 continued

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Question 9 continued

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Question 9 continued

Q9

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

END

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