Write your name here

Surname	Other names

Pearson
Edexcel GCE

Centre Number | Candidate Number

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Friday 20 May 2016 – Morning
Time: 1 hour 30 minutes

Paper Reference
6667/01

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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PEARSON

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1. Given that k is a real number and that

$$\mathbf{A} = \begin{pmatrix} 1+k & k \\ k & 1-k \end{pmatrix}$$

find the exact values of k for which $\mathbf{A}$ is a singular matrix. Give your answers in their simplest form.

(3)

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Question 1 continued

Q1

(Total 3 marks)

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2. $$f(x) = 3x^{\frac{3}{2}} - 25x^{-\frac{1}{2}} - 125, \qquad x > 0$$

(a) Find $f'(x)$.

(2)

The equation $f(x) = 0$ has a root α in the interval [12, 13].

(b) Using $x_0 = 12.5$ as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to find a second approximation to α, giving your answer to 3 decimal places.

(4)

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Question 2 continued

Q2

(Total 6 marks)

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3. (a) Using the formula for $\sum_{r=1}^{n} r^2$ write down, in terms of n only, an expression for

$$\sum_{r=1}^{3n} r^2$$

(1)

(b) Show that, for all integers n, where $n > 0$

$$\sum_{r=2n+1}^{3n} r^2 = \frac{n}{6}(an^2 + bn + c)$$

where the values of the constants a, b and c are to be found.

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Question 3 continued

Q3

(Total 5 marks)

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4.

$$z = \frac{4}{1 + \mathrm{i}}$$

Find, in the form $a + \mathrm{i}b$ where $a, b \in \mathbb{R}$

(a) z

(2)

(b) z^2

(2)

Given that z is a complex root of the quadratic equation $x^2 + px + q = 0$, where p and q are real integers,

(c) find the value of p and the value of q.

(3)

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Question 4 continued

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Question 4 continued

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Question 4 continued

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Q4

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5. Points $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$, where $p^2 \neq q^2$, lie on the parabola $y^2 = 4ax$.

(a) Show that the chord PQ has equation

$$y(p + q) = 2x + 2apq$$

(5)

Given that this chord passes through the focus of the parabola,

(b) show that $pq = -1$

(1)

(c) Using calculus find the gradient of the tangent to the parabola at P.

(2)

(d) Show that the tangent to the parabola at P and the tangent to the parabola at Q are perpendicular.

(2)

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Question 5 continued

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Question 5 continued

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Question 5 continued

Q5

(Total 10 marks)

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6.

$$\mathbf{P} = \begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

(a) Describe fully the single geometrical transformation U represented by the matrix **P**. **(2)**

The transformation U maps the point A, with coordinates (p, q), onto the point B, with coordinates $(6\sqrt{2}, 3\sqrt{2})$.

(b) Find the value of p and the value of q. **(3)**

The transformation V, represented by the 2×2 matrix **Q**, is a reflection in the line with equation $y = x$.

(c) Write down the matrix **Q**. **(1)**

The transformation U followed by the transformation V is the transformation T. The transformation T is represented by the matrix **R**.

(d) Find the matrix **R**. **(3)**

(e) Deduce that the transformation T is self-inverse. **(1)**

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Question 6 continued

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Question 6 continued

Question 6 continued

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Q6

(Total 10 marks)

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7. A complex number z is given by

$$z = a + 2\text{i}$$

where a is a non-zero real number.

(a) Find $z^2 + 2z$ in the form $x + \text{i}y$ where x and y are real expressions in terms of a.
(4)

Given that $z^2 + 2z$ is real,

(b) find the value of a.
(1)

Using this value for a,

(c) find the values of the modulus and argument of z, giving the argument in radians, and giving your answers to 3 significant figures.
(3)

(d) Show the points P, Q and R, representing the complex numbers z, z^2 and $z^2 + 2z$ respectively, on a single Argand diagram with origin O.
(3)

(e) Describe fully the geometrical relationship between the line segments OP and QR.
(2)

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Question 7 continued

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Question 7 continued

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Q7

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8. (i) Prove by induction that, for $n \in \mathbb{Z}^+$

$$\sum_{r=1}^{n} \frac{2r+1}{r^2(r+1)^2} = 1 - \frac{1}{(n+1)^2}$$

(5)

(ii) A sequence of positive rational numbers is defined by

$$u_1 = 3$$

$$u_{n+1} = \frac{1}{3}u_n + \frac{8}{9}, \qquad n \in \mathbb{Z}^+$$

Prove by induction that, for $n \in \mathbb{Z}^+$

$$u_n = 5 \times \left(\frac{1}{3}\right)^n + \frac{4}{3}$$

(5)

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Question 8 continued

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Question 8 continued

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Q8

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9. The rectangular hyperbola, H, has cartesian equation $xy = 25$

(a) Show that an equation of the normal to H at the point $P\left(5p, \frac{5}{p}\right)$, $p \neq 0$, is

$$y - p^2x = \frac{5}{p} - 5p^3$$

(5)

This normal meets the line with equation $y = -x$ at the point A.

(b) Show that the coordinates of A are

$$\left(-\frac{5}{p} + 5p, \frac{5}{p} - 5p\right)$$

(3)

The point M is the midpoint of the line segment AP.
Given that M lies on the positive x-axis,

(c) find the exact value of the x coordinate of point M.

(3)

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Question 9 continued

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Question 9 continued

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Question 9 continued

Question 9 continued

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Q9

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

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