Write your name here

Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number | | | | | |

Candidate Number | | | | |

Core Mathematics C34

Advanced

Friday 12 June 2015 – Morning
Time: 2 hours 30 minutes

Paper Reference
WMA02/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1. A curve has equation

$$4x^2 - y^2 + 2xy + 5 = 0$$

The points P and Q lie on the curve.

Given that $\frac{dy}{dx} = 2$ at P and at Q,

(a) use implicit differentiation to show that $y - 6x = 0$ at P and at Q.

(6)

(b) Hence find the coordinates of P and Q.

(3)

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Question 1 continued

Q1

(Total 9 marks)

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2. Given that

$$\frac{4(x^2 + 6)}{(1 - 2x)(2 + x)^2} \equiv \frac{A}{(1 - 2x)} + \frac{B}{(2 + x)} + \frac{C}{(2 + x)^2}$$

(a) find the values of the constants A and C and show that $B = 0$

(4)

(b) Hence, or otherwise, find the series expansion of

$$\frac{4(x^2 + 6)}{(1 - 2x)(2 + x)^2}, \qquad |x| < \frac{1}{2}$$

in ascending powers of x, up to and including the term in x^2, simplifying each term.

(5)

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Question 2 continued

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Question 2 continued

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Question 2 continued

Q2

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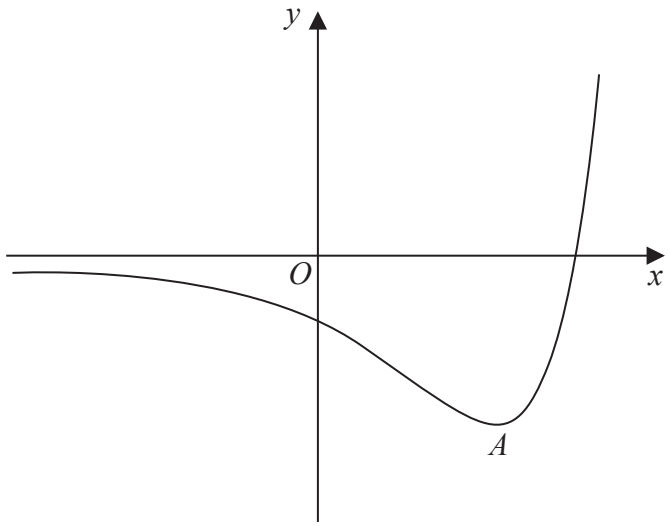

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = (2x - 5)e^x, \qquad x \in \mathbb{R}$$

The curve has a minimum turning point at A.

(a) Use calculus to find the exact coordinates of A.

(5)

Given that the equation $f(x) = k$, where k is a constant, has exactly two roots,

(b) state the range of possible values of k.

(2)

(c) Sketch the curve with equation $y = |f(x)|$.

Indicate clearly on your sketch the coordinates of the points at which the curve crosses or meets the axes.

(3)

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Question 3 continued

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Question 3 continued

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Question 3 continued

Q3

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4.

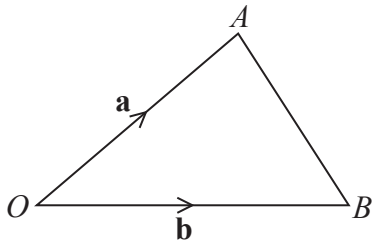

Figure 2

Figure 2 shows the points A and B with position vectors **a** and **b** respectively, relative to a fixed origin O.

Given that $|\mathbf{a}| = 5$, $|\mathbf{b}| = 6$ and $\mathbf{a}.\mathbf{b} = 20$

(a) find the cosine of angle AOB,

(2)

(b) find the exact length of AB.

(2)

(c) Show that the area of triangle OAB is $5\sqrt{5}$

(3)

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Question 4 continued

Q4

(Total 7 marks)

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5. (i) Find the x coordinate of each point on the curve $y = \dfrac{x}{x+1}$, $x \neq -1$, at which the gradient is $\dfrac{1}{4}$

(4)

(ii) Given that

$$\int_a^{2a} \frac{t+1}{t}\,\mathrm{d}t = \ln 7 \qquad a > 0$$

find the exact value of the constant a.

(4)

Leave blank

Question 5 continued

Q5

(Total 8 marks)

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6. The mass, m grams, of a radioactive substance t years after first being observed, is modelled by the equation

$$m = 25e^{1-kt}$$

where k is a positive constant.

(a) State the value of m when the radioactive substance was first observed.

(1)

Given that the mass is 50 grams, 10 years after first being observed,

(b) show that $k = \frac{1}{10}\ln\left(\frac{1}{2}e\right)$

(4)

(c) Find the value of t when $m = 20$, giving your answer to the nearest year.

(3)

Leave blank

Question 6 continued

Q6

(Total 8 marks)

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7. (a) Use the substitution $t = \tan x$ to show that the equation

$$4\tan 2x - 3\cot x \sec^2 x = 0$$

can be written in the form

$$3t^4 + 8t^2 - 3 = 0$$

(4)

(b) Hence solve, for $0 \leqslant x < 2\pi$,

$$4\tan 2x - 3\cot x \sec^2 x = 0$$

Give each answer in terms of π. You must make your method clear.

(4)

Leave blank

Question 7 continued

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Question 7 continued

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Question 7 continued

Q7

(Total 8 marks)

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8. (a) Prove by differentiation that

$$\frac{\mathrm{d}}{\mathrm{d}y}\left(\ln \tan 2y\right) = \frac{4}{\sin 4y}, \qquad 0 < y < \frac{\pi}{4}$$

(4)

(b) Given that $y = \frac{\pi}{6}$ when $x = 0$, solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 2\cos x \sin 4y, \qquad 0 < y < \frac{\pi}{4}$$

Give your answer in the form $\tan 2y = A\mathrm{e}^{B\sin x}$, where A and B are constants to be determined.

(6)

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Question 8 continued

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Question 8 continued

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Question 8 continued

Q8

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9.

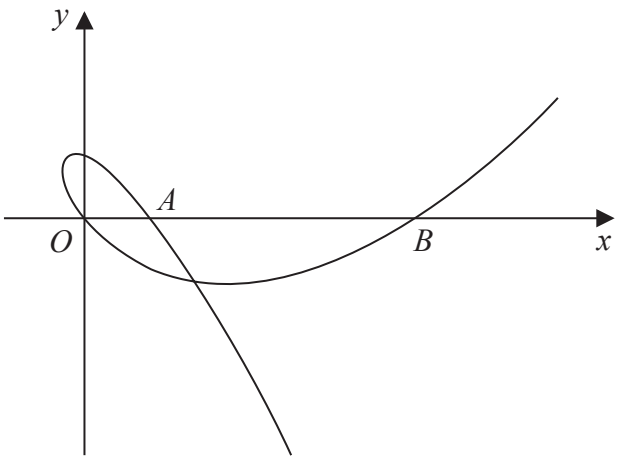

Figure 3

Figure 3 shows a sketch of part of the curve with parametric equations

$$x = t^2 + 2t, \qquad y = t^3 - 9t, \qquad t \in \mathbb{R}$$

The curve cuts the x-axis at the origin and at the points A and B as shown in Figure 3.

(a) Find the coordinates of point A and show that point B has coordinates (15, 0).

(3)

(b) Show that the equation of the tangent to the curve at B is $9x - 4y - 135 = 0$

(5)

The tangent to the curve at B cuts the curve again at the point X.

(c) Find the coordinates of X.

(5)

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

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Question 9 continued

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Question 9 continued

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Question 9 continued

Q9

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10.

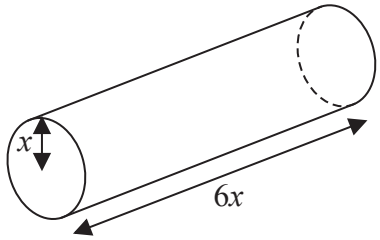

Figure 4

Figure 4 shows a right circular cylindrical rod which is expanding as it is heated.

At time t seconds the radius of the rod is x cm and the length of the rod is $6x$ cm.

Given that the **cross-sectional area** of the rod is increasing at a constant rate of $\frac{\pi}{20}$ cm^2 s^{-1}, find the rate of increase of the volume of the rod when $x = 2$

Write your answer in the form $k\pi$ cm^3 s^{-1} where k is a rational number.

(6)

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Question 10 continued

Q10

(Total 6 marks)

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11. (a) Express $1.5\sin\theta - 1.2\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the value of R and the value of α to 3 decimal places.

(3)

The height, H metres, of sea water at the entrance to a harbour on a particular day, is modelled by the equation

$$H = 3 + 1.5\sin\left(\frac{\pi t}{6}\right) - 1.2\cos\left(\frac{\pi t}{6}\right), \quad 0 \leqslant t < 12$$

where t is the number of hours after midday.

(b) Using your answer to part (a), calculate the minimum value of H predicted by this model and the value of t, to 2 decimal places, when this minimum occurs.

(4)

(c) Find, to the nearest minute, the times when the height of sea water at the entrance to the harbour is predicted by this model to be 4 metres.

(6)

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Question 11 continued

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Question 11 continued

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Question 11 continued

Q11

(Total 13 marks)

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12. (i) Relative to a fixed origin O, the line l_1 is given by the equation

$$l_1: \ \mathbf{r} = \begin{pmatrix} -5 \\ 1 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} \text{ where } \lambda \text{ is a scalar parameter.}$$

The point P lies on l_1. Given that $\overrightarrow{OP}$ is perpendicular to l_1, calculate the coordinates of P.

(5)

(ii) Relative to a fixed origin O, the line l_2 is given by the equation

$$l_2: \ \mathbf{r} = \begin{pmatrix} 4 \\ -3 \\ 12 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ -3 \\ 4 \end{pmatrix} \text{ where } \mu \text{ is a scalar parameter.}$$

The point A **does not** lie on l_2. Given that the vector $\overrightarrow{OA}$ is parallel to the line l_2 and $|\overrightarrow{OA}| = \sqrt{2}$ units, calculate the possible position vectors of the point A.

(5)

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Question 12 continued

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Question 12 continued

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Question 12 continued

Q12

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13.

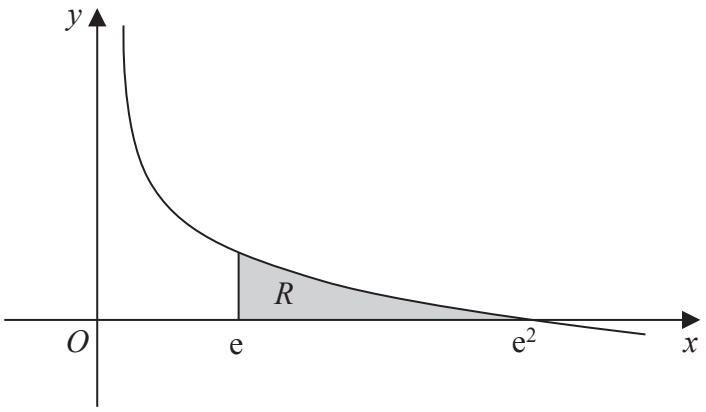

Figure 5

Figure 5 shows a sketch of part of the curve with equation $y = 2 - \ln x, \quad x > 0$

The finite region R, shown shaded in Figure 5, is bounded by the curve, the x-axis and the line with equation $x = e$.

The table below shows corresponding values of x and y for $y = 2 - \ln x$

x	e	$\frac{e + e^2}{2}$	e^2
y	1		0

(a) Complete the table giving the value of y to 4 decimal places.

(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 3 decimal places.

(3)

(c) Use integration by parts to show that $\int (\ln x)^2 \, dx = x (\ln x)^2 - 2x \ln x + 2x + c$

(4)

The area R is rotated through 360° about the x-axis.

(d) Use calculus to find the exact volume of the solid generated.

Write your answer in the form $\pi e(pe + q)$, where p and q are integers to be found.

(6)

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Question 13 continued

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Question 13 continued

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Question 13 continued

Q13

(Total 14 marks)

TOTAL FOR PAPER: 125 MARKS

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