Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 7 / 0 1 R	Signature	

Paper Reference(s)

6667/01R

Edexcel GCE

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Monday 10 June 2013 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total	

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 10 questions in this question paper. The total mark for this paper is 75.
There are 36 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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P 4 2 8 2 8 A 0 1 3 6

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PEARSON

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1. The complex numbers z and w are given by

$$z = 8 + 3\text{i}, \quad w = -2\text{i}$$

Express in the form $a + b\text{i}$, where a and b are real constants,

(a) $z - w$,

(1)

(b) zw.

(2)

Leave blank

Question 1 continued

(Total 3 marks)

Q1

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2. (i) $$\mathbf{A} = \begin{pmatrix} 2k+1 & k \\ -3 & -5 \end{pmatrix}, \quad \text{where } k \text{ is a constant}$$

Given that

$$\mathbf{B} = \mathbf{A} + 3\mathbf{I}$$

where $\mathbf{I}$ is the 2×2 identity matrix, find

(a) $\mathbf{B}$ in terms of k,

(2)

(b) the value of k for which $\mathbf{B}$ is singular.

(2)

(ii) Given that

$$\mathbf{C} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}, \quad \mathbf{D} = (2 \;\; -1 \;\; 5)$$

and

$$\mathbf{E} = \mathbf{CD}$$

find $\mathbf{E}$.

(2)

Leave blank

Question 2 continued

Leave blank

Question 2 continued

Leave blank

Question 2 continued

Q2

(Total 6 marks)

Leave blank

3. $$\mathrm{f}(x) = \frac{1}{2}x^4 - x^3 + x - 3$$

(a) Show that the equation $\mathrm{f}(x) = 0$ has a root α between $x = 2$ and $x = 2.5$

(2)

(b) Starting with the interval $[2, 2.5]$ use interval bisection twice to find an interval of width 0.125 which contains α.

(3)

The equation $\mathrm{f}(x) = 0$ has a root β in the interval $[-2, -1]$.

(c) Taking -1.5 as a first approximation to β, apply the Newton-Raphson process once to $\mathrm{f}(x)$ to obtain a second approximation to β.
Give your answer to 2 decimal places.

(5)

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Q3

(Total 10 marks)

Leave blank

4. $$f(x) = (4x^2 + 9)(x^2 - 2x + 5)$$

(a) Find the four roots of $f(x) = 0$

(4)

(b) Show the four roots of $f(x) = 0$ on a single Argand diagram.

(2)

Leave blank

Question 4 continued

Q4

(Total 6 marks)

P 4 2 8 2 8 A 0 1 3 3 6

Turn over

5.

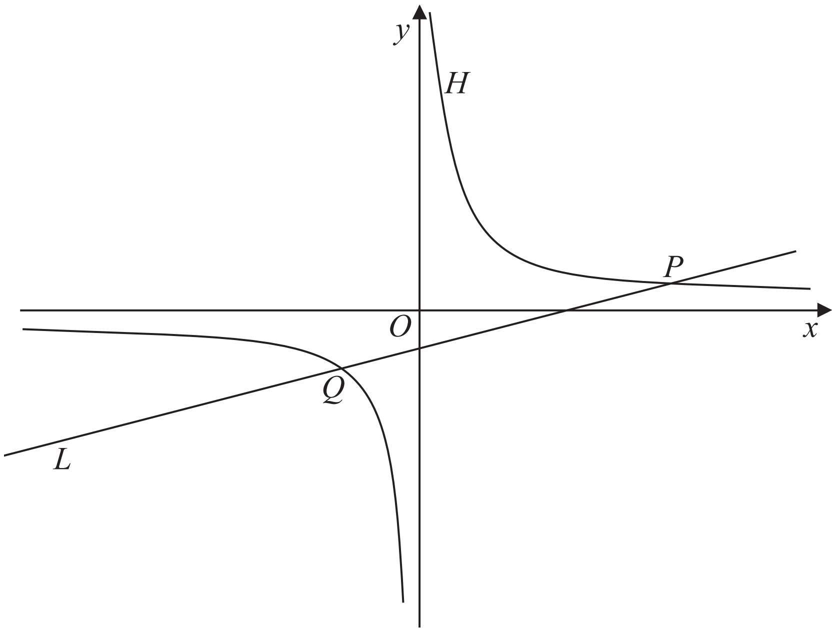

Figure 1

Figure 1 shows a rectangular hyperbola H with parametric equations

$$x = 3t, \quad y = \frac{3}{t}, \quad t \neq 0$$

The line L with equation $6y = 4x - 15$ intersects H at the point P and at the point Q as shown in Figure 1.

(a) Show that L intersects H where $4t^2 - 5t - 6 = 0$

(3)

(b) Hence, or otherwise, find the coordinates of points P and Q.

(5)

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P 4 2 8 2 8 A 0 1 4 3 6

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Question 5 continued

Leave blank

Question 5 continued

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Question 5 continued

(Total 8 marks)

Q5

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6. $$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$$

The transformation represented by **B** followed by the transformation represented by **A** is equivalent to the transformation represented by **P**.

(a) Find the matrix **P**.

(2)

Triangle T is transformed to the triangle T' by the transformation represented by **P**.

Given that the area of triangle T' is 24 square units,

(b) find the area of triangle T.

(3)

Triangle T' is transformed to the original triangle T by the matrix represented by **Q**.

(c) Find the matrix **Q**.

(2)

Leave blank

Question 6 continued

Q6

(Total 7 marks)

P 4 2 8 2 8 A 0 1 9 3 6

Turn over

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7. The parabola C has equation $y^2 = 4ax$, where a is a positive constant.

The point $P(at^2, 2at)$ is a general point on C.

(a) Show that the equation of the tangent to C at $P(at^2, 2at)$ is

$$ty = x + at^2$$

(4)

The tangent to C at P meets the y-axis at a point Q.

(b) Find the coordinates of Q.

(1)

Given that the point S is the focus of C,

(c) show that PQ is perpendicular to SQ.

(3)

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Question 7 continued

P 4 2 8 2 8 A 0 2 1 3 6

Turn over

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Question 7 continued

Question 7 continued

Leave blank

(Total 8 marks)

Q7

P 4 2 8 2 8 A 0 2 3 3 6

Turn over

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8. (a) Prove by induction, that for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} r(2r-1) = \frac{1}{6}n(n+1)(4n-1)$$

(6)

(b) Hence, show that

$$\sum_{r=n+1}^{3n} r(2r-1) = \frac{1}{3}n(an^2 + bn + c)$$

where a, b and c are integers to be found.

(4)

Leave blank

Question 8 continued

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Question 8 continued

Question 8 continued

(Total 10 marks)

Q8

Leave blank

9. The complex number w is given by

$$w = 10 - 5\mathrm{i}$$

(a) Find $|w|$.

(1)

(b) Find arg w, giving your answer in radians to 2 decimal places.

(2)

The complex numbers z and w satisfy the equation

$$(2 + \mathrm{i})(z + 3\mathrm{i}) = w$$

(c) Use algebra to find z, giving your answer in the form $a + b\mathrm{i}$, where a and b are real numbers.

(4)

Given that

$$\arg(\lambda + 9\mathrm{i} + w) = \frac{\pi}{4}$$

where λ is a real constant,

(d) find the value of λ.

(2)

Leave blank

Question 9 continued

P 4 2 8 2 8 A 0 2 9 3 6

Turn over

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Question 9 continued

Question 9 continued

Leave blank

Q9

(Total 9 marks)

P 4 2 8 2 8 A 0 3 1 3 6

Leave blank

10. (i) Use the standard results for $\sum_{r=1}^{n} r^3$ and $\sum_{r=1}^{n} r$ to evaluate

$$\sum_{r=1}^{24}(r^3 - 4r)$$

(2)

(ii) Use the standard results for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that

$$\sum_{r=0}^{n}(r^2 - 2r + 2n + 1) = \frac{1}{6}(n+1)(n+a)(bn+c)$$

for all integers $n \geqslant 0$, where a, b and c are constant integers to be found.

(6)

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Question 10 continued

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Question 10 continued

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Question 10 continued

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Question 10 continued

Q10

(Total 8 marks)

TOTAL FOR PAPER: 75 MARKS

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