| Centre No. | | | | | | Paper Reference | | | | | | | Surname | Initial(s) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Candidate No. | | | | | | 6 | 6 | 7 | 6 | / | 0 | 1 | Signature | |

Paper Reference(s)

**6676/01**

# Edexcel GCE

## Further Pure Mathematics FP3

## Advanced/Advanced Subsidiary

Wednesday 3 February 2010 – Morning

Time: 1 hour 30 minutes

Examiner's use only

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Team Leader's use only

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| Question Number | Leave Blank |
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**Materials required for examination**
Mathematical Formulae (Green)

**Items included with question papers**
Nil

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

### Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

### Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 7 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

### Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.



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**1.** $$\frac{dy}{dx} = x^2 + 2\sin y$$

It is given that $y = 1$ at $x = 0$.

Use the approximation $\frac{y_1 - y_0}{h} = \left(\frac{dy}{dx}\right)_0$ with a step length of $h = 0.1$ to find estimates of $y$ at $x = 0.1$ and at $x = 0.2$, giving your answers to 4 decimal places.

**(5)**

Leave blank

**Question 1 continued**

**(Total 5 marks)**

**Q1**

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**2.** Given that $y = x^3 \ln x$,

(a) find $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$.

**(5)**

(b) Find the Taylor series expansion of $x^3 \ln x$ in ascending powers of $(x-1)$ up to and including the term in $(x-1)^3$.

**(3)**

**Question 2 continued**

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**Question 2 continued**

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**Question 2 continued**

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**(Total 8 marks)**

**Q2**

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**3.** (a) Use De Moivre's theorem to show that

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$$

**(5)**

(b) Hence find the two positive solutions of

$$32x^5 - 40x^3 + 10x + 1 = 0,$$

giving your answers to 3 decimal places.

**(6)**

**Question 3 continued**

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**Question 3 continued**

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**Question 3 continued**

**(Total 11 marks)**

**Q3**

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**4.** For $n \in \mathbb{Z}^+$, show, using mathematical induction, that

(i) $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}^n = \begin{pmatrix} 1 & 0 & 0 \\ n & 1 & 0 \\ n(n+2) & 2n & 1 \end{pmatrix}$,

**(5)**

(ii) $(2^{3n+1} + 5)$ is divisible by 7.

**(5)**

**Question 4 continued**

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**Question 4 continued**

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**Question 4 continued**

**(Total 10 marks)**

**Q4**

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**5.** Given that

$$\mathbf{a} = \mathbf{i} + 7\mathbf{j} + 9\mathbf{k} \quad \text{and} \quad \mathbf{b} = -\mathbf{i} + 3\mathbf{j} + \mathbf{k},$$

(a) show that $\mathbf{a} \times \mathbf{b} = c(2\mathbf{i} + \mathbf{j} - \mathbf{k})$, and state the value of the constant $c$.

**(2)**

The plane $\Pi_1$ passes through the point (3, 1, 3) and the vector $\mathbf{a} \times \mathbf{b}$ is perpendicular to $\Pi_1$.

(b) Find a cartesian equation for the plane $\Pi_1$.

**(2)**

The line $l_1$ has equation $\mathbf{r} = \mathbf{i} - 2\mathbf{k} + \lambda\mathbf{a}$.

(c) Show that the line $l_1$ lies in the plane $\Pi_1$.

**(2)**

The line $l_2$ has equation $\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + \mu\mathbf{b}$.
The line $l_2$ lies in a plane $\Pi_2$, which is parallel to the plane $\Pi_1$.

(d) Find a cartesian equation of the plane $\Pi_2$.

**(2)**

(e) Find the distance between the planes $\Pi_1$ and $\Pi_2$.

**(3)**

**Question 5 continued**

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**Question 5 continued**

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**Question 5 continued**

**(Total 11 marks)**

**Q5**

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**6.**

$$\mathbf{M} = \begin{pmatrix} 11 & -5\sqrt{3} \\ -5\sqrt{3} & 1 \end{pmatrix}$$

Given that $\lambda_1$ and $\lambda_2$ are the eigenvalues of **M** and $\lambda_1 > \lambda_2$,

(a) show that $\lambda_1 = 16$ and find the value of $\lambda_2$.

**(4)**

(b) Find eigenvectors corresponding to the eigenvalues $\lambda_1$ and $\lambda_2$.

**(4)**

Given that there is an orthogonal matrix **P** such that $\mathbf{P}^{-1}\mathbf{MP}$ is the diagonal matrix **D**, where $\mathbf{D} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$,

(c) find the matrix **P**,

**(2)**

(d) verify that $\mathbf{P}^{-1}\mathbf{MP} = \mathbf{D}$.

**(4)**

**Question 6 continued**

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**Question 6 continued**

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**Question 6 continued**

**(Total 14 marks)**

**Q6**

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**7.** The point $P$ represents the complex number $z$ in an Argand diagram.
Point $P$ moves on the curve $C$ given by the equation

$$|z-4+4\mathrm{i}|=2|z-1+\mathrm{i}|$$

(a) Show that $C$ is a circle whose equation may be written $|z|=k$, giving the exact value of $k$.

**(5)**

(b) Draw an Argand diagram showing the circle $C$ and the points representing the complex numbers $1-\mathrm{i}$ and $4-4\mathrm{i}$.

**(3)**

(c) For the points on the circle $C$, find the maximum and minimum values of $|z-4+4\mathrm{i}|$.

**(3)**

The transformation $T$ from the $z$-plane to the $w$-plane is given by $w=z+\frac{8}{z}$.

(d) Show that $T$ maps the curve $C$ onto a line segment in the $w$-plane and define this line segment by giving its equation and the coordinates of its end points.

**(5)**

**Question 7 continued**

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**Question 7 continued**

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**Question 7 continued**

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**Q7**

**(Total 16 marks)**

**TOTAL FOR PAPER: 75 MARKS**

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