Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	7	6	/	0	1	Signature	

Paper Reference(s)

6676/01

Edexcel GCE

Further Pure Mathematics FP3

Advanced/Advanced Subsidiary

Tuesday 23 June 2009 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions. You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

This publication may be reproduced only in accordance with Edexcel Limited copyright policy.
©2009 Edexcel Limited.

Printer's Log. No.
N34272A
W850/R6676/57570 3/3/3/3/3

N 3 4 2 7 2 A 0 1 2 8

Turn over

edexcel
advancing learning, changing lives

Leave blank

1.

$$\frac{dy}{dx} = xy^2 + 3$$

It is given that $y = 0$ at $x = 0$.

Use the approximation $\frac{y_1 - y_0}{h} = \left(\frac{dy}{dx}\right)_0$, with a step length of $h = 0.1$, to find estimates of y at $x = 0.1$, 0.2 and 0.3

(5)

N 3 4 2 7 2 A 0 2 2 8

Leave blank

Question 1 continued

(Total 5 marks)

Q1

N 3 4 2 7 2 A 0 3 2 8

Turn over

Leave blank

2.

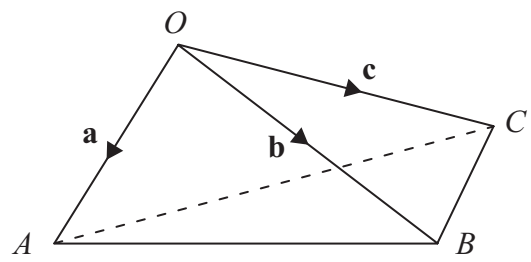

Figure 1

The points A, B and C have position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively, relative to a fixed origin O, as shown in Figure 1.
It is given that

$$\mathbf{a} = \mathbf{i} + \mathbf{j}, \quad \mathbf{b} = 3\mathbf{i} - \mathbf{j} + \mathbf{k} \quad \text{and} \quad \mathbf{c} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}.$$

Calculate

(a) $\mathbf{b} \times \mathbf{c}$,

(3)

(b) $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$,

(2)

(c) the area of triangle OBC,

(2)

(d) the volume of the tetrahedron $OABC$.

(1)

N 3 4 2 7 2 A 0 4 2 8

Leave blank

Question 2 continued

(Total 8 marks)

Q2

N 3 4 2 7 2 A 0 5 2 8

Turn over

Leave blank

3.

$$\mathbf{M} = \begin{pmatrix} 6 & 1 & -1 \\ 0 & 7 & 0 \\ 3 & -1 & 2 \end{pmatrix}$$

(a) Show that 7 is an eigenvalue of the matrix **M** and find the other two eigenvalues of **M**.
(5)

(b) Find an eigenvector corresponding to the eigenvalue 7.
(4)

N 3 4 2 7 2 A 0 6 2 8

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Q3

(Total 9 marks)

Leave blank

4.

$$\frac{d^2y}{dx^2} + (1 + y^2)\frac{dy}{dx} + y = e^{2x}$$

At $x = 0$, $y = 1$ and $\frac{dy}{dx} = 2$.

(a) Find the value of $\frac{d^3y}{dx^3}$ at $x = 0$.

(6)

Given that the value of $\frac{d^4y}{dx^4}$ at $x = 0$ is 40,

(b) express y as a series in ascending powers of x, up to and including the term in x^4.

(4)

N 3 4 2 7 2 A 0 1 0 2 8

Leave blank

Question 4 continued

(Total 10 marks)

Q4

N 3 4 2 7 2 A 0 1 1 2 8

Turn over

Leave blank

5. (a) Use de Moivre's theorem to show that

$$\cos 6\theta = 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1.$$

(5)

(b) Hence, or otherwise, solve the equation $\cos 6\theta = \cos 2\theta$, $0 \leqslant \theta \leqslant \pi$.

(6)

N 3 4 2 7 2 A 0 1 2 2 8

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Leave blank

Question 5 continued

(Total 11 marks)

Q5

N 3 4 2 7 2 A 0 1 5 2 8

Leave blank

6. Prove by induction that, for $n \in \mathbb{Z}^+$,

(a) $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}^n = \begin{pmatrix} \cos n\theta & -\sin n\theta \\ \sin n\theta & \cos n\theta \end{pmatrix}$,

(5)

(b) $(4n + 3)5^n - 3$ is divisible by 16.

(5)

N 3 4 2 7 2 A 0 1 6 2 8

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

(Total 10 marks)

Q6

N 3 4 2 7 2 A 0 1 9 2 8

Leave blank

7. The lines l_1 and l_2 have equations

$$\mathbf{r}=\begin{pmatrix}1\\-1\\2\end{pmatrix}+\lambda\begin{pmatrix}-1\\3\\4\end{pmatrix} \quad \text{and} \quad \mathbf{r}=\begin{pmatrix}\alpha\\-4\\0\end{pmatrix}+\mu\begin{pmatrix}0\\3\\2\end{pmatrix}.$$

If the lines l_1 and l_2 intersect, find

(a) the value of α,

(4)

(b) an equation for the plane containing the lines l_1 and l_2, giving your answer in the form $ax + by + cz + d = 0$, where a, b, c and d are constants.

(4)

For other values of α, the lines l_1 and l_2 do not intersect and are skew lines.

Given that $\alpha = 2$,

(c) find the shortest distance between the lines l_1 and l_2.

(3)

N 3 4 2 7 2 A 0 2 0 2 8

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

(Total 11 marks)

Q7

Leave blank

8. The point P represents the complex number z on an Argand diagram. The locus of P is the curve C given by the equation

$$|z-3| = 2|z-4\mathrm{i}|.$$

(a) Show that C is a circle and give the coordinates of its centre and the value of its radius.

(6)

The point Q represents the complex number w. The point Q is related to the point P by $w = \dfrac{12}{z}$.

As P describes the curve C,

(b) show that the locus of Q is given by the equation

$$|w-a| = k|w-\mathrm{i}b|, \text{ where } a, b \text{ and } k \in \mathbb{R},$$

stating the values of a, b and k.

(5)

N 3 4 2 7 2 A 0 2 4 2 8

Leave blank

Question 8 continued

Leave blank

Question 8 continued

N 3 4 2 7 2 A 0 2 6 2 8

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

END

N 3 4 2 7 2 A 0 2 8 2 8