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Mark Scheme (Results)
January 2009

GCE

GCE Mathematics (6674/01)

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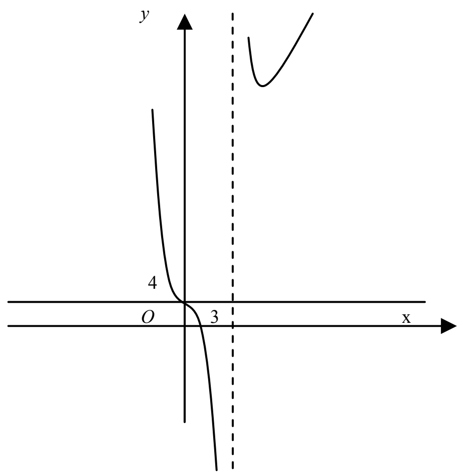
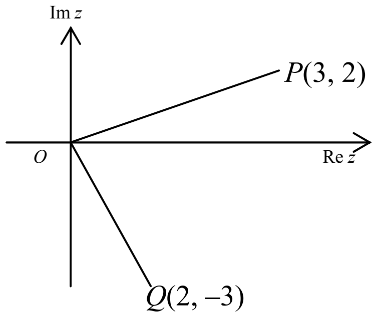
January 2009
6674 Further Pure Mathematics FP1 (legacy)
Mark Scheme

Question Number	Scheme	Marks
1 (a)	$\sum_{r=1}^{n} r^2 - \sum_{r=1}^{n} r - \sum_{r=1}^{n} 1 = \frac{1}{6}n(n+1)(2n+1) - \frac{1}{2}n(n+1) - n$	M1, A1
	Simplifying this expression	M1
	$= \frac{1}{3}n(n^2-4)$ (✱)	A1 (4) cso
(b)	$\sum_{r=1}^{20}(r^2-r-1) - \sum_{r=1}^{9}(r^2-r-1) = \frac{1}{3}\times 20\times(20^2-4) - \frac{1}{3}\times 9\times(9^2-4)$	M1
	$= 2409$	A1 (2)
Alt (b)	$\sum_{r=1}^{20}(r^2-r-1) - \sum_{r=1}^{9}(r^2-r-1) =$	
	$\left(\frac{1}{6}\times 20\times 21\times 41 - \frac{1}{2}\times 20\times 21 - 20\right) - \left(\frac{1}{6}\times 9\times 10\times 19 - \frac{1}{2}\times 9\times 10 - 9\right)$ M1	
	$= 2409$ A1	**[6]**
Notes	(a) 1st M: Separating, substituting set results, at least two correct. 2nd M: Either "eliminate" brackets totally or factor x [......] where any product of brackets inside [....] has been reduced to a single bracket 2nd A: ANSWER GIVEN. No wrong working seen; must have been an intermediate step, e.g. $\frac{1}{6}n(2n^2+3n+1-3n-3-6)$. (b) M : Must be $\sum_{r=1}^{20}(....) - \sum_{r=1}^{9}(....)$ applied. If list terms and add, allow M1 if **11 terms** with **at most two wrong**: [89, 109, 131, 155, 181, 209, 239, 271, 305, 341, 379]	

Question Number	Scheme	Marks
2	3 – i is a root (seen anywhere)	B1
	Attempt to multiply out $[x-(3+i)][x-(3-i)]$ $\{= x^2 - 6x + 10\}$ $f(x) = (x^2 - 6x + 10)(2x^2 - 2x + 1)$	M1 M1, A1
	$x = \dfrac{2 \pm \sqrt{4-8}}{4}$, $x = \dfrac{1 \pm i}{2}$	*M1, A1 **[6]**
Notes	1st M: Using the two roots to form a quadratic factor. 2nd M: Complete method to find second quadratic factor $2x^2 + ax$ (+ b). 3rd *M: Correct method, as far as $x = \ldots$, for solving candidate's second quadratic, DEPENDENT on both previous M marks	
Alt	(i)$f(x)/\{x-(3+i)\} = 2x^3 + (-8+2i)x^2 + (7-2i)x - 3 + i$ $\{=g(x)\}$ $g(x)/\{x-(3-i)\} = (2x^2 - 2x + 1)$ Attempt at complete process M2; A1 (ii)(2)$(x - a + ib)(x - a - ib)$("$x^2 - 6x + 10$") = $f(x)$ and compare ≥ 1 coeff. M1 Either $-2a - 6 = -7$, or two of $10(b^2 + a^2) = 5$ or $-6(a^2 + b^2) - 20a = -13$, $20 + 2(b^2 + a^2) + 24a = 33$ A1; Complete method for a **and** b, M1; AnswerA1	Lines 2 and 3 Lines 3 and 4

Question Number	Scheme	Marks
3	Identifying 3 as critical value e.g. used in soln Identifying 0 as critical value e.g. used in soln	B1 B1
	$\dfrac{x^3+5x-12-4(x-3)}{x-3}>0$ or $(x^3+5x-12)(x-3)>4(x-3)^2$ o.e.	M1
	$\dfrac{x(x^2+1)}{x-3}>0$ or $(x-3)(x^3+x)>0$	A1
	Using their critical values to **obtain** inequalities. $x<0$ or $x>3$	M1 A1 cso
Notes	1^{st} M must be a valid opening strategy. Sketching $y=\dfrac{x}{x-3}$ or $y=\dfrac{x(x^2+1)}{x-3}$ should mark as scheme. The result $0>x>3$ (poor notation) can gain final M but not A.	
Alt		
	Identifying 3 as critical value e.g. $x=3$ seen as asymp. Identifying 0 as critical value e.g. pt of intersection on y-axis of $y=\dfrac{x^3+5x-12}{x-3}$ and $y=4$	B1 B1
	M1 $y=\dfrac{x^3+5x-12}{x-3}$ sketched for $x<3$ or $y=\dfrac{x^3+5x-12}{x-3}$ sketched for $x>3$ A1 All correct including y = 4 drawn	M1, A1
	Using the graph values to obtain one or more inequalities $x<0$ or $x>3$	M1 A1

Question Number	Scheme	Marks
4 (a)	At st. pt $f'(x) = 0$, $\therefore x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$ is undefined or at st. pt, **tan.** // to x-axis, or **tan.** does not cross x-axis, o.e.	B1 (1)
(b)	$f'(x) = -1 - 2x\cos(x^2)$ (may be seen in body of work)	M1, A1
	$f(0.6) = 0.0477\ldots$, $f'(0.6) = -2.123\ldots$ (may be implied by correct answer)	A1
	Attempt to use $(x_1) = 0.6 - \dfrac{f(0.6)}{f'(0.6)}$ $\left[0.6 - \dfrac{0.0477\ldots}{-2.123\ldots}\right]$	M1
	$= 0.622$ (3 dp) (0.6224795…)	A1 (5)
(c)	$f(0.6215) = 1.77\ldots\times10^{-3} > 0$, $f(0.6225) = -3.807\times\ldots\times10^{-4} < 0$	M1
	Change of sign in $f(x)$ in (0.6215, 0.6225) "so 0.622 correct"	A1 (2)
		[8]
Notes	(b) 2ndM: If the N-R statement applied to 0.6 **not** seen, can be implied if answer correct; otherwise M0 If no values for $f(0.6)$, $f'(0.6)$ seen, they can be implied if final answer correct. (c) M: For candidates x_1, calculate $f(x_1 - 0.0005)$ and $f(x_1 + 0.0005)$ (or a tighter interval) A: Requires correct values of $f(0.6215)$ and $f(0.6225)$ (or their acceptable values) [may be rounded, e.g. 2×10^{-3}, or truncated, e.g -3.80×10^{-4}], sign change stated or >0, <0 seen, and conclusion.	

Question Number	Scheme	Marks
5 (a)	$z_2 = \frac{12-5\text{i}}{3+2\text{i}} \times \frac{3-2\text{i}}{3-2\text{i}} = \frac{36-24\text{i}-15\text{i}-10}{13}$	M1
	$= 2 - 3\text{i}$	A1 (2)
(b)	 P: B1, Q: B1ft from (a)	B1, B1ft (2)
(c)	grad. OP × grad. $OQ = (\frac{2}{3} \times -\frac{3}{2})$	M1
	$= -1 \quad \Rightarrow \angle POQ = \frac{\pi}{2}$ (✱)	A1 (2)
Alt (c)	(i) $\angle \text{POX} = \tan^{-1}\frac{2}{3}$, $\angle \text{QOX} = \tan^{-1}\frac{3}{2}$	
	$\text{Tan}(\angle \text{POQ}) = \frac{\frac{2}{3}+\frac{3}{2}}{1-\frac{2}{3}\times\frac{3}{2}}$ M1	
	$\Rightarrow \angle POQ = \frac{\pi}{2}$ (✱) A1	
(d)	$z = \frac{3+2}{2} + \frac{2+(-3)}{2}\text{i}$	M1
	$= \frac{5}{2} - \frac{1}{2}\text{i}$	A1 (2)
(e)	$r = \sqrt{\left(\frac{5}{2}\right)^2 + \left(-\frac{1}{2}\right)^2}$	M1
	$= \frac{\sqrt{26}}{2}$ or exact equivalent	A1 (2) **[10]**
Notes	(a) M: Multiplying num. and den. by 3–2i and attempt to simplify num. and denominator . If (c + id)(3 + 2i) = 12 –5i used, need to find 2 equations in c and d and then solve for c and d. (b) Coords seen or clear from labelled axes. S.C: If only P and Q seen(no coords) or correct coords given but P and Q interchanged allow B1B0 (c) If separate arguments are found and then added, allow M1 but **not** A1 for decimals used e.g. 1.570796327.. = ½π. Alts: Appropriate transformation matrix applied to one point M1; A1 Scalar product used correctly M1; 0 and conclusion A1 Pythagoras' theorem, congruent triangles are other methods seen. (d) M: Any complete method for finding centre. A: Must be complex number; coordinates not sufficient. (e) M: Correct method for radius, or diameter, for candidate's answer to (d)	

Question Number	Scheme	Marks
6 (a)	$r=\sqrt{x^2+y^2}$, $y= r\sin\theta$	
	$\therefore \sqrt{x^2+y^2} = \dfrac{6y}{\sqrt{x^2+y^2}}$ or $x^2+y^2=6y$ o.e.	M1, A1 (2)
(b)	$r= 9\sqrt{6}(1-2\sin^2\theta)$ o.e.	B1 (1)
(c)	$y= r\sin\theta = 9\sqrt{6}(\sin\theta - 2\sin^3\theta) \Rightarrow \dfrac{dy}{d\theta}=$; $9\sqrt{6}\cos\theta(1-6\sin^2\theta)$ o.e.	M1;A1
	Or $y=9\sqrt{6}\sin\theta\cos 2\theta \Rightarrow \dfrac{dy}{d\theta}=9\sqrt{6}(\cos 2\theta\cos\theta - 2\sin\theta\sin 2\theta)$ o.e.	
	$\dfrac{dy}{d\theta}=0$ [$\Rightarrow \cos\theta(1-6\sin^2\theta)=0$] **and** attempt to solve	M1
	($0\le\theta\le\dfrac{\pi}{4}$) $\therefore \sin\theta=\dfrac{1}{\sqrt{6}}$ (✱)	A1 (4)
(d)	$r=9\sqrt{6}\left(1-2\times\dfrac{1}{6}\right)$	M1
	$=6\sqrt{6}$ or 14.7 (awrt)	A1 (2)
(e)	C_2: tan. // to initial line is $y= r\sin\theta = 6\sqrt{6}\times\dfrac{1}{\sqrt{6}}=6$	B1
	C_1: Circle, centre (0, 3) (cartesian) or (3, $\frac{\pi}{2}$) (polar), passing through (0,0).	M1
	$\therefore$ tangent // to initial line has eqn $y=6 \Rightarrow y=6$ is a common tangent	A1 (3) **[12]**
Notes	(a) M1: Use of $r=\sqrt{x^2+y^2}$ or $r^2=x^2+y^2$, and $y= r\sin\theta$ (allow $x= r\sin\theta$) to form cartesian equation. (b) May be scored in (c) (c) 1st M: Finds y and attempts to find $\dfrac{dy}{d\theta}$ Working with $r\cos\theta$ instead of $r\sin\theta$, can score the M marks. If $\dfrac{dy}{dx}=\dfrac{dy}{d\theta}/\dfrac{dx}{d\theta}$ used throughout, $\dfrac{dy}{dx}=0$ etc. all marks may be gained (d) M: Using $\sin\theta=\dfrac{1}{\sqrt{6}}$ to find r (e) Alt. for C_1: M:Find $y=6\sin^2\theta$, ($\dfrac{dy}{d\theta}=12\sin\theta\cos\theta$) and solve $\dfrac{dy}{d\theta}=0$ A: Find $\theta=\dfrac{\pi}{2}$ and conclude that $y=6$, so common tangent	

Question Number	Scheme	Marks
7 (a)	$\frac{dy}{dx} = \lambda xe^x + \lambda e^x$ Use of the product rule	M1
	$\frac{d^2y}{dx^2} = \lambda xe^x + \lambda e^x + \lambda e^x$	A1
	$\lambda xe^x + 2\lambda e^x + 4\lambda xe^x + 4\lambda e^x - 5\lambda xe^x = 4e^x$	*M1
	$\lambda = \frac{2}{3}$	A1 (4)
	$(\therefore$ P.I. is $\frac{2}{3}xe^x)$	
(b)	Aux. eqn. $m^2 + 4m - 5 = 0$	
	$(m-1)(m+5) = 0$	
	$m = 1$ or $m = -5$	M1
	C.F. is $y = Ae^x + Be^{-5x}$	M1 A1
	Gen. soln. is $(y =)\ \frac{2}{3}xe^x + Ae^x + Be^{-5x}$ [f.t: Candidate's C.F + P.I.]	A1ft (4)
(c)	$-\frac{2}{3} = A + B$	M1
	$\frac{dy}{dx} = \frac{2}{3}xe^x + \frac{2}{3}e^x + Ae^x - 5Be^{-5x}$	M1
	$-\frac{4}{3} = \frac{2}{3} + A - 5B$ A1 two correct unsimplified eqns.	A1
	$-2 = A - 5B$	
	$\frac{4}{3} = 6B$	
	$B = \frac{2}{9},\ A = -\frac{8}{9}$	M1
	$y = \frac{2}{3}xe^x - \frac{8}{9}e^x + \frac{2}{9}e^{-5x}$	A1 (5)
		[13]
Notes	(a) 2nd M dependent on first M. (b) 1st M: Attempt to solve A.E. 2nd M: Only allow C.F. of form $Ae^{ax} + Be^{bx}$, where a and b are real. If seen in (a), award marks there. PI must be of form λxe^x ($\lambda \neq 0$) to gain final A1 f.t. (c) 1st M: Using $x = 0, y = -⅔$ in their **general solution**. 2nd M: Differentiating their **general solution** {C.F. + P.I. } (must have term in λxe^x) (condone slips) and using $x = 0, \frac{dy}{dx} = -\frac{4}{3}$ to find an equation in A and B. 3rd M: Solving simultaneous equations to find a value of A and a value of B. Can be awarded if only C.F. found. Insist on $y =$ .... in this part.	

Question Number	Scheme	Marks
8 (a)	$\frac{dy}{dx} = -\frac{1}{t^2} \times \frac{dt}{dx}$ o.e.	M1, A1
	$\sin x \times -\frac{1}{t^2} \times \frac{dt}{dx} + \frac{1}{t}\cos x = \frac{1}{t^2}$	M1
	$\frac{dt}{dx} - t\cot x = -\operatorname{cosec} x$ (✱)	A1 cso(4)
(b)	$I = e^{\int -\cot x\, dx}$	M1
	$= e^{-\ln \sin x}$	
	$= \frac{1}{\sin x}$ or cosec$x$	A1
	$\frac{1}{\sin x}\frac{dt}{dx} - t\frac{\cos x}{\sin^2 x} = -\operatorname{cosec}^2 x$	M1
	$\frac{t}{\sin x} = \int -\operatorname{cosec}^2 x\, dx$ or $\frac{d}{dx}\left(\frac{t}{\sin x}\right) = -\operatorname{cosec}^2 x$	A1f.t.
	$\frac{t}{\sin x} = \cot x \ (+c)$ o.e.	A1 cso (5)
(c)	$t = \cos x + c\sin x \Rightarrow y = \frac{1}{\cos x + c\sin x}$ (✱)	M1, A1 (2)
(d)	$\frac{\sqrt{2}}{3} = \frac{1}{\frac{1}{\sqrt{2}} + \frac{c}{\sqrt{2}}}$	M1
	$\sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{c}{\sqrt{2}}\right) = 3$	
	$c = 2$	A1
	$x = \frac{\pi}{2}, \quad y = \frac{1}{2}$ ft on their c	A1ft (3)
		[14]
Notes	(a) 1[st] M: Use of $\frac{dy}{dt}.\frac{dt}{dx}$ (even if integrated 1/t) 2[nd] M: Substituting for $\frac{dy}{dx}, y, y^2$ to form d.e. in x and t only (b) 1[st] M: For $e^{\int -\cot x\, dx}$ (allow $e^{\int \cot x\, dx}$) and attempt at integrating 2[nd*] M: Multiplying by integrating factor (requires at least two terms "correct" for their IF.) (can be implied) 3rdA1f.t: is only for those who have I.F. = sin$x$ or $-\sin x$ $\frac{d}{dx}(t\sin x) = -1$ equivalent integral (c) M: Substituting to find t = 1/y in their solution to (b) (d) M: Using y = $\frac{\sqrt{2}}{3}, x = \frac{\pi}{4}$ to find a value for c.	