Mark Scheme (Results)
Summer 2008

GCE Mathematics (6664/01)

GCE

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June 2008
Core Mathematics C2
Mark Scheme

Question number	Scheme	Marks
1.	(a) Attempt to find f(–4) or f(4). $\left(f(-4) = 2(-4)^3 - 3(-4)^2 - 39(-4) + 20\right)$ $\left(= -128 - 48 + 156 + 20\right) = 0,$ so $(x + 4)$ is a factor.	M1 A1 (2)
	(b) $2x^3 - 3x^2 - 39x + 20 = (x+4)(2x^2 - 11x + 5)$	M1 A1
	$.....(2x-1)(x-5)$ (The 3 brackets need not be written together) or $.....\left(x - \frac{1}{2}\right)(2x - 10)$ or equivalent	M1 A1cso (4)
		6
	(a) Long division scores no marks in part (a). The factor theorem is required. However, the first two marks in (b) can be earned from division seen in (a)… … but if a different long division result is seen in (b), the work seen in (b) takes precedence for marks in (b). A1 requires zero and a simple conclusion (even just a tick, or Q.E.D.), or may be scored by a preamble, e.g. 'If f(–4) = 0, $(x + 4)$ is a factor…..' (b) First M requires use of $(x + 4)$ to obtain $(2x^2 + ax + b)$, $a \neq 0, b \neq 0$, even with a remainder. Working need not be seen… this could be done 'by inspection'. Second M for the attempt to factorise their three-term quadratic. Usual rule: $(kx^2 + ax + b) = (px + c)(qx + d)$, where $\lvert cd \rvert = \lvert b \rvert$ and $\lvert pq \rvert = \lvert k \rvert$. If 'solutions' appear before or after factorisation, ignore… … but factors must be seen to score the second M mark. Alternative (first 2 marks): $(x+4)(2x^2 + ax + b) = 2x^3 + (8+a)x^2 + (4a+b)x + 4b = 0$, then compare coefficients to find values of a and b. [M1] $a = -11,\ b = 5$ [A1] Alternative: Factor theorem: Finding that $f\left(\frac{1}{2}\right) = 0$ $\therefore$ factor is, $(2x - 1)$ [M1, A1] Finding that $f(5) = 0$ $\therefore$ factor is, $(x - 5)$ [M1, A1] "Combining" all 3 factors is not required. If just one of these is found, score the first 2 marks M1 A1 M0 A0. Losing a factor of 2: $(x+4)\left(x - \frac{1}{2}\right)(x-5)$ scores M1 A1 M1 A0. Answer only, one sign wrong: e.g. $(x+4)(2x-1)(x+5)$ scores M1 A1 M1 A0	

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2.	(a) 1.732, 2.058, 5.196 awrt (One or two correct B1 B0, All correct B1 B1)	B1 B1 (2)
	(b) $\frac{1}{2} \times 0.5$	B1
	......$\{(1.732 + 5.196) + 2(2.058 + 2.646 + 3.630)\}$	M1 A1ft
	$= 5.899$ (awrt 5.9, allowed even after minor slips in values)	A1 (4)
		6
	(a) Accept awrt (but <u>less</u> accuracy loses these marks). Also accept <u>exact</u> answers, e.g. $\sqrt{3}$ at $x = 0$, $\sqrt{27}$ or $3\sqrt{3}$ at $x = 2$. (b) For the M mark, the first bracket must contain the 'first and last' values, and the second bracket must have no additional values. If the only mistake is to <u>omit</u> one of the values from the second bracket, this can be considered as a slip and the M mark can be allowed. Bracketing mistake: i.e. $\frac{1}{2} \times 0.5(1.732 + 5.196) + 2(2.058 + 2.646 + 3.630)$ scores B1 M1 A0 A0 <u>unless</u> the final answer implies that the calculation has been done correctly (then full marks can be given). <u>x values</u>: M0 if the values used in the brackets are x values instead of y values. <u>Alternative</u>: Separate trapezia may be used, and this can be marked equivalently. $\left[\frac{1}{4}(1.732 + 2.058) + \frac{1}{4}(2.058 + 2.646) + \frac{1}{4}(2.646 + 3.630) + \frac{1}{4}(3.630 + 5.196)\right]$	

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3.	(a) $(1+ax)^{10} = 1 + 10ax......$ (<u>Not</u> unsimplified versions)	B1
	$+\frac{10\times 9}{2}(ax)^2 + \frac{10\times 9\times 8}{6}(ax)^3$ Evidence from <u>one</u> of these terms is sufficient	M1
	$+45(ax)^2, +120(ax)^3$ or $+45a^2x^2, +120a^3x^3$	A1, A1 (4)
	(b) $120a^3 = 2\times 45a^2$ $a = \frac{3}{4}$ or equiv. $\left(\text{e.g.} \frac{90}{120}, 0.75\right)$ Ignore $a = 0$, if seen	M1 A1 (2)
		6
	(a) The terms can be 'listed' rather than added. M1: Requires correct structure: 'binomial coefficient' (perhaps from Pascal's triangle) and the correct power of x. (The M mark can also be given for an expansion in <u>descending</u> powers of x). Allow 'slips' such as: $\frac{10\times 9}{2}ax^2$, $\frac{10\times 9}{3\times 2}(ax)^3$, $\frac{10\times 9}{2}x^2$, $\frac{9\times 8\times 7}{3\times 2}a^3x^3$ However, $45 + a^2x^2 + 120 + a^3x^3$ or similar is M0. $\binom{10}{2}$ and $\binom{10}{3}$ or equivalent such as ${}^{10}C_2$ and ${}^{10}C_3$ are acceptable, and even $\left(\frac{10}{2}\right)$ and $\left(\frac{10}{3}\right)$ are acceptable for the method mark. 1st A1: Correct x^2 term. 2nd A1: Correct x^3 term (These <u>must</u> be simplified). If simplification is not seen in (a), but correct simplified terms are seen in (b), these marks can be awarded. However, if <u>wrong</u> simplification is seen in (a), this takes precedence. <u>Special case</u>: If $(ax)^2$ and $(ax)^3$ are seen within the working, but then lost… … A1 A0 can be given if $45ax^2$ and $120ax^3$ are <u>both</u> achieved. (b) M: Equating their coefficent of x^3 to twice their coefficient of x^2 … … <u>or</u> equating their coefficent of x^2 to twice their coefficient of x^3. (… or coefficients can be <u>correct</u> coefficients rather than their coefficients). Allow this mark even if the equation is trivial, e.g. $120a = 90a$. An equation in a alone is required for this M mark, although… …condone, e.g. $120a^3x^3 = 90a^2x^2 \Rightarrow (120a^3 = 90a^2 \Rightarrow)\ a = \frac{3}{4}$. <u>Beware</u>: $a = \frac{3}{4}$ following $120a = 90a$, which is A0.	

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4.	(a) $x = \frac{\log 7}{\log 5}$ or $x = \log_5 7$ (i.e. correct method up to $x = \ldots$)	M1	
	1.21 Must be this answer (3 s.f.)	A1	(2)
	(b) $(5^x - 7)(5^x - 5)$ Or another variable, e.g. $(y-7)(y-5)$, even $(x-7)(x-5)$	M1 A1	
	$(5^x = 7$ or $5^x = 5)$ $x = 1.2$ (awrt) ft from the answer to (a), if used	A1ft	
	$x = 1$ (allow 1.0 or 1.00 or 1.000)	B1	(4)
			6
	(a) 1.21 with no working: M1 A1 (even if it left as $5^{1.21}$). Other answers which round to 1.2 with no working: M1 A0. (b) M: Using the <u>correct</u> quadratic equation, attempt to factorise $(5^x \pm 7)(5^x \pm 5)$, or attempt quadratic formula. Allow $\log_5 7$ or $\frac{\log 7}{\log 5}$ instead of 1.2 for A1ft. No marks for simply substituting a decimal answer from (a) into the given equation (perhaps showing that it gives approximately zero). <u>However</u>, note the following <u>special case</u>: Showing that $5^x = 7$ satisfies the given equation, therefore 1.21 is a solution scores 0, 0, 1, 0 (and could score <u>full marks</u> if the $x = 1$ were also found). e.g. If $5^x = 7$, then $5^{2x} = 49$, and $5^{2x} - 12(5^x) + 35 = 49 - 84 + 35 = 0$, so one solution is $x = 1.21$ ('conclusion' must be seen). To score this special case mark, values substituted into the equation must be <u>exact</u>. Also, the mark would <u>not</u> be scored in the following case: e.g. If $5^x = 7$, $5^{2x} - 84 + 35 = 0 \Rightarrow 5^{2x} = 49 \Rightarrow x = 1.21$ (Showing no appreciation that $5^{2x} = (5^x)^2$) B1: Do not award this mark if $x = 1$ clearly follows from <u>wrong</u> working.		

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5.	(a) $(8-3)^2+(3-1)^2$ or $\sqrt{(8-3)^2+(3-1)^2}$	M1 A1
	$(x\pm 3)^2+(y\pm 1)^2=k$ or $(x\pm 1)^2+(y\pm 3)^2=k$ (k a positive value)	M1
	$(x-3)^2+(y-1)^2=29$ (Not $\left(\sqrt{29}\right)^2$ or 5.39^2)	A1 (4)
	(b) Gradient of radius $=\frac{2}{5}$ (or exact equiv.) Must be seen or used in (b)	B1
	Gradient of tangent $=\frac{-5}{2}$ (Using perpendicular gradient method)	M1
	$y-3=\frac{-5}{2}(x-8)$ (ft gradient of radius, dependent upon both M marks)	M1 A1ft
	$5x+2y-46=0$ (Or equiv., equated to zero, e.g. $92-4y-10x=0$) (Must have integer coefficients)	A1 (5) **9**
	(a) For the M mark, condone one slip inside a bracket, e.g. $(8-3)^2+(3+1)^2$, $(8-1)^2+(1-3)^2$ The first two marks may be gained implicitly from the circle equation. (b) 2nd M: Eqn. of line through (8, 3), in any form, with any grad.(except 0 or ∞). If the 8 and 3 are the 'wrong way round', this M mark is only given if a correct general formula, e.g. $y-y_1=m(x-x_1)$, is quoted. Alternative: 2nd M: Using (8, 3) and an m value in $y=mx+c$ to find a value of c. A1ft: as in main scheme. (Correct substitution of 8 and 3, then a wrong c value will still score the A1ft) (b) Alternatives for the first 2 marks: (but in these 2 cases the 1st A mark is not ft) (i) Finding gradient of tangent by implicit differentiation $2(x-3)+2(y-1)\frac{dy}{dx}=0$ (or equivalent) B1 Subs. $x=8$ and $y=3$ into a 'derived' expression to find a value for dy/dx M1 (ii) Finding gradient of tangent by differentiation of $y=1+\sqrt{20+6x-x^2}$ $\frac{dy}{dx}=\frac{1}{2}\left(20+6x-x^2\right)^{-\frac{1}{2}}(6-2x)$ (or equivalent) B1 Subs. $x=8$ into a 'derived' expression to find a value for dy/dx M1 Another alternative: Using $xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$ $x^2+y^2-6x-2y-19=0$ B1 $8x+3y,\quad -3(x+8)-(y+3)-19=0$ M1, M1 A1ft (ft from circle eqn.) $5x+2y-46=0$ A1	

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6.	(a) $T_{20} = 5\times\left(\frac{4}{5}\right)^{19} = 0.072$ (Accept awrt) Allow $5\times\frac{4}{5}^{19}$ for M1	M1 A1	(2)
	(b) $S_\infty = \frac{5}{1-0.8} = 25$	M1 A1	(2)
	(c) $\frac{5(1-0.8^k)}{1-0.8} > 24.95$ (Allow with = or <)	M1	
	$1-0.8^k > 0.998$ (or equiv., see below) (Allow with = or <)	A1	
	$k\log 0.8 < \log 0.002$ or $k > \log_{0.8} 0.002$ (Allow with = or <)	M1	
	$k > \frac{\log 0.002}{\log 0.8}$ (*)	A1cso	(4)
	(d) $k = 28$ (Must be this integer value) <u>Not</u> $k > 27$, or $k < 28$, or $k > 28$	B1	(1)
			9
	(a) and (b): Correct answer without working scores both marks. (a) M: Requires use of the correct formula ar^{n-1}. (b) M: Requires use of the correct formula $\frac{a}{1-r}$ (c) 1st M: The sum may have already been 'manipulated' (perhaps wrongly), but this mark can still be allowed. 1st A: A 'numerically correct' version that has dealt with $(1-0.8)$ denominator, e.g. $1-\left(\frac{4}{5}\right)^k > 0.998$, $5(1-0.8^k) > 4.99$, $25(1-0.8^k) > 24.95$, $5-5(0.8^k) > 4.99$. In any of these, $\frac{4}{5}$ instead of 0.8 is fine, and condone $\frac{4}{5}^k$ if correctly treated later. 2nd M: Introducing logs and using laws of logs correctly (this must include dealing with the power k so that $p^k = k\log p$). 2nd A: An <u>incorrect</u> statement (including equalities) at any stage in the working loses this mark (this is often identifiable at the stage $k\log 0.8 > \log 0.002$). (So a fully correct method with inequalities is required.)		

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7.	(a) $r\theta = 7 \times 0.8 = 5.6$ (cm)	M1 A1	(2)
	(b) $\frac{1}{2}r^2\theta = \frac{1}{2} \times 7^2 \times 0.8 = 19.6$ (cm^2)	M1 A1	(2)
	(c) $BD^2 = 7^2 + (\text{their } AD)^2 - (2 \times 7 \times (\text{their } AD) \times \cos 0.8)$	M1	
	$BD^2 = 7^2 + 3.5^2 - (2 \times 7 \times 3.5 \times \cos 0.8)$ (or awrt 46° for the angle)	A1	
	$(BD = 5.21)$		
	Perimeter = (their *DC*) + "5.6" + "5.21" = 14.3 (cm) (Accept awrt)	M1 A1	(4)
	(d) $\Delta ABD = \frac{1}{2} \times 7 \times (\text{their } AD) \times \sin 0.8$ (or awrt 46° for the angle) (ft their *AD*)	M1 A1ft	
	$(= 8.78\ldots)$		
	(If the correct formula $\frac{1}{2}ab\sin C$ is <u>quoted</u> the use of any two of the sides of ΔABD as *a* and *b* scores the M mark).		
	Area = "19.6" – "8.78…" = 10.8 (cm^2) (Accept awrt)	M1 A1	(4)
			12
	Units (cm or cm^2) are not required in any of the answers. (a) and (b): Correct answers without working score both marks. (a) M: Use of $r\theta$ (with θ in radians), or equivalent (could be working in degrees with a correct degrees formula). (b) M: Use of $\frac{1}{2}r^2\theta$ (with θ in radians), or equivalent (could be working in degrees with a correct degrees formula). (c) 1st M: Use of the (correct) cosine rule formula to find BD^2 or BD. Any other methods need to be complete methods to find BD^2 or BD. 2nd M: Adding their *DC* to their arc *BC* and their *BD*. <u>Beware</u>: If 0.8 is used, but calculator is in degree mode, this can still earn M1 A1 (for the required expression), but this gives *BD* = 3.50… so the perimeter may appear as 3.5 + 5.6 + 3.5 (earning M1 A0). (d) 1st M: Use of the (correct) area formula to find ΔABD. Any other methods need to be complete methods to find ΔABD. 2nd M: Subtracting their ΔABD from their sector *ABC*. Using segment formula $\frac{1}{2}r^2(\theta - \sin\theta)$ scores no marks in part (d).		

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8.	(a) $\left(\frac{dy}{dx}=\right) 8+2x-3x^2$ (M: $x^n \to x^{n-1}$ for one of the terms, <u>not</u> just $10 \to 0$)	M1 A1
	$3x^2-2x-8=0$ $(3x+4)(x-2)=0$ $x=2$ (Ignore other solution) (*)	A1cso (3)
	(b) Area of triangle = $\frac{1}{2}\times 2\times 22$ (M: Correct method to find area of triangle)	M1 A1
	(Area = 22 with no working is acceptable)	
	$\int 10+8x+x^2-x^3\,dx = 10x+\frac{8x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}$ (M: $x^n \to x^{n+1}$ for one of the terms)	M1 A1 A1
	Only one term correct: M1 A0 A0 2 or 3 terms correct: M1 A1 A0 Integrating the <u>gradient function</u> loses this M mark.	
	$\left[10x+\frac{8x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}\right]_0^2 = \ldots\ldots$ (Substitute limit 2 into a 'changed function')	M1
	$\left(=20+16+\frac{8}{3}-4\right)$ (This M can be awarded even if the other limit is wrong)	
	Area of $R = 34\frac{2}{3}-22=\frac{38}{3}$ $\left(=12\frac{2}{3}\right)$ (Or $12.\dot{6}$)	M1 A1 (8)
	M: <u>Dependent on use of calculus in (b) and correct overall 'strategy'</u>: subtract either way round. A: Must be <u>exact</u>, not 12.67 or similar. A negative area at the end, even if subsequently made positive, loses the A mark.	
		11
	(a) The final mark may also be scored by <u>verifying</u> that $\frac{dy}{dx}=0$ at $x=2$.	
	(b) <u>Alternative</u>:	
	Eqn. of line $y=11x$. (Marks dependent on subsequent use in integration) (M1: Correct method to find equation of line. A1: Simplified form $y=11x$)	M1 A1
	$\int 10+kx+x^2-x^3\,dx = 10x+\frac{kx^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}$ (k perhaps -3)	M1 A1 A1
	$\left[10x+\frac{kx^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}\right]_0^2 = \ldots\ldots$ (Substitute limit 2 into a 'changed function')	M1
	Area of $R = \left[10x-\frac{3x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}\right]_0^2 = 20-6+\frac{8}{3}-4=\frac{38}{3}$ $\left(=12\frac{2}{3}\right)$	M1 A1 (8)
	Final M1 for $\int(\text{curve})-\int(\text{line})$ or $\int(\text{line})-\int(\text{curve})$.	

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9.	(a) 45 (α) (This mark can be implied by an answer 65)	B1
	$180-\alpha$, Add 20 (for at least one angle)	M1, M1
	65 155	A1 (4)
	(b) 120 or 240 (β): (This mark can be implied by an answer 40 or 80) (Could be achieved by working with 60, using $180-60$ and/or $180+60$)	B1
	$360-\beta$, $360+\beta$ (or 120 + an angle that has been divided by 3) Dividing by 3 (for at least one angle)	M1, M1 M1
	40 80 160 200 280 320 First A1: at least 3 correct	A1 A1 (6) **10**
	(a) Extra solution(s) in range: Loses the A mark. Extra solutions outside range: Ignore (whether correct or not). Common solutions: 65 (only correct solution) will score B1 M0 M1 A0 (2 marks) 65 and 115 will score B1 M0 M1 A0 (2 marks) 44.99 (or similar) for α is B0, and 64.99, 155.01 (or similar) is A0. (b) Extra solution(s) in range: Loses the final A mark. Extra solutions outside range: Ignore (whether correct or not). Common solutions: 40 (only correct solution) will score B1 M0 M0 M1 A0 A0 (2 marks) 40 and 80 (only correct solutions) B1 M1 M0 M1 A0 A0 (3 marks) 40 and 320 (only correct solutions) B1 M0 M0 M1 A0 A0 (2 marks) <u>Answers without working</u>: Full marks can be given (in both parts), B and M marks by implication. <u>Answers given in radians</u>: Deduct a maximum of 2 marks (misread) from B and A marks. (Deduct these at first and second occurrence.) <u>Answers that begin</u> with statements such as $\sin(x-20)=\sin x-\sin 20$ or $\cos x=-\frac{1}{6}$, then go on to find a value of 'α' or 'β', however badly, <u>can</u> continue to earn the first M mark in either part, but will score <u>no further marks</u>. <u>Possible misread</u>: $\cos 3x=\frac{1}{2}$, giving 20, 100, 140, 220, 260, 340 Could score up to 4 marks B0 M1 M1 M1 A0 A1 for the above answers.	