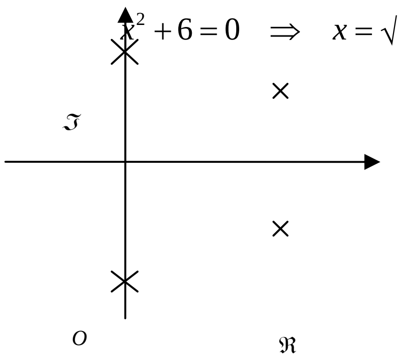
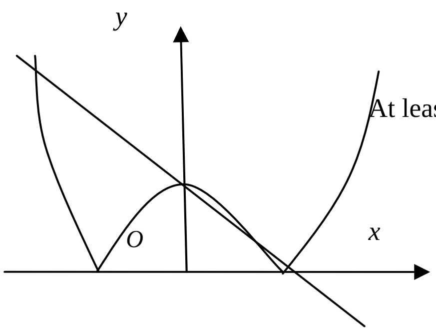
June 2006
6674 Further Pure Mathematics FP1
Mark Scheme

Question Number	Scheme	Marks
1.	(a) $2z + \mathrm{i}w = -1$	
	$\mathrm{i}z - \mathrm{i}w = 3\mathrm{i} - 3$	
	Adding $2z + \mathrm{i}z = -4 + 3\mathrm{i}$ Eliminating either variable	M1
	$z = \dfrac{-4+3\mathrm{i}}{2+\mathrm{i}}$	A1
	$z = \dfrac{-4+3\mathrm{i}}{2+\mathrm{i}} \times \dfrac{2-\mathrm{i}}{2-\mathrm{i}}$	M1
	$= \dfrac{-8+3+4\mathrm{i}+6\mathrm{i}}{5}$	
	$= -1 + 2\mathrm{i}$	A1 (4)
	(b) $\arg z = \pi - \underline{\arctan 2}$	<u>M1</u>
	≈ 2.03 cao	A1 (2)
		[6]
2.	Use of $\frac{1}{2}\int r^2\,\mathrm{d}\theta$	B1
	Limits are $\dfrac{\pi}{8}$ and $\dfrac{\pi}{4}$	B1
	$16a^2\cos^2 2\theta = 8a^2(1+\cos 4\theta)$	M1
	$\int(1+\cos 4\theta)\,\mathrm{d}\theta = \theta + \dfrac{\sin 4\theta}{4}$	*M1 A1*
	$A = 4a^2\left[\theta + \dfrac{\sin 4\theta}{4}\right]_{\pi/8}^{\pi/4}$	
	$= a^2\left[4\left(\dfrac{\pi}{4} - \dfrac{\pi}{8}\right) + (0-1)\right]$	*M1*
		A1
	$= a^2\left(\dfrac{\pi}{2} - 1\right) = \dfrac{1}{2}a^2(\pi - 2)$ * cso	*(7)*
		[7]

Question Number	Scheme	Marks
3.	(a) $y' = 3\sin 2x + 6x\cos 2x$	M1
	$y'' = 12\cos 2x - 12x\sin 2x$	A1
	Substituting $12\cos 2x - 12x\sin 2x + 12x\sin 2x = k\cos 2x$	M1
	$k = 12$	A1 (4)
	(b) General solution is $y = A\cos 2x + B\sin 2x + 3x\sin 2x$	B1
	$(0, 2) \Rightarrow A = 2$	B1
	$\left(\frac{\pi}{4}, \frac{\pi}{2}\right) \Rightarrow \frac{\pi}{2} = B + \frac{3\pi}{4} \Rightarrow B = -\frac{\pi}{4}$	M1
	$y = 2\cos 2x - \frac{\pi}{4}\sin 2x + 3x\sin 2x$ Needs $y = \ldots$	A1 (4)
		[8]
4.	(a) 3 + 2i is a solution	B1
	$(x-3-2\mathrm{i})(x-3+2\mathrm{i}) = x^2 - 6x + 13$	M1
	$\mathrm{f}(x) = (x^2 - 6x + 13)(x^2 + ax + b)$	
	$b = 6$	B1
	Coefficients of x^3 $a - 6 = -6$ or equivalent	M1
	$a = 0$	*A1*
	$x^2 + 6 = 0 \Rightarrow x = \sqrt{6}\mathrm{i}, -\sqrt{6}\mathrm{i}$	*M1A1*
		(7)
	(b) 	
	Conjugate complex pair on imaginary axis	*B1*
	Conjugate complex pair in correct quadrants	*B1*
		(2)
		[9]

Question Number	Scheme	Marks
5.	(a) $(2r+1)^3 = 8r^3 + 12r^2 + 6r + 1$	
	$(2r-1)^3 = 8r^3 - 12r^2 + 6r - 1$	
	$(2r+1)^3 - (2r-1)^3 = 24r^2 + 2 \qquad \left(A = 24,\ B = 2\right)$	M1 A1 (2)
	Accept $r = 0 \Rightarrow B = 2$ and $r = 1 \Rightarrow A + B = 26 \Rightarrow A = 24$	
	M1 for both	
	(b) $\cancel{3^3} - 1^3 = 24\times 1^2 + 2$	
	$5^3 - \cancel{3^3} = 24\times 2^2 + 2$	
	M	
	$\left(2n+1\right)^3 - \cancel{\left(2n-1\right)^3} = 24\times n^2 + 2$	
	$\left(2n+1\right)^3 - 1^3 = 24\sum_{r=1}^{n} r^2 + \underline{2n}$ <u>ft</u> their *B*	M1 A1 <u>A1ft</u>
	$\sum_{r=1}^{n} r^2 = \dfrac{8n^3 + 12n^2 + 4n}{24}$	M1
	$= \dfrac{1}{6}n\left(2n^2 + 3n + 1\right) = \dfrac{1}{6}n\left(n+1\right)\left(2n+1\right)$ ∗ cso	A1 (5)
	(c) $\sum_{r=1}^{40}(3r-1)^2 = \sum_{r=1}^{40}(9r^2 - 6r + 1)$	M1
	$= 9\times\dfrac{1}{6}\times 40\times 41\times 81 - 6\times\dfrac{1}{2}\times 40\times 41 + 40$	M1
	$= 194380$	A1 (3)
		[10]

Question Number	Scheme	Marks
6.	(a) $\text{f}(0.24) \approx -0.058, \ \text{f}(0.28) = 0.089$ accept 1sf	M1
	Change of sign (and continuity) $\Rightarrow \ \alpha \in (0.24, 0.28)$	A1 (2)
	(b) $\text{f}(0.26) \approx 0.017 \quad (\Rightarrow \quad \alpha \in (0.24, 0.26))$ accept 1sf	M1
	$\text{f}(0.25) \approx -0.020 \quad (\Rightarrow \quad \alpha \in (0.25, 0.26))$	
	$\text{f}(0.255) \approx -0.001 \quad \Rightarrow \quad \alpha \in (0.255, 0.26)$	M1 A1 (3)
	(c) $\text{f}(11) \approx 0.0534$ at least 3sf	B1
	$\text{f}'(x) = \frac{2\cos\sqrt{x}}{\sqrt{x}} + \frac{1}{4}$	M1 A1
	$\text{f}'(11) \approx -0.3438$ at least 2sf	A1
	$\beta \approx 11 + \frac{0.0534}{0.3438} \approx 11.16$ cao	M1 A1 (6)
		[11]
	If $\text{f}'(11) \approx -0.3438$ is produced without working, this is to be accepted for three marks M1 A1 A1.	

Question Number	Scheme	Marks
7.	(a) $2x^2+x-6=6-3x$	*M1*
	Leading to $x^2+2x-6=0$	
	$(x+1)^2=7 \Rightarrow x=-1\pm\sqrt{7}$ surds required	*M1 A1*
	$-2x^2-x+6=6-3x$	*M1*
	Leading to $2x^2-2x=0 \Rightarrow x=0, 1$	*A1, A1* *(6)*
	(b) Accept if parts (a) and (b) done in reverse order	
		
	Curved shape	B1
	Line	B1
	At least 3 intersections	B1 (3)
	(c) Using all 4 CVs and getting all into inequalities	*M1*
	$x>\sqrt{7}-1, \quad x<-\sqrt{7}-1$ both	*A1ft*
	ft their greatest positive and their least negative CVs	
	$0<x<1$	*A1* *(3)*
		[12]

Question Number	Scheme	Marks
8.	(a) $\int \frac{2}{120-t}\,\mathrm{d}t = -2\ln(120-t)$	B1
	$\mathrm{e}^{-2\ln(120-t)} = (120-t)^{-2}$	M1 A1
	$\frac{1}{(120-t)^2}\frac{\mathrm{d}S}{\mathrm{d}t} + \frac{2S}{(120-t)^3} = \frac{1}{4(120-t)^2}$	
	$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{S}{(120-t)^2}\right) = \frac{1}{4(120-t)^2}$ or integral equivalent	M1
	$\frac{S}{(120-t)^2} = \frac{1}{4(120-t)}(+C)$	M1 A1
	$(0, 6) \Rightarrow 6 = 30 + 120^2 C \Rightarrow C = -\frac{1}{600}$	M1
	$S = \frac{120-t}{4} - \frac{(120-t)^2}{600}$ accept C = awrt -0.0017	A1 (8)
	(b) $\frac{\mathrm{d}S}{\mathrm{d}t} = -\frac{1}{4} + \frac{2(120-t)}{600}$	M1
	$\frac{\mathrm{d}S}{\mathrm{d}t} = 0 \Rightarrow t = 45$	M1 A1
	Substituting $S = 9\frac{3}{8}$ (kg)	A1 (4)
		[12]

Question Number	Scheme	Marks
8.Contd.	Alternative forms for S are $S=6+\frac{3t}{20}-\frac{t^2}{600}=\frac{(t+30)(120-t)}{600}$ $=\frac{3600+90t-t^2}{600}=\frac{5625-(t-45)^2}{600}$	
	Alternative for part (b)	
	S can be found without finding t	
	Using $\frac{dS}{dt}=0$ in the original differential equation	
	$\frac{2S}{120-t}=\frac{1}{4}$	M1
	Substituting for t into the answer to part (a)	
	$S=2S-\frac{64S^2}{600}$	M1 A1
	Solving to $S=9\frac{3}{8}$ (kg)	A1 (4)