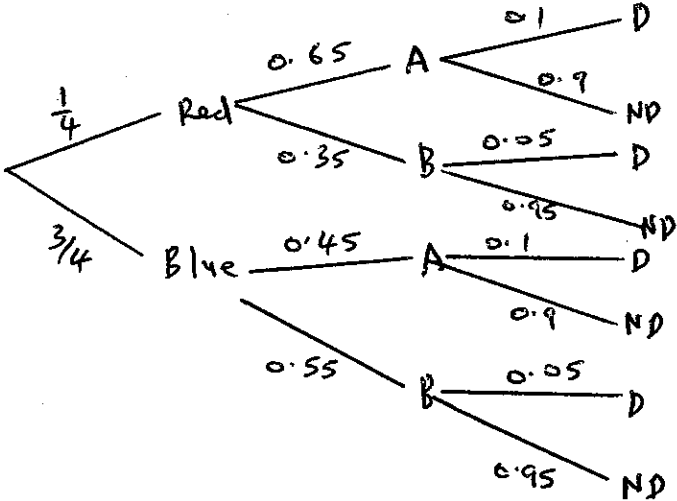


January 2006  
6687 Statistics S5  
Mark Scheme

Question Number	Scheme	Marks
1 (a)	Geometric	B1 (1)
(b)(i)	$p = \frac{1}{5}$ $\therefore P(J=4) = \left(\frac{4}{5}\right)^3 \left(\frac{1}{5}\right) = \frac{64}{625} = 0.1024$	B1 M1A1 (3)
(ii)	$P(J \geq 3) = (1-p)^2 = \left(\frac{4}{5}\right)^2 = \frac{16}{25} = 0.64$	M1A1A1 (3)
(c)	Assume <u>shots</u> are <u>independent</u> and <u>probability of a hit</u> is <u>constant</u> .	B1 B1 (2)
TOTAL 9		
2		
(a)	$P(D) = \left(\frac{1}{4} \times 0.65 \times 0.1\right) + \left(\frac{1}{4} \times 0.35 \times 0.05\right) + \left(\frac{3}{4} \times 0.45 \times 0.1\right) + \left(\frac{3}{4} \times 0.55 \times 0.05\right)$ $= 0.075$	4 cases 2 terms $\frac{1}{4}$ 2 terms $\frac{3}{4}$ A1 (4)
(b)	$P(A D) = \frac{\left(\frac{1}{4} \times 0.65 \times 0.1\right) + \left(\frac{3}{4} \times 0.45 \times 0.1\right)}{0.075}$ $= \frac{2}{3} \text{ or } 0.\dot{6}$	Bayes M1(M1A1) A1 (5)
TOTAL 9		

Question Number	Scheme	Marks
3 (a)  (b)         (c)	<p>Uniform / Rectangular</p> $M_x(t) = \int_0^4 0.25 e^{tx} dx$ $= \left[ \frac{0.25 e^{tx}}{t} \right]_0^4$ $= \frac{0.25 e^{4t}}{t} - \frac{0.25}{t}$ $\mu = \frac{1}{4t} (e^{4t} - 1) \quad * \text{ AG}$ $M_x(t) = \frac{4t + \frac{(4t)^2}{2!} + \frac{(4t)^3}{3!} + \dots}{4t}$ $= 1 + 2t + \frac{8t^2}{3} + \dots$ $\mu = M'(0) = 2; \quad \sigma^2 = M''(0) - (M'(0))^2 = \frac{4}{3}$	B1 (1) M1A1 A1 A1 CS= A1 (4) M1 A1 A1 A1 (4) TOTAL 9
4(a)(i)  (ii)  (b)  (c)	$\binom{6}{1} (0.15)(0.85)^5 (0.15) = 0.059900\dots$ $\frac{3}{0.15} = 20$ <p>Probability that John wins a coconut in a game is constant Games are independent</p> $\frac{r}{p} = 18; \quad \frac{r(1-p)}{p^2} = 36$ $\therefore 18(1-p) = 36p$ $p = \frac{1}{3} > 0.15 \Rightarrow \text{Sue}$	M1A1 (2) B1 (1) B1 B1 (2) B1 B1 M1 A1 A1 ✓ (5) TOTAL 10

<p>5(a)</p> <p>Let <math>X</math> represent the no. of power cuts per month.</p> <p><math>X \sim Po(0.25)</math></p> <p><math>P(X=1) = 0.25e^{-0.25} = 0.1947</math></p> <p>(b)</p> <p><math>P(0 \leq T \leq t) = \int_0^t 0.25e^{-0.25x} dx = \left[ -e^{-0.25x} \right]_0^t</math></p> <p><math>\ast = 1 - e^{-t/4} \ast Ag \text{ cso}</math></p> <p>[OR]</p> <p><math>P(0 \leq T \leq t) = P(\text{At least one cut in } (0, t))</math></p> <p><math>= 1 - P(\text{No cuts in } (0, t))</math></p> <p><math>= 1 - e^{-t/4}</math></p> <p>(c)(i)</p> <p><math>P(4 \text{ months or less}) = 1 - e^{-4/4} = 0.63212</math></p> <p>(ii)</p> <p><math>P(\text{Between 3 and 4 months}) = 0.63212 - (1 - e^{-3/4})</math></p> <p><math>= 0.10447\dots</math></p> <p>(d)</p> <p><math>P(\text{No cuts}) = 0.75</math></p> <p><math>e^{-t/4} = 0.75</math></p> <p><math>t = 1.150728\dots</math></p>	<p>M1</p> <p>A1 (2)</p> <p>M1 [A1]</p> <p>A1 (3)</p> <p>B1</p> <p>M1</p> <p>A1 (3)</p> <p>M1 A1</p> <p>M1</p> <p>A1 (4)</p> <p>M1</p> <p>M1</p> <p>A1 (3)</p>	<p>(2)</p> <p>[A1]</p> <p>(3)</p> <p>(3)</p> <p>(4)</p> <p>(3)</p> <p>TOTAL 12</p>
<p>6 (a)</p> <p><math>P(\text{Batch accepted after 1st sample}) = (1 - 0.05)^{10} = 0.598731\dots</math></p> <p>(b)</p> <p><math>P(9 \text{ pass}) + P(8 \text{ pass}) = 10(0.95)^9(0.05) + 45(0.95)^8(0.05)^2</math></p> <p><math>= 0.389759\dots</math></p> <p>(c)</p> <p><math>P(\text{Accepted after 2nd sample}) = 10(0.95)^9(0.05)(0.95)^{10} + [10(0.95)^9(0.05)]^2</math></p> <p><math>+ 45(0.95)^8(0.05)^2(0.95)^{10}</math></p> <p><math>= 0.1866768\dots + 0.09930357\dots + 0.044684\dots</math></p> <p><math>= 0.332666991\dots</math></p> <p><math>P(\text{Acceptance}) = 0.598731\dots + 0.332666991\dots</math></p> <p><math>= 0.93140393</math></p>	<p>B1</p> <p>(1)</p> <p>M1 A1 A1</p> <p>A1 (4)</p> <p>M1 A1 A1 A1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1 (7)</p>	<p>(1)</p> <p>(4)</p> <p>(7)</p> <p>TOTAL 12</p>

7(a)	$G_X(t) = 1$ $k(1+2t+2t^2)^2 = 1 \Rightarrow k = \frac{1}{25} \neq AG$	CSO M1 A1 (2)
(b)	$\frac{1}{25}(1+2t+2t^2)(1+2t+2t^2) = \frac{1}{25}(1+4t+8t^2+\dots)$ $P(X=2) = \frac{8}{25}$	M1 A1 (2)
(c)	$G'_X(t) = \frac{2}{25}(1+2t+2t^2)(2+4t)$ $G'_X(1) = \frac{12}{5} \Rightarrow E(X) = \frac{12}{5}$ $G''_X(t) = \frac{2}{25}(2+4t)(2+4t) + \frac{2}{25}(1+2t+2t^2) \cdot 4$ $G''_X(1) = \frac{112}{25}$ $Var(X) = \frac{112}{25} + \frac{12}{5} - \left(\frac{12}{5}\right)^2 = \frac{28}{25}$	M1A1 A1 M1A1 A1 M1A1 (8)
(d)	$\frac{t}{25}(1+2t^2+2t^4)^2$	B1B1 (2)
		TOTAL 14