Paper Reference(s)

6673/01

Edexcel GCE

Pure Mathematics P3

Advanced/Advanced Subsidiary

Tuesday 10 January 2006 – Afternoon
Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Lilac)	Nil

Candidates may only use one of the basic scientific calculators approved by the Qualifications and Curriculum Authority.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Pure Mathematics P3), the paper reference (6673), your surname, initials and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions on this paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. (*a*) Find the first four terms in the expansion, in ascending powers of *x*, of

$$(1-2x)^{-\frac{1}{2}}, \qquad |2x| < 1,$$

giving each term in its simplest form.

(4)

(*b*) Hence write down the first four terms in the expansion, in ascending powers of *x*, of

$$(100-200x)^{-\frac{1}{2}}, \qquad |2x| < 1.$$

(2)

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2. $f(x) = 2x^3 - x^2 + ax + b$, where a and b are constants.

It is given that $(x - 2)$ is a factor of $f(x)$.

When $f(x)$ is divided by $(x + 1)$, the remainder is 6.

Find the value of a and the value of b.

(7)

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3. A circle C has radius $\sqrt{5}$ and has its centre at the point with coordinates (4, 3).

(*a*) Prove that an equation of the circle C is $x^2 + y^2 - 8x - 6y + 20 = 0$.

(3)

The line l, with equation $y = 2x$, is a tangent to the circle C.

(*c*) Find the coordinates of the point where the line l touches C.

(4)

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4. $$f(x) = (x^2 + 1)\ln x, \qquad x > 0.$$

(*a*) Use differentiation to find the value of $f'(x)$ at $x = e$, leaving your answer in terms of e.

(4)

(*b*) Find the exact value of $\int_1^e f(x)\ dx$.

(5)

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5. $$\mathrm{f}(x) = \frac{9+4x^2}{9-4x^2}, \qquad x \neq \pm\tfrac{3}{2}.$$

(*a*) Find the values of the constants A, B and C such that

$$\mathrm{f}(x) = A + \frac{B}{3+2x} + \frac{C}{3-2x}, \qquad x \neq \pm\tfrac{3}{2}.$$

(4)

(*b*) Hence find the exact value of

$$\int_{-1}^{1} \frac{9+4x^2}{9-4x^2}\,\mathrm{d}x.$$

(5)

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6. The rate of decrease of the concentration of a drug in the bloodstream is proportional to the concentration C of the drug which is present at that time. The time t is measured in hours from the administration of the drug and C is measured in micrograms per litre.

(*a*) Show that the process is described by the differential equation $\frac{\mathrm{d}C}{\mathrm{d}t} = -kC$, explaining why k is a positive constant.

(1)

(*b*) Find the general solution of the differential equation, in the form $C = \mathrm{f}(t)$.

(3)

After 4 hours, the concentration of the drug in the bloodstream is reduced to 10% of its starting value C_0.

(*c*) Find the exact value of k.

(4)

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7. The points A and B have position vectors $\mathbf{i} - \mathbf{j} + p\mathbf{k}$ and $7\mathbf{i} + q\mathbf{j} + 6\mathbf{k}$ respectively, where p and q are constants.

The line l_1, passing through the points A and B, has equation

$$\mathbf{r} = 9\mathbf{i} + 7\mathbf{j} + 7\mathbf{k} + \lambda(2\mathbf{i} + 2\mathbf{j} + \mathbf{k}), \text{ where } \lambda \text{ is a parameter.}$$

(*a*) Find the value of p and the value of q.

(4)

(*b*) Find a unit vector in the direction of $\overrightarrow{AB}$.

(2)

A second line l_2 has vector equation

$$\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \mu(2\mathbf{i} + \mathbf{j} + 2\mathbf{k}), \text{ where } \mu \text{ is a parameter.}$$

(*c*) Find the cosine of the acute angle between l_1 and l_2.

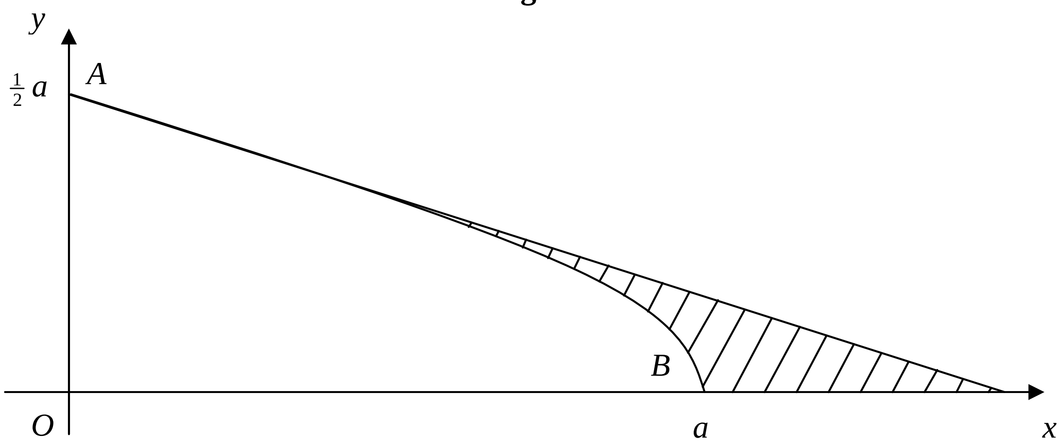
(3)

(*d*) Find the coordinates of the point where the two lines meet.

(5)

8.

Figure 1



The curve shown in Figure 1 has parametric equations

$$x = a\cos 3t, \quad y = a\sin t, \qquad 0 \leq t \leq \frac{\pi}{6}.$$

The curve meets the axes at points A and B as shown in Figure 1.

The straight line shown is part of the tangent to the curve at the point A.

Find, in terms of a,

(a) an equation of the tangent at A,

(6)

(b) an exact value for the area of the finite region between the curve, the tangent at A and the x-axis, shown shaded in Figure 1.

(9)

TOTAL FOR PAPER: 75 MARKS

END