EDEXCEL
June 2005
6675 Further Pure P5
Mark Scheme

Publication.

Question number	Scheme	Marks
1.	(a) $\int \frac{1+x}{\sqrt{(1-4x^2)}}dx = \int \frac{1}{\sqrt{(1-4x^2)}}dx + \int \frac{x}{\sqrt{(1-4x^2)}}dx$	M1
	$\frac{1}{2}\arcsin 2x, + \; 2\times\frac{-1}{8}\sqrt{(1-4x^2)}$ one correct	M1 A1,M1
	$= \frac{1}{2}\arcsin 2x - \frac{1}{4}\sqrt{(1-4x^2)}\ (+C)$	A1 (5)
	Alternative Let $x=\frac{1}{2}\sin\theta$, $\int \frac{1+0.5\sin\theta}{\cos\theta}\times\frac{1}{2}\cos\theta d\theta$ M1	
	$= \frac{1}{2}\theta - \frac{1}{4}\cos\theta\ (+C)$, $= \frac{1}{2}\arcsin 2x - \frac{1}{4}\sqrt{(1-4x^2)}\ (+C)$ M1A1, M1 A1	
	(b) $\int_0^{0.3}\frac{1+x}{\sqrt{(1-4x^2)}}dx = 0.5\arcsin 0.6 - 0.25\sqrt{0.64} + 0.25 = 0.372$	M1 A1 (2)
		(7)
2.	(a) $\cosh 2x = \frac{e^{2x}+e^{-2x}}{2} = \frac{e^{2\ln k}+e^{-2\ln k}}{2}$ (or use $e^x = k$)	M1
	$= \frac{k^2+k^{-2}}{2} = \frac{k^4+1}{2k^2}$ (*)	M1 A1 (3)
	(b) $f'(x) = p - 2\text{sech}^2 2x$	M1 A1
	For $x=\ln 2$, $\cosh 2x = \frac{2^4+1}{2\times2^2} = \frac{17}{8}$	B1
	$p - \frac{2}{\cosh^2 2x} = 0$, $p = 2\times\frac{64}{289} = \frac{128}{289}$	A1 (4)
		(7)

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3.	$\frac{dx}{dt} = -3a\cos^2 t \sin t \quad \frac{dy}{dt} = 3a\sin^2 t\cos t$		M1 A1
	Area $= 2\pi \int a\sin^3 t\sqrt{9a^2(\cos^4 t\sin^2 t + \sin^4 t\cos^2 t)}dt$		M1 A1
	$= 6\pi a^2 \int \sin^3 t \sin t \cos t dt = 6\pi a^2\left[\frac{\sin^5 t}{5}\right]_0^{\pi/2} = \frac{6\pi a^2}{5}$		M1 A1 A1 (7)
			(7)
4.	(a) $I_n = \frac{1}{2}x^n e^{2x} - \frac{n}{2}\int x^{n-1}e^{2x}dx, \quad I_n = \frac{1}{2}(x^n e^{2x} - nI_{n-1})$	(*)	M1 A1 A1 (3)
	(b) $\int_0^1 x^2 e^{2x}dx = I_2 = \left[\frac{1}{2}x^2 e^{2x}\right]_0^1 - I_1 = \frac{1}{2}e^2 - I_1$	one correct start	M1
	$I_1 = \left[\frac{1}{2}xe^{2x}\right]_0^1 - \frac{I_0}{2} = \frac{1}{2}e^2 - \frac{1}{2}I_0$	linking all three	M1 A1
	$I_0 = \int_0^1 e^{2x}dx = \left[\frac{e^{2x}}{2}\right]_0^1 = \frac{e^2}{2} - \frac{1}{2}$		
	$I_2 = \frac{e^2}{2} - \left(\frac{e^2}{2} - \left(\frac{e^2}{4} - \frac{1}{4}\right)\right) = \frac{1}{4}(e^2 - 1)$	use of limits	M1 A1 (5)
			(8)

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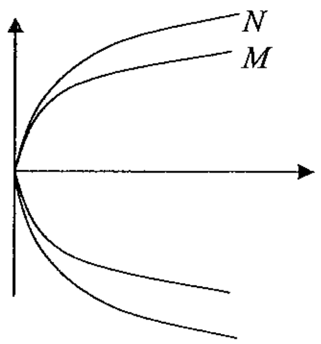
5.	(a)	$2y\frac{\mathrm{d}y}{\mathrm{d}x} = 4a \quad \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4a}{2y} = \frac{1}{p}$		B1	
		$y - 2ap = \frac{1}{p}\left(x - ap^2\right), \qquad py = x + ap^2$	(*)	M1,A1	(3)
	(b)	At Q, parameter = $4p \qquad 4py = x + 16ap^2$		M1 A1	(2)
	(c)	Tangents intersect: $3py = 15ap^2 \quad y = 5ap \quad x = 4ap^2$		M1 A1	
		$p = \frac{y}{5a} \quad x = 4a\frac{y^2}{25a^2} \qquad 4y^2 = 25ax$	(*)	M1 A1	(4)
	(d)	Focus $\left(\frac{25a}{16}, 0\right)$		B1	
		Directrix $x = -\frac{25a}{16}$		B1√	(2)
	(e)			B1 B1	(2)
					(13)

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6.	$\int x\text{arcosh}x\,dx = \frac{x^2}{2}\text{arcosh}x - \int \frac{x^2}{2\sqrt{x^2-1}}dx$		M1 A1	
	$\left[\frac{x^2}{2}\text{arcosh}x\right]_1^2 = 2\,\text{arcosh}2$			
	Let $x = \cosh\theta$ $\int \frac{\cosh^2\theta}{2\sinh\theta}\sinh\theta\,d\theta$		M1 A1	
	$= \int \frac{\cosh^2\theta}{2}d\theta = \int \frac{1+\cosh 2\theta}{4}d\theta, = \frac{\theta}{4} + \frac{\sinh 2\theta}{8}$		M1, M1 A1	
	$= \left[\frac{\theta}{4} + \frac{\sinh 2\theta}{8}\right]_0^{\text{arcosh}2} = \frac{1}{4}\text{arcosh}2 + \frac{2\times\sqrt{3}\times 2}{8}$	limits	M1 A1	
	$\text{Area} = \frac{7}{4}\text{arcosh}\,2 - \frac{\sqrt{3}}{2} = \frac{7}{4}\ln(2+\sqrt{3}) - \frac{\sqrt{3}}{2}$	(*)	A1	(10)
				(10)

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7.	(a)	$\frac{dx}{dt} = 1 + \cos t \quad \frac{dy}{dt} = \sin t$		B1	
		$s = \int_0^t \sqrt{(1+\cos t)^2 + \sin^2 t}\, dt, = \int_0^t \sqrt{2}\sqrt{1+\cos t}\, dt$		M1, A1	
		$= \int_0^t 2\cos \tfrac{t}{2}\, dt = 4\sin \tfrac{t}{2}$	(*)	A1	(4)
	(b)	$\frac{dy}{dx} = \frac{\sin t}{1+\cos t} \quad = \frac{2\sin \tfrac{t}{2} \cos \tfrac{t}{2}}{2\cos^2 \tfrac{t}{2}} = \tan \tfrac{t}{2}$		M1 A1	
		$\tan\psi = \tan \tfrac{t}{2} \qquad t = 2\psi \qquad s = 4\sin\psi$	(*)	M1 A1	(4)
	(c)	$\frac{ds}{d\psi} = 4\cos\psi = 4\cos\frac{\pi}{6} = 2\sqrt{3}$		M1 A1	(2)
					(10)

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8.	(a) $\ln\left(\frac{1-\sqrt{(1-x^2)}}{x}\right)=\ln\left(\frac{1-\sqrt{(1-x^2)}}{x}\times\frac{1+\sqrt{(1-x^2)}}{1+\sqrt{(1-x^2)}}\right)$	M1
	$=\ln\left(\frac{1-(1-x^2)}{x\left(1+\sqrt{(1-x^2)}\right)}\right)=-\ln\left(\frac{1+\sqrt{(1-x^2)}}{x}\right)$ (*)	M1 A1 (3)
	(b) Let $y=\text{arsech}\, x$ $\quad \text{sech}\, y=\frac{2}{e^y+e^{-y}}$	B1
	$xe^y+xe^{-y}=2 \quad xe^{2y}-2e^y+x=0$	M1
	$e^y=\frac{2\pm\sqrt{4-4x^2}}{2x}=\frac{1\pm\sqrt{1-x^2}}{x}$	M1 A1
	$y=\ln\left(\frac{1\pm\sqrt{1-x^2}}{x}\right)=(\pm)\ \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right)$ (*)	A1 (5)
	(c) $3\left(1-\text{sech}^2 x\right)-4\text{sech}x+1=0$	M1
	$(3\text{sech}x-2)(\text{sech}x+2)=0 \quad \text{sech}x=\frac{2}{3}$	M1 A1
	$x=\pm\ln\left(\frac{3}{2}\left(1+\sqrt{\frac{5}{9}}\right)\right)=\pm\ln\left(\frac{3+\sqrt{5}}{2}\right)$	M1 A1 (5)
		(13)