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Edexcel GCE

Mathematics

Mechanics M6 6682

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Confidential Mark Scheme

Edexcel GCE Mathematics

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GENERAL CERTIFICATE OF EDUCATION

Advanced Subsidiary/Advanced Level

Mechanics M6

MARKING SCHEME

June 2005

Principal Examiner:

Dr CM Tuckett
Celsea Cottage
1 Wallingford Road
Cholsey
Wallingford
OX10 9LQ

Tel.: 01491 659091

Marking should be completed by 18 July 2005.

General Instructions

1. The total number of marks for the paper is 75.

2. Method (M) marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.

3. Accuracy (A) marks can only be awarded if the relevant method (M) marks have been earned.

4. (B) marks are independent of method marks.

5. Method marks should not be subdivided.

6. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. Indicate this action by 'MR' in the body of the script (but see also note 10).

7. If a candidate makes more than one attempt at any question:

   (a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.

   (b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

8. Marks for each question, or part of a question, must appear in the right-hand margin and, in addition, total marks for each question, even where zero, must be ringed and appear in the right-hand margin and on the grid on the front of the answer book. It is important that a check is made to ensure that the totals in the right-hand margin of the ringed marks and of the unringed marks are equal. The total mark for the paper must be put on the top right-hand corner of the front cover of the answer book.

9. For methods of solution not in the mark scheme, allocate the available M and A marks in as closely equivalent a way as possible, and indicate this by the letters 'OS' (outside scheme) put alongside in the body of the script.

10. All A marks are 'correct answer only' (c.a.o.) unless shown, for example, as A1 f.t. to indicate that previous wrong working is to be followed through. In the body of the script the symbol $\surd$ should be used for correct f.t. and $\not\surd$ for incorrect f.t. After a misread, however, the subsequent A marks affected are treated as A f.t., but manifestly absurd answers should never be awarded A marks.

11. Ignore wrong working or incorrect statements following a correct answer.

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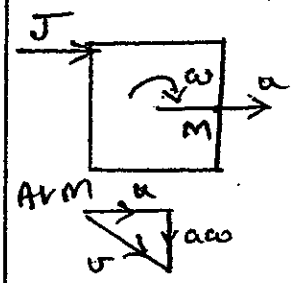
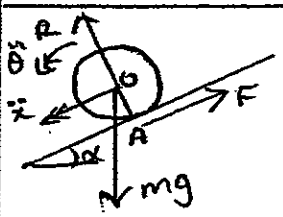
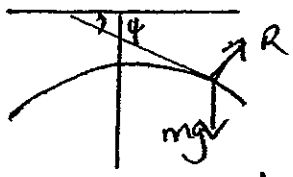
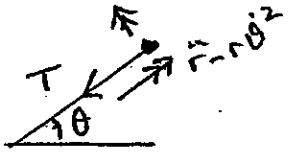
June 2005
6682 Mechanics M6
Mark Scheme

Question Number	Scheme	Marks
1.	 Lin. mom^m : $J = 4mu$	M1 A1
	Ang mom^m : $Ja = \frac{16}{3}ma^2.\omega$	M1 A1
	$v^2 = u^2 + (a\omega)^2$	M1 A1
	$v = \sqrt{\left(\frac{J}{4m}\right)^2 + \left(\frac{3J}{16m}\right)^2}$	↓ M1
	$= \frac{5J}{16m}$	A1 (8)
2.	 Ar A $\dot{x} = a\dot{\theta} \Rightarrow \ddot{x} = a\ddot{\theta}$	B1
	R(↖) $R = mg\cos\alpha$	B1
	R(↗) $m\ddot{x} = mg\sin\alpha - F$ —(1)	M1 A1
	M(O) $Fa = \frac{2}{3}ma^2\ddot{\theta}$	↓ M1 A1
	Eliminate $\ddot{x}$ & $\ddot{\theta}$: $mg\sin\alpha - F = \frac{3F}{2}$	↓ M1
	$\Rightarrow F = \frac{2}{5}mg\sin\alpha$	↓ A1
	$F \le \mu R \Rightarrow \frac{2}{5}mg\sin\alpha \le \mu mg\cos\alpha$	↓ M1
	$\Rightarrow \mu \ge \frac{2}{5}\tan\alpha$ (*)	A1
	From (1) $m\ddot{x} = mg\sin\alpha - \frac{2}{5}mg\sin\alpha$	M1
	$\Rightarrow \ddot{x} = \frac{3}{5}g\sin\alpha$	A1 (12)

Question Number	Scheme	Marks
3.	(a) Impulse from wall is ← & does not affect vert^{l} motion	M1
	(↑): $0 = U\sin\alpha\, t - \frac{1}{2}gt^2$	M1 A1
	$t = (0 \text{ or }) \dfrac{2U\sin\alpha}{g}$ (*) cso	A1 (4)
	(b) At wall (⇔) speed of rebound $= eU\cos\alpha$	M1 A1
	Time from O to wall $= \dfrac{d}{U\cos\alpha}$	B1
	Time from wall to O $= \dfrac{d}{eU\cos\alpha}$	B1
	$\Rightarrow \dfrac{2U\sin\alpha}{g} = \dfrac{d}{U\cos\alpha} + \dfrac{d}{eU\cos\alpha}$	M1 A1
	$\Rightarrow 2U^2\sin\alpha\cos\alpha = gd\left(1+\frac{1}{e}\right)$	M1
	$\Rightarrow U^2\sin 2\alpha = gd\left(1+\frac{1}{e}\right)$ (*)	A1 (8) (12)
4.	 R(↙) $mg\cos\psi - R = \dfrac{mv^2}{\rho}$ —(1)	M1 A1
	$s = a\tan\psi \Rightarrow \rho = \dfrac{ds}{d\psi} = a\sec^2\psi$	M1 A1
	$\dfrac{dy}{d\psi} = \sin\psi \Rightarrow dy = \sin\psi \dfrac{ds}{d\psi} d\psi$	M1
	$y = a\int \sin\psi \sec^2\psi\, d\psi$	|
	$= a\int \sec\psi \tan\psi\, d\psi = a\sec\psi + C$	M1 A1
	$\psi = 0, y = a \Rightarrow C = 0$	|
	Energy: $\frac{1}{2}mv^2 = mg(y-a)$	M1
	$\Rightarrow v^2 = 2ga(\sec\psi - 1)$	A1
	Put $R=0$ in (1) & sub for v^2 & ρ: $mg\cos\psi = \dfrac{m\cdot 2ga(\sec\psi-1)}{a\sec^2\psi}$	M1 A1
	$\Rightarrow \cos\psi = 2(\cos\psi - \cos^2\psi)$	
	$\cos\psi \neq 0 \Rightarrow \cos\psi = \frac{1}{2} \Rightarrow \psi = \pi/3$ (*)	A1 (12)

5. (a)



$R(\perp)\quad 0 = 2\cdot\frac{1}{r}\frac{d}{dt}(r^2\dot{\theta}) \Rightarrow r^2\dot{\theta}=h$ (cons M) — m1 A1

$t=0, r=5, \dot{r}=1.\cos\alpha=\frac{3}{5}, \quad r\dot{\theta}=1.\sin\alpha=\frac{4}{5}$ — m1

$\Rightarrow h = 5.\frac{4}{5} = \underline{4}$ ⊛ — A1 (4)

(b)

$R(\nearrow)\quad -\frac{10}{r^2} = 2(\ddot{r}-r\dot{\theta}^2)$ — m1 A1

$-\frac{10}{r^2} = 2\left(\ddot{r} - r.\left(\frac{4}{r^2}\right)^2\right)$ — m1

$\ddot{r} = -\frac{5}{r^2} + \frac{16}{r^3}$

$\int \dot{r}\,d\dot{r} = \int -\frac{5}{r^2} + \frac{16}{r^3}\,dr$ — m1

$\frac{1}{2}\dot{r}^2 = \frac{5}{r} - \frac{8}{r^2} + C$ — A1

$t=0, r=5\ \dot{r}=\frac{3}{5}: \quad \frac{9}{50} = \frac{5}{5} - \frac{8}{25} + C \Rightarrow C = -\frac{1}{2}$ — m1 A1

P moving $\perp r$ F when $\dot{r}=0 \Rightarrow \frac{5}{r} - \frac{8}{r^2} - \frac{1}{2} = 0$ — m1

$\Rightarrow r^2 - 10r + 16 = 0$ — m1

$(r-2)(r-8)=0$

$r = 2$ or 8 — A1 (10)

(14)

6. (a) $r\dot{\theta} = u$ B1

$r = a\sec^2\theta/2 \Rightarrow \dot{r} = (2a\sec\theta/2)\cdot\frac{1}{2}\cdot\sec\theta/2\tan\theta/2\,\dot{\theta}$ M1 A1

$= a\sec^2\theta/2\tan\theta/2\left(\frac{u}{a\sec^2\theta/2}\right)$ M1

$= u\tan\theta/2$

Hence $|\dot{r}| = |u\tan\theta/2|$ (*) A1 (5)

(b) $\ddot{r} - r\dot{\theta}^2 = \frac{u}{2}\sec^2\theta/2\cdot\dot{\theta} - r\left(\frac{u}{r}\right)^2$ M1 A1

$= \frac{u}{2}\cdot\frac{r}{a}\cdot\frac{u}{r} - \frac{u^2}{r}$

$= \frac{u^2}{2a}(1 - 2\cos^2\theta/2) = -\frac{u^2\cos\theta}{2a}$ M1 A1 (4)

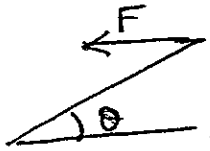
(c) $\frac{1}{r}\frac{d}{dt}(r^2\dot{\theta}) = \frac{1}{r}\frac{d}{dt}(ur) = \frac{u}{r}\dot{r}$ M1

$= \frac{u^2\tan\frac{1}{2}\theta}{a\sec^2\theta/2}$ A1 (2)

$= \frac{u^2}{a}\sin\theta/2\cos\theta/2 = \frac{u^2}{2a}\sin\theta$

(d) $|\underline{F}| = m\sqrt{\left(-\frac{u^2\cos\theta}{2a}\right)^2 + \left(\frac{u^2}{2a}\sin\theta\right)^2}$ M1 A1√

$= \frac{mu^2}{2a}$ Show indep^t of θ M1 A1 cso (4)

(e) 

$\perp^r$ to v_0 $\theta = 0$ M1

Dir^n A1 (2)

(17)