

| Question Number                   | Scheme  | Marks                                 |
|-----------------------------------|---|---------------------------------------|
| <p><b>1(a)</b></p>                | $\frac{3(x+1)}{(x+2)(x-1)} \equiv \frac{A}{x+2} + \frac{B}{x-1},$ <p>and correct method for finding A or B</p> <p><math>A = 1, B = 2</math></p>   | <p>M1</p> <p>A1, A1<br/>(3)</p>       |
| <p><b>(b)</b></p>                 | $f'(x) = -\frac{1}{(x+2)^2} - \frac{2}{(x-1)^2}$ <p>Argument for negative, including statement that square terms are positive for all values of x. (f.t. on wrong values of A and B)</p>                                    | <p>M1 A1</p> <p>A1ft✓<br/>(3)</p>     |
| <p><b>2</b></p> <p><b>(a)</b></p> | <p><math>a = 4, b = 5</math> (both are required)</p>  | <p>B1<br/>(1)</p>                     |
| <p><b>(b)</b></p>                 | $(x-4)^2 + (y-5)^2 = 25$  | <p>M1A1ft<br/>(2)</p>                 |
| <p><b>(c)</b></p>                 | <p>Finding the distance between centre and <math>(8, 17)</math>, <math>\sqrt{[(8-a)^2 + (17-b)^2]}</math></p> <p>Complete method to find PT, i.e. use Pythagoras theorem and subtraction,</p> <p><math>PT = 11.6</math></p> | <p>M1</p> <p>M1</p> <p>A1<br/>(3)</p> |

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| <p><b>3(a)</b></p> <p><b>(b)</b></p> <p><b>(c)</b></p> | <p>Using <math>f(\pm 2) = 3</math></p> <p>Showing that <math>p = 6</math> ★, with no wrong working seen.</p> <p><b>S.C.</b> If <math>p = 6</math> used and the remainder is shown to be 3 award B1</p> <p>Attempt to find quotient when dividing <math>(n + 2)</math> into <math>f(n)</math> or attempting to equate coefficients.</p> <p>Quotient = <math>n^2 + 4n + 3</math>, or finding either <math>q = 1</math> or <math>r = 3</math></p> <p>Finding both <math>q = 1</math> and <math>r = 3</math></p> <p>The product of three consecutive numbers must be divisible by 3</p> <p>Complete argument</p> | <p>M1</p> <p>A1 (2)</p> <p>M1</p> <p>A1</p> <p>A1 (3)</p> <p>M1</p> <p>A1 (2)</p> |
| <p><b>4. (a)</b></p> <p><b>(b)</b></p>                 | <p><math>(1 + 3x)^{-2} = 1 + (-2)(3x) + \frac{(-2)(-3)}{2!}(3x)^2 + \frac{(-2)(-3)(-4)}{3!}(3x)^3 + \dots</math></p> <p><math>= 1, -6x, +27x^2 \dots (-108x^3)</math></p> <p>Using (a) to expand <math>(x + 4)(1 + 3x)^{-2}</math> or complete method to find coefficients [e.g. Maclaurin or <math>\frac{1}{3}(1 + 3x)^{-1} + \frac{11}{3}(1 + 3x)^{-2}</math>].</p> <p><math>= 4 - 23x, +102x^2, -405x^3 = 4, -23x, +102x^2 \dots (-405x^3)</math></p>   | <p>M1</p> <p>B1, A1, A1 (4)</p> <p>M1</p> <p>A1, A1ft, A1ft (4)</p>               |

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| <b>6(a)</b>     | $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k} \pm \lambda(4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k})$ or $\mathbf{r} = 5\mathbf{i} - 3\mathbf{j} \pm \lambda(4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k})$<br>(or any equivalent vector equation)  | M1A1 (2)   |
| <b>(b)</b>      | Show that $\mu = -3$  | B1 (1)   |
| <b>(c)</b>      | Using $\cos \theta = \frac{(4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}) \cdot (\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})}{\sqrt{(4^2 + 5^2 + 3^2)}\sqrt{(1^2 + 2^2 + 2^2)}}$<br><br>$= \frac{20}{15\sqrt{2}} = \frac{4}{3\sqrt{2}}$ (ft on $4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$ )   | M1<br><br>num, denom. A1ft A1ft<br><br>A1 (4)              |
| <b>(d)</b>      | Shortest distance = $AC \sin \theta$<br><br>$AC = \sqrt{((a-1)^2 + 2^2 + (b+3)^2)}$ (= 3)<br><br>Shortest distance = 1 unit<br><br><i>Alternatives</i><br>Since $X = (1+4\lambda, 2-5\lambda, -3+3\lambda)$<br>$\mathbf{CX} = (-1+4\lambda)\mathbf{i} + (2-5\lambda)\mathbf{j} + (-2+3\lambda)\mathbf{k}$<br>Use Scalar product $\mathbf{CX} \cdot (4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}) = 0$ , OR differentiate $ \mathbf{CX} $ or $ \mathbf{CX} ^2$<br>and equate to zero,<br><br>to obtain $\lambda = 0.4$<br>and thus $ \mathbf{CX}  = 1$ | M1<br><br>M1A1<br><br>A1 (4)<br><br>M1<br><br>A1<br>A1 (4) |

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| 5. (a)          | $\frac{dV}{dt} = 30 - \frac{2}{15}V$ $\Rightarrow -15 \frac{dV}{dt} = -450 + 2V, \quad \text{no wrong working seen}$  | M1A1<br>A1* (3)                          |
| (b)             | Separating the variables $\Rightarrow -\frac{15}{2V-450}dV = dt$<br>Integrating to obtain $-\frac{15}{2}\ln 2V-450 =t$ OR $-\frac{15}{2}\ln V-225 =t$<br>Using limits correctly or finding $c$ ( $-\frac{15}{2}\ln 1550$ OR $-\frac{15}{2}\ln 775$ )<br>$\ln \frac{2V-450}{1550} = -\frac{2}{15}t$ , or equivalent<br>Rearranging to give $V = 225 + 775e^{-\frac{2}{15}t}$ . | M1<br>dM1 A1<br>M1<br>A1<br>dM1A1<br>(7) |
| (c)             | $V = 225$   | B1 (1)                                   |

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| <b>7(a)</b>     | $\frac{dy}{dx} = -2e^{-2x}\sqrt{x} + \frac{e^{-2x}}{2\sqrt{x}}$ <p>Putting <math>\frac{dy}{dx} = 0</math> and attempting to solve</p> $x = \frac{1}{4}$   | M1 A1 A1<br><br>dM1<br><br>A1 (5)                   |
| <b>(b)</b>      | $\text{Volume} = \pi \int_0^1 (\sqrt{x}e^{-2x})^2 dx = \pi \int_0^1 xe^{-4x} dx$ $\int xe^{-4x} dx = -\frac{1}{4}xe^{-4x} + \int \frac{1}{4}e^{-4x} dx$ $= -\frac{1}{4}xe^{-4x} - \frac{1}{16}e^{-4x}$ $\text{Volume} = \pi \left[ -\frac{1}{4}e^{-4} - \frac{1}{16}e^{-4} \right] - \left[ -\frac{1}{16} \right] = \frac{\pi}{16} [1 - 5e^{-4}]$ | M1 A1<br><br>M1 A1<br><br>A1 ft<br><br>M1 A1<br>(7) |

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| <b>8 (a)</b>    | $\cos(A + A) = \cos^2 A - \sin^2 A$ $= \cos^2 A - (1 - \cos^2 A) = 2\cos^2 A - 1$   | M1<br>A1 (2)                              |
| <b>(b)</b>      | $[x = 2, \theta = \frac{\pi}{4}; x = \sqrt{6}, \theta = \frac{\pi}{3}]$ $x = 2\sqrt{2} \sin \theta, \frac{dx}{d\theta} = 2\sqrt{2} \cos \theta$ $\int \sqrt{8 - x^2} dx = \int 2\sqrt{2} \cos \theta \cdot 2\sqrt{2} \cos \theta d\theta = \int 8 \cos^2 \theta d\theta$ <p>Using <math>\cos 2\theta = 2\cos^2 \theta - 1</math> to give <math>\int 4(1 + \cos 2\theta) d\theta</math><br/> <math>= 4\theta + 2 \sin 2\theta</math></p> <p>Substituting limits to give <math>\frac{1}{3}\pi + \sqrt{3} - 2</math> or given result</p> | B1<br>B1<br>M1A1<br>dM1<br>A1ft<br>A1 (7) |
| <b>(c)</b>      | $\frac{dy}{d\theta} = \frac{-2 \sin 2\theta}{1 + \cos 2\theta}$ <p>Using the chain rule, with <math>\frac{dx}{d\theta} = \sec \theta \tan \theta</math> to give <math>\frac{dy}{dx} (= -2 \cos \theta)</math></p> <p>Gradient at the point where <math>\theta = \frac{\pi}{3}</math> is <math>-1</math>.</p> <p>Equation of tangent is <math>y + \ln 2 = -(x - 2)</math> (o.a.e.)</p>   | B1<br>M1<br>A1ft<br>M1A1 (5)              |