Paper Reference(s)

6673

Edexcel GCE

Pure Mathematics P3
Advanced/Advanced Subsidiary

Thursday 9 January 2003 – Afternoon
Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Answer Book (AB16)	Nil
Mathematical Formulae (Lilac)	
Graph Paper (ASG2)	

Candidates may only use one of the basic scientific calculators approved by the Qualifications and Curriculum Authority.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Pure Mathematics P3), the paper reference (6673), your surname, other name and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has eight questions. Pages 6, 7 and 8 are blank.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. The function f is given by

$$f(x) = \frac{3(x+1)}{(x+2)(x-1)},\ x \in \mathbb{R},\ x \neq -2,\ x \neq 1.$$

(*a*) Express f(x) in partial fractions.

(3)

(*b*) Hence, or otherwise, prove that $f'(x) < 0$ for all values of x in the domain.

(3)

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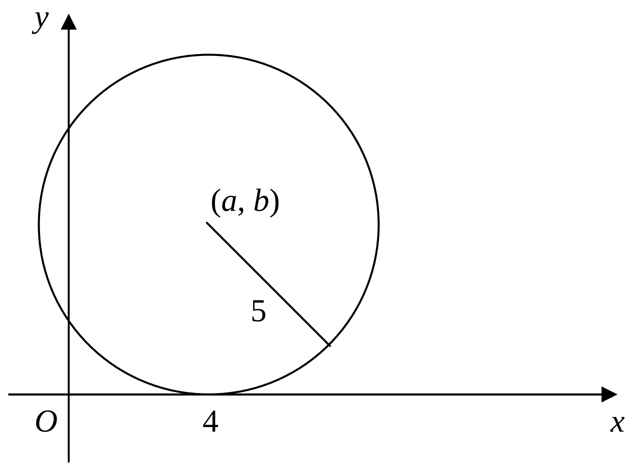
2. **Figure 1**



The circle C, with centre (a, b) and radius 5, touches the x-axis at (4, 0), as shown in Fig. 1.

(*a*) Write down the value of a and the value of b.

(1)

(*b*) Find a cartesian equation of C.

(2)

A tangent to the circle, drawn from the point $P(8, 17)$, touches the circle at T.

(*c*) Find, to 3 significant figures, the length of PT.

(3)

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3. $\mathrm{f}(n) = n^3 + pn^2 + 11n + 9$, where p is a constant.

(a) Given that $\mathrm{f}(n)$ has a remainder of 3 when it is divided by $(n + 2)$, prove that $p = 6$.

(2)

(b) Show that $\mathrm{f}(n)$ can be written in the form $(n + 2)(n + q)(n + r) + 3$, where q and r are integers to be found.

(3)

(c) Hence show that $\mathrm{f}(n)$ is divisible by 3 for all positive integer values of n.

(2)

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4. (a) Expand $(1 + 3x)^{-2}$, $|x| < \frac{1}{3}$, in ascending powers of x up to and including the term in x^3, simplifying each term.

(4)

(b) Hence, or otherwise, find the first three terms in the expansion of $\dfrac{x+4}{(1+3x)^2}$ as a series in ascending powers of x.

(4)

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5. Liquid is poured into a container at a constant rate of 30 $\mathrm{cm}^3\ \mathrm{s}^{-1}$. At time t seconds liquid is leaking from the container at a rate of $\frac{2}{15}V$ $\mathrm{cm}^3\ \mathrm{s}^{-1}$, where V cm^3 is the volume of liquid in the container at that time.

(a) Show that

$$-15\frac{\mathrm{d}V}{\mathrm{d}t} = 2V - 450.$$

(3)

Given that $V = 1000$ when $t = 0$,

(b) find the solution of the differential equation, in the form $V = \mathrm{f}(t)$.

(7)

(c) Find the limiting value of V as $t \to \infty$.

(1)

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6. Referred to a fixed origin O, the points A and B have position vectors $(\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$ and $(5\mathbf{i} - 3\mathbf{j})$ respectively.

(*a*) Find, in vector form, an equation of the line l_1 which passes through A and B.

(2)

The line l_2 has equation $\mathbf{r} = (4\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}) + \mu(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$, where μ is a scalar parameter.

(*b*) Show that A lies on l_2.

(1)

(*c*) Find, in degrees, the acute angle between the lines l_1 and l_2.

(4)

The point C with position vector $(2\mathbf{i} - \mathbf{k})$ lies on l_2.

(*d*) Find the shortest distance from C to the line l_1.

(4)

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7. **Figure 2**

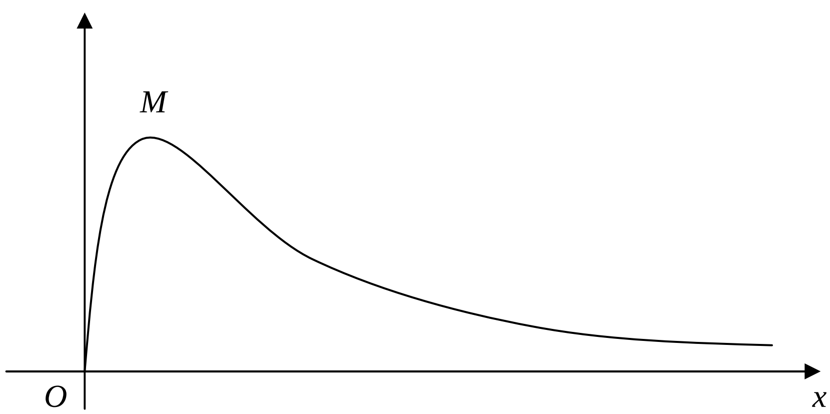

Figure 2 shows the curve with equation $y = x^{\frac{1}{2}} \mathrm{e}^{-2x}$.

(*a*) Find the x-coordinate of M, the maximum point of the curve.

(5)

The finite region enclosed by the curve, the x-axis and the line $x = 1$ is rotated through 2π about the x-axis.

(*b*) Find, in terms of π and e, the volume of the solid generated.

(7)

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8. (*a*) Use the identity for cos $(A + B)$ to prove that $\cos 2A = 2\cos^2 A - 1$.

(2)

(*b*) Use the substitution $x = 2\sqrt{2}\sin\theta$ to prove that

$$\int_2^{\sqrt{6}} \sqrt{(8 - x^2)}\ \mathrm{d}x = \tfrac{1}{3}(\pi + 3\sqrt{3} - 6).$$

(7)

A curve is given by the parametric equations

$$x = \sec\theta,\quad y = \ln(1 + \cos 2\theta),\quad 0 \leq \theta < \frac{\pi}{2}.$$

(*c*) Find an equation of the tangent to the curve at the point where $\theta = \dfrac{\pi}{3}$.

(5)

END