Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
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**Pearson Edexcel Level 3 GCE**

Centre Number | Candidate Number

# Wednesday 7 October 2020

Afternoon (Time: 2 hours) | Paper Reference **9MA0/01**

# Mathematics

## Advanced

## Paper 1: Pure Mathematics 1

**You must have:**
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

**Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 16 questions in this question paper. The total mark for this paper is 100.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

*Turn over*


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Pearson

**1.** (a) Find the first four terms, in ascending powers of $x$, of the binomial expansion of

$$(1 + 8x)^{\frac{1}{2}}$$

giving each term in simplest form.

**(3)**

(b) Explain how you could use $x = \frac{1}{32}$ in the expansion to find an approximation for $\sqrt{5}$

There is no need to carry out the calculation.

**(2)**

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**Question 1 continued**

**(Total for Question 1 is 5 marks)**

**2.** By taking logarithms of both sides, solve the equation

$$4^{3p-1} = 5^{210}$$

giving the value of $p$ to one decimal place.

**(3)**

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**Question 2 continued**

**(Total for Question 2 is 3 marks)**

**3.** Relative to a fixed origin $O$

- point $A$ has position vector $2\mathbf{i} + 5\mathbf{j} - 6\mathbf{k}$
- point $B$ has position vector $3\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}$
- point $C$ has position vector $2\mathbf{i} - 16\mathbf{j} + 4\mathbf{k}$

(a) Find $\overrightarrow{AB}$

**(2)**

(b) Show that quadrilateral $OABC$ is a trapezium, giving reasons for your answer.

**(2)**

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**Question 3 continued**

**(Total for Question 3 is 4 marks)**

**4.** The function f is defined by

$$\mathrm{f}(x) = \frac{3x - 7}{x - 2} \qquad x \in \mathbb{R}, x \neq 2$$

(a) Find $\mathrm{f}^{-1}(7)$

**(2)**

(b) Show that $\mathrm{ff}(x) = \dfrac{ax + b}{x - 3}$ where $a$ and $b$ are integers to be found.

**(3)**

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**Question 4 continued**

**(Total for Question 4 is 5 marks)**

**5.** A car has six forward gears.

The fastest speed of the car

- in 1st gear is 28 km $h^{-1}$
- in 6th gear is 115 km $h^{-1}$

Given that the fastest speed of the car in successive gears is modelled by an **arithmetic sequence**,

(a) find the fastest speed of the car in 3rd gear.

**(3)**

Given that the fastest speed of the car in successive gears is modelled by a **geometric sequence**,

(b) find the fastest speed of the car in 5th gear.

**(3)**

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**Question 5 continued**

**(Total for Question 5 is 6 marks)**

**6.** (a) Express $\sin x + 2\cos x$ in the form $R\sin(x + \alpha)$ where $R$ and $\alpha$ are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of $R$ and give the value of $\alpha$ in radians to 3 decimal places.

**(3)**

The temperature, $\theta$°C, inside a room on a given day is modelled by the equation

$$\theta = 5 + \sin\left(\frac{\pi t}{12} - 3\right) + 2\cos\left(\frac{\pi t}{12} - 3\right) \qquad 0 \leqslant t < 24$$

where $t$ is the number of hours after midnight.

Using the equation of the model and your answer to part (a),

(b) deduce the maximum temperature of the room during this day,

**(1)**

(c) find the time of day when the maximum temperature occurs, giving your answer to the nearest minute.

**(3)**

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**Question 6 continued**

**Question 6 continued**

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**Question 6 continued**

**(Total for Question 6 is 7 marks)**

**7.**

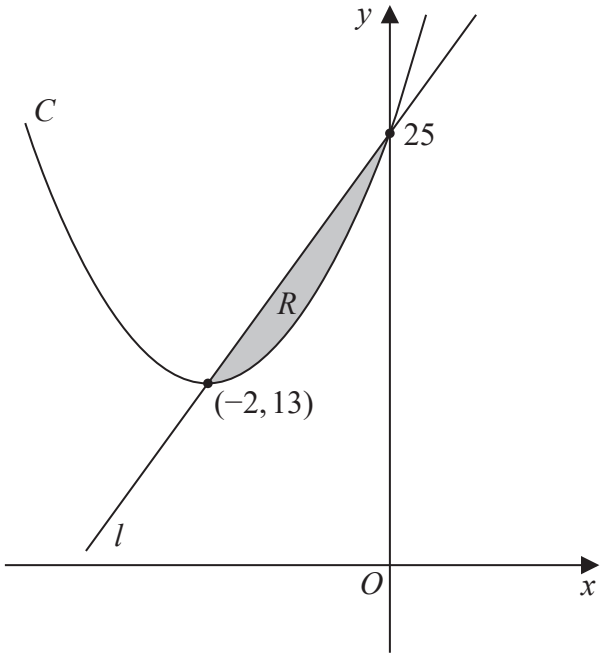


**Figure 1**

Figure 1 shows a sketch of a curve $C$ with equation $y = \mathrm{f}(x)$ and a straight line $l$.

The curve $C$ meets $l$ at the points $(-2, 13)$ and $(0, 25)$ as shown.

The shaded region $R$ is bounded by $C$ and $l$ as shown in Figure 1.

Given that

- $\mathrm{f}(x)$ is a quadratic function in $x$
- $(-2, 13)$ is the minimum turning point of $y = \mathrm{f}(x)$

use inequalities to define $R$.

**(5)**

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**Question 7 continued**

**Question 7 continued**

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**Question 7 continued**

**(Total for Question 7 is 5 marks)**

**8.** A new smartphone was released by a company.

The company monitored the total number of phones sold, $n$, at time $t$ days after the phone was released.

The company observed that, during this time,

the rate of increase of $n$ was proportional to $n$

Use this information to write down a suitable equation for $n$ in terms of $t$.

(*You do not need to evaluate any unknown constants in your equation.*)

**(2)**

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**Question 8 continued**

**(Total for Question 8 is 2 marks)**

**9.**

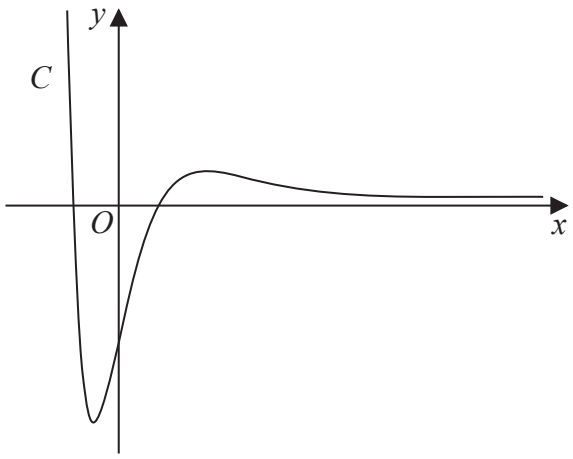


**Figure 2**

Figure 2 shows a sketch of the curve $C$ with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 4(x^2 - 2)\mathrm{e}^{-2x} \qquad x \in \mathbb{R}$$

(a) Show that $\mathrm{f}'(x) = 8(2 + x - x^2)\mathrm{e}^{-2x}$

**(3)**

(b) Hence find, in simplest form, the exact coordinates of the stationary points of $C$.

**(3)**

The function g and the function h are defined by

$$\mathrm{g}(x) = 2\mathrm{f}(x) \qquad x \in \mathbb{R}$$

$$\mathrm{h}(x) = 2\mathrm{f}(x) - 3 \qquad x \geqslant 0$$

(c) Find (i) the range of g

(ii) the range of h

**(3)**

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**Question 9 continued**

**Question 9 continued**

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**Question 9 continued**

**(Total for Question 9 is 9 marks)**

**10.** (a) Use the substitution $x = u^2 + 1$ to show that

$$\int_5^{10} \frac{3\,\mathrm{d}x}{(x-1)\left(3+2\sqrt{x-1}\right)} = \int_p^q \frac{6\,\mathrm{d}u}{u(3+2u)}$$

where $p$ and $q$ are positive constants to be found.

**(4)**

(b) Hence, using algebraic integration, show that

$$\int_5^{10} \frac{3\,\mathrm{d}x}{(x-1)\left(3+2\sqrt{x-1}\right)} = \ln a$$

where $a$ is a rational constant to be found.

**(6)**

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**Question 10 continued**

**Question 10 continued**

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**Question 10 continued**

**(Total for Question 10 is 10 marks)**

**11.**

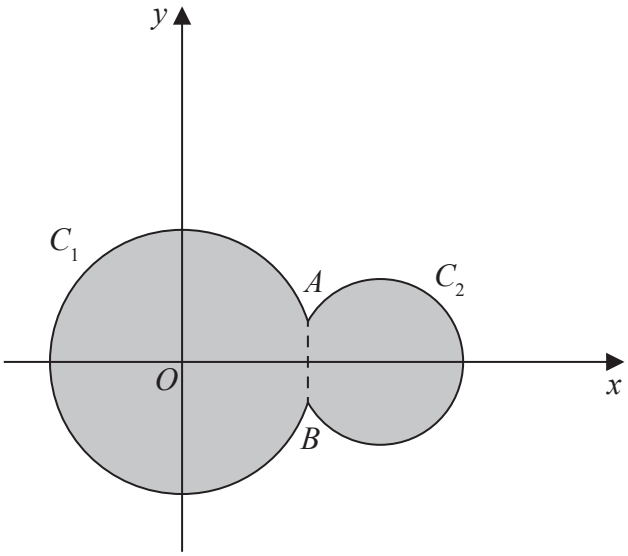


**Figure 3**

Circle $C_1$ has equation $x^2 + y^2 = 100$

Circle $C_2$ has equation $(x - 15)^2 + y^2 = 40$

The circles meet at points $A$ and $B$ as shown in Figure 3.

(a) Show that angle $AOB = 0.635$ radians to 3 significant figures, where $O$ is the origin.

**(4)**

The region shown shaded in Figure 3 is bounded by $C_1$ and $C_2$

(b) Find the perimeter of the shaded region, giving your answer to one decimal place.

**(4)**

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**Question 11 continued**

**Question 11 continued**

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**Question 11 continued**

**(Total for Question 11 is 8 marks)**

**12.** **In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

(a) Show that

$$\operatorname{cosec}\theta - \sin\theta \equiv \cos\theta\cot\theta \qquad \theta \neq (180n)^\circ \qquad n \in \mathbb{Z}$$

**(3)**

(b) Hence, or otherwise, solve for $0 < x < 180^\circ$

$$\operatorname{cosec}x - \sin x = \cos x \cot(3x - 50^\circ)$$

**(5)**

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**Question 12 continued**

**Question 12 continued**

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**Question 12 continued**

**(Total for Question 12 is 8 marks)**

**13.** A sequence of numbers $a_1, a_2, a_3, \ldots$ is defined by

$$a_{n+1} = \frac{k(a_n + 2)}{a_n} \qquad n \in \mathbb{N}$$

where $k$ is a constant.

Given that

- the sequence is a periodic sequence of order 3
- $a_1 = 2$

(a) show that

$$k^2 + k - 2 = 0$$

**(3)**

(b) For this sequence explain why $k \neq 1$

**(1)**

(c) Find the value of

$$\sum_{r=1}^{80} a_r$$

**(3)**

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**Question 13 continued**

**Question 13 continued**

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**Question 13 continued**

**(Total for Question 13 is 7 marks)**

**14.** A large spherical balloon is deflating.

At time $t$ seconds the balloon has radius $r$ cm and volume $V$ cm$^3$

The volume of the balloon is modelled as decreasing at a constant rate.

(a) Using this model, show that

$$\frac{\mathrm{d}r}{\mathrm{d}t} = -\frac{k}{r^2}$$

where $k$ is a positive constant.

**(3)**

Given that

- the initial radius of the balloon is 40 cm
- after 5 seconds the radius of the balloon is 20 cm
- the volume of the balloon continues to decrease at a constant rate until the balloon is empty

(b) solve the differential equation to find a complete equation linking $r$ and $t$.

**(5)**

(c) Find the limitation on the values of $t$ for which the equation in part (b) is valid.

**(2)**

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**Question 14 continued**

**Question 14 continued**

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**Question 14 continued**

**(Total for Question 14 is 10 marks)**

**15.** The curve $C$ has equation

$$x^2 \tan y = 9 \qquad 0 < y < \frac{\pi}{2}$$

(a) Show that

$$\frac{dy}{dx} = \frac{-18x}{x^4 + 81}$$

**(4)**

(b) Prove that $C$ has a point of inflection at $x = \sqrt[4]{27}$

**(3)**

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**Question 15 continued**

**Question 15 continued**

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**Question 15 continued**

**(Total for Question 15 is 7 marks)**

**16.** Prove by contradiction that there are no positive integers $p$ and $q$ such that

$$4p^2 - q^2 = 25$$

**(4)**

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**Question 16 continued**

**Question 16 continued**

**(Total for Question 16 is 4 marks)**

**TOTAL FOR PAPER IS 100 MARKS**

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