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Introduction

This paper generally discriminated well between students of all abilities and there were sufficient opportunities for a typical E grade student to gain some marks across many of the questions. There were some testing questions involving proof, mathematical modelling, differential equations, differentiation and integration that allowed the paper to discriminate well between the higher grades.

In a significant number of cases, students' solutions, particularly to Q6(b), Q8, Q10, Q13 and Q14, proved difficult to read. This was a result of either poor handwriting, incoherent working, disorganised presentation or a combination of all three.

In the new specification, more emphasis is placed on using mathematical models and formulating them from real-life situations in context. It was clear, from looking at responses to Q8(a) and Q10(a), that a significant number of students struggled to formulate a correct mathematical model for each of the given situations. Consequently, these students then lost access to a significant number of marks.

In summary, Q1(a), Q1(c), Q2, Q4, Q5, Q6(a), Q7(i), Q9, Q11(a) and Q12(a) were a good source of marks for the average student, mainly testing standard ideas and techniques whereas Q1(b), Q6(b), Q6(c), Q7(ii), Q8, Q10, Q11(b), Q12(b) and Q13 were discriminating at the higher grades. Q14 proved to be the most challenging question on the paper.

Comments on individual questions

Question 1

This question proved accessible to all, but only the most able students scored full marks. In part (a), most students found the value of $\mathrm{g}(5)$ and substituted this value back into $\mathrm{g}(x)$, with nearly all of them being successful in finding the correct $\mathrm{gg}(5) = \frac{40}{9}$. A minority found $\mathrm{gg}(x)$ algebraically and substituted $x = 5$ into the result.

In part (b), only a few students stated the correct range $2 < \mathrm{g}(x) \leq \frac{15}{2}$. The most common incorrect answers included $\mathrm{g}(x) \leq \frac{15}{2}$, $\mathrm{g}(x) < \frac{15}{2}$, $\mathrm{g}(x) > \frac{15}{2}$ and $2 < x \leq \frac{15}{2}$.

In part (c), most students used a correct process to find $y = \frac{3x+5}{x-2}$ with some finding $y = \frac{11}{x-2} + 3$. A few students, who misunderstood the notation g^{-1}, tried to either differentiate $\mathrm{g}(x)$ or write down the reciprocal of $\mathrm{g}(x)$. Some students lost the final mark by stating neither $\mathrm{g}^{-1}(x) = \frac{3x+5}{x-2}$ nor its domain in correct notation. Some students made no attempt to write down the domain of g^{-1}.

Question 2

This was a well-answered question with most students scoring at least 4 of the 5 marks available.

Part (a) was often completed successfully. Most students found $\overrightarrow{AB}$ and used the given result $\overrightarrow{AB} = \overrightarrow{BD}$ to deduce and apply $\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{AB}$. A few students applied $\overrightarrow{OD} = \overrightarrow{OA} + 2\overrightarrow{AB}$. There were some sign, manipulation and bracketing errors seen in this part and students who used column vectors were generally more successful than those who used the $\mathbf{i}, \mathbf{j}, \mathbf{k}$ notation.

In part (b), most students found the difference between $\overrightarrow{OC}$ and $\overrightarrow{OA}$, and applied Pythagoras' Theorem to $\left|\overrightarrow{AC}\right| = 4$ to give the correct $(a-2)^2 + (2)^2 + (2)^2 = (4)^2$. The resulting quadratic equation was generally solved correctly but not always efficiently. Too often, students expanded to form a 3-term quadratic rather than spotting that their quadratic was already in a 'completed square' form. Despite the question stating that $a < 0$, some students gave both solutions $a = 2 \pm 2\sqrt{2}$ as their final answer, while others mistakenly rejected $a = 2 - 2\sqrt{2}$ to leave $a = 2 + 2\sqrt{2}$.

Question 3

This question discriminated well between students of all abilities with some students struggling to access the final mark in part (b).

In part (a), most students provided a complete solution to disprove the statement given in the question. Common errors included: choosing inappropriate values for m, n (e.g. $\sqrt{2}$ and $\sqrt{3}$ which were sometimes seen); choosing two rational numbers; attempting to disprove the general statement by using algebra; and not giving a conclusion, despite choosing appropriate values for m, n.

In part (b)(i), most students drew a V-shaped graph in the correct position with many labelling 3 or $(0, 3)$ as the y-intercept.

In part (b)(ii), those students who superimposed the graph of $y = |x+3|$ over the graph of $y = |x| + 3$ were usually more successful in explaining why $|x| + 3 \geq |x+3|$ for all real values of x. Otherwise, a variety of explanations were seen, of which some were detailed, usually containing at least one correct statement. Common errors included explanations which did not consider the case $x = 0$ or explanations including the statement that the graphs of $y = |x| + 3$ and $y = |x+3|$ were parallel for all $x < 0$. A significant number of students who only tested the statement $|x| + 3 \geq |x+3|$ for specific values of x (usually one positive and one negative) did not receive any credit in this part.

Question 4

This was an accessible question with part (ii) more successfully answered than part (i).

In part (i), some students attempted to find $\sum_{r=1}^{16}(3+5r+2^r)$ by applying the strategy (3×16) + $\sum_{r=1}^{16}5r$ + $\sum_{r=1}^{16}2^r$, while other students applied the strategy $\sum_{r=1}^{16}(3+5r)$ + $\sum_{r=1}^{16}2^r$. Most students applied the sum formula for an arithmetic series with $a=5,\ d=5$ to (3×16) + $\sum_{r=1}^{16}5r$ to give (3×16) + $\frac{16}{2}(2(5)+15(5))$ while some applied $a=8,\ d=5$ to $\sum_{r=1}^{16}(3+5r)$ to give $\frac{16}{2}(2(8)+15(5))$. A few students used $\sum_{r=1}^{n}r=\frac{1}{2}n(n+1)$ to help them to evaluate $5\sum_{r=1}^{16}r$. Students were generally less successful with using the formula for the sum of a geometric series to find $\sum_{r=1}^{16}2^r$. Common errors included assuming the first term, a, was 1 or writing down the value 131070 with no method (perhaps by working back from the given answer).

A significant number of students listed all 16 terms and added them to achieve the given result. Those students who did not list all the terms, but just wrote e.g. $10+17+26+39+60+...+65619=131798,$ did not receive any credit. Some students, who attempted to find the first, second and third differences between the terms of the series $10,\ 17,\ 26,\ 39,\ 60,...,$ usually made no creditable progress.

Part (ii) was very well attempted, even by those who failed to gain any credit in part (i). Almost all students wrote down the next two terms $u_2,\ u_3$ correctly. Many deduced that they were working with an alternating sequence and found the correct exact value for $\sum_{r=1}^{100}u_r$. Some students attempted to round their answer, usually to 108.3. A few students attempted to find $\sum_{r=1}^{100}u_r$ by using the formula for the sum of an arithmetic series or the formula for the sum of a geometric series.

Question 5

This was a well-answered question with part (a) and part (c) providing most of the discrimination.

In part (a), most students substituted a defined $\mathrm{f}(x_n) = 2x_n^{\,3} + x_n^{\,2} - 1$ and a correct $\mathrm{f}'(x_n) = 6x_n^{\,2} + 2x_n$ into the Newton-Raphson formula $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$. Many students used correct algebra to arrive at the given result. Some students moved directly from $x_{n+1} = x_n - \dfrac{2x_n^{\,3} + x_n^{\,2} - 1}{6x_n^{\,2} + 2x_n}$ to $x_{n+1} = \dfrac{4x_n^{\,3} + x_n^{\,2} + 1}{6x_n^{\,2} + 2x_n}$ without showing the correct intermediate step of rationalising the denominator. There were bracketing errors, sign errors and the error of omitting '$x_{n+1} =$' to write only the right-hand side of the Newton-Raphson formula.

In part (b), nearly all students wrote down a correct $x_2 = \dfrac{3}{4}$, $x_3 = \dfrac{2}{3}$.

In part (c), only a few students gave an acceptable explanation that detailed why the Newton-Raphson method did not work for this question. Some of these students stated 'because there is a stationary point at $x = 0$, then the method could not work' while others explained that a tangent to the curve at $x = 0$ would never meet the x-axis. The explanation, 'not being able to divide by zero in the formula', by itself, was not sufficient to gain credit in part (c).

Question 6

This was a well-answered question with part (b) and part (c) providing most of the discrimination.

In part (a), nearly all students found $\mathrm{f}(2) = 0$ and many used this result to deduce that $(x - 2)$ was a factor of $\mathrm{f}(x)$. The method of long division was the most popular and successful method to find the quadratic factor of $\mathrm{f}(x)$. Those who used the more time-efficient method of comparing coefficients were slightly more prone to making sign or manipulation errors. Most students arrived at the correct answer $\mathrm{f}(x) = (x-2)(-3x^2 + 2x - 5)$, with some writing the equivalent $\mathrm{f}(x) = (2 - x)(3x^2 - 2x + 5)$. There were a few students who found both correct factors but did not express their final answer as a product of two algebraic factors.

In part (b), many students used part (a) to write down $(y^2 - 2)(-3y^4 + 2y^2 - 5) = 0$ and most deduced the two real solutions $y = \pm\sqrt{2}$. Surprisingly a few students solved $y^2 = 2$ to give $y = \pm\ 2$. A significant number of students stated, without proof, that $(-3x^2 + 2x - 5)$ or $(-3y^4 + 2y^2 - 5)$ where $x = y^2$, had no real solutions. Some students applied '$b^2 - 4ac = (2)^2 - 4(-3)(-5) = -56 < 0$' with an appropriate conclusion to show that $(-3y^4 + 2y^2 - 5) = 0$ had no real solutions. Common errors included just stating '$b^2 - 4ac < 0$' with no reference to a discriminant calculation or the omission of '< 0' after a correct discriminant calculation. Only a minority of students were able to bring the two strands of the proof together to show that the given equation had only two real solutions.

In part (c), some students correctly deduced that the given equation had 3 real solutions for $7\pi \le \theta < 10\pi$, with a few students incorrectly deducing that there were 6 real solutions. Most students misunderstood the requirement of the question by finding $\theta = 23.1,\ 26.2,\ 29.4,$ with those students who did not go on to state that there were 3 real solutions losing the mark for this part.

Question 7

Part (i) was accessible to most students, whereas some students struggled to gain access to part (ii).

In part (i), many students replaced $\sec x$ with $\dfrac{1}{\cos x}$ and arrived at $4\sin x\cos x = 1.$ A few students replaced $4\sin x\cos x$ with $\sin 4x,$ but most used the identity $\sin 2x \equiv 2\sin x\cos x$ to give the correct $\sin 2x = \dfrac{1}{2}.$ Many students obtained both correct exact solutions in radians, with only a few omitting the second solution $\dfrac{5\pi}{12}.$

Some students made no attempt at part (ii) while a significant number of other students did not employ a correct strategy for solving $5\sin\theta - 5\cos\theta = 2.$ The majority of students, however, attempted one of several correct strategies. The most popular strategy was to square both sides of the given equation and apply the identities $\cos^2\theta + \sin^2\theta \equiv 1$ and $\sin 2\theta \equiv 2\sin\theta\cos\theta$ to achieve $\sin 2\theta = \dfrac{21}{25}.$ A second strategy was to square both sides of $5\sin\theta = 2 + 5\cos\theta$ and apply the identity $\cos^2\theta + \sin^2\theta \equiv 1$ to achieve a quadratic equation in either $\cos\theta$ or $\sin\theta.$ In both of these strategies, extra (incorrect) solutions were generated as a result of squaring both sides of their equation. Only a few students rejected these extra solutions to leave the correct solutions θ = 61.4°, 208.6°. A third strategy was to rewrite the given equation as $\sqrt{50}\sin(\theta - 45°) = 2$ or even $\sqrt{50}\cos(\theta + 45°) = -2,$ which were usually solved to give θ = 61.4°, 208.6°. A few students applied other correct strategies which led to solving quadratic equations in either $\tan\theta$ or $\cot\theta.$

Question 8

This question discriminated well between the average and more able students. There was a significant minority of students, however, that made no creditable progress.

In part (a), a minority of students struggled to write down a suitable form of the quadratic model. Some students, who tried to model the height of the rugby ball as a negative quadratic cutting the x-axis at $x=0$ and $x=40$, wrote down either $H=-x(x-40)$ or $H=x(40-x)$, but struggled, due to the lack of a constant, to fit the condition $(20, 12)$ to their model. Those who wrote $H=Ax(40-x)$ usually applied $(20, 12)$ to their model to give the correct $H=\frac{3}{100}x(40-x)$. Other students who adopted a model of the form $H=ax^2+bx+c$ were usually successful in using the conditions $(0, 0)$, $(40, 0)$ and $(20, 12)$ to give the correct $H=-0.03x^2+1.2x$. Only a few students used the model $H=12-\lambda(x-20)^2$ with condition $(40, 0)$ to find a correct $H=12-\frac{3}{100}(x-20)^2$. A few other elaborate methods were seen, including one where $(40, 0)$ and $\left(\frac{-b}{2a}, 12\right)$ were applied to $H=ax^2+bx$, which usually resulted in a correct $H=-0.03x^2+1.2x$.

In part (b), many students substituted $H=3$ into their quadratic model and most applied a correct method for solving their resulting 3-term quadratic equation or their quadratic equation of the form $(x-20)^2=k,\ k\neq 0$. Many students found the largest solution in the range $(0, 40)$ and gave their answer, usually to 3 significant figures, in metres.

Students gave a variety of acceptable limitations for the model in part (c) with many stating that 'the model assumes that there is no wind or air resistance acting on the ball'. No credit was given for answers that discussed what might have happened to the rugby ball after it had hit the ground.

Question 9

This was a well-answered question considering that this was the first time that differentiation from first principles has been tested on the new specification.

Most students wrote down a correct expression for the gradient of the chord joining the points $(\theta, \cos\theta)$ and $(\theta + h, \cos(\theta + h))$ for a small value h. Many students simplified their gradient expression to give a correct $-\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta$ and applied the limiting arguments given in the question to deduce that $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = -\sin\theta$. Some students lost the final mark for not applying limiting arguments correctly. Common errors included: stating that $\frac{\sin h}{h} = 1$, $\frac{\cos h - 1}{h} = 0$; not using a limiting argument; or applying the limiting argument inconsistently. Other students lost the final mark for the use of inconsistent notation in their proof. These students, who usually referred to both x and θ in their proof, wrote incorrect statements such as $\frac{\mathrm{d}}{\mathrm{d}x}(\cos\theta) = -\sin\theta$ or $\mathrm{f}(x) = \cos\theta$.

Question 10

This question discriminated well between the average and more able students. There waere a significant minority of students, however, that made no creditable progress.

In part (a), many students struggled to translate the description of the model into a mathematical statement. Some incorrect statements, such as $\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{1}{r^2}$, $\frac{\mathrm{d}r}{\mathrm{d}t}=r^2$ or $\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{1}{r}$, did not contain a constant of proportionality; while other incorrect statements $\frac{\mathrm{d}r}{\mathrm{d}t}=kr^2$, $\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{r^2}{k}$ or $\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{k}{r}$ contained a constant of proportionality. Many students used a correct method to separate the variables and most proceeded to integrate their $\int r^2\,\mathrm{d}r = \int k\,\mathrm{d}t$ to give an equation linking r and t which contained a constant of integration. At this stage, some students incorrectly integrated r^2 to give $2r$. The majority of students applied the boundary conditions $t=0$, $r=5$ and $t=4$, $r=3$ (or $t=240$, $r=3$) to find both constants and proceeded to give an equation linking r with t. Some students who worked in minutes obtained correct answers of either $r=\sqrt[3]{\frac{250-49t}{2}}$, $r^3=-\frac{49}{2}t+125$ or $t=\frac{250-2r^3}{49}$. A few students who worked in seconds obtained correct answers of either $r=\sqrt[3]{\frac{15000-49t}{120}}$, $r^3=-\frac{49}{120}t+125$ or $t=\frac{15000-120r^3}{49}$. Many students lost the final mark in this part by not including sufficient detail in their definition of their variables. Some students defined r as the radius of the mint, in mm, but it was rare to see the time, t, defined from a correct starting point (i.e. from when the mint was placed in the mouth) and with the correct unit of time used.

In part (b), many students substituted $r=0$ in their equation from part (a) and rearranged to find a value for t, with a significant number obtaining the correct answer, 5 minutes 6 seconds. Some students lost the final mark by giving their final answer as either 5.10 minutes, $\frac{250}{49}$ minutes or 306 seconds.

Part (c) was generally well-answered with a variety of valid limitations given. Students commented on the shape of the mint while it was being sucked; on how the mint was sucked; or that the model predicted a negative radius for the mint after a time of 5 minutes 6 seconds after the mint was placed in the mouth. Some students who stated that ‘the radius of the mint may not decrease at a constant rate’ did not receive any credit because the radius of the mint in the original model was not decreasing at a constant rate.

Question 11

Part (a) was accessible to most students, whereas some students struggled to gain access to part (b).

In part (a), most students used the given $\frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv A+\frac{B}{(x-3)}+\frac{C}{(1-2x)}$ to write down a correct identity $1+11x-6x^2 \equiv A(1-2x)(x-3)+B(1-2x)+C(x-3)$. Some students used a method of long division to deduce that $A=3$ and formulated the correct identity $-10x+10 \equiv B(1-2x)+C(x-3)$. Only a few students wrote down the incorrect identity $1+11x-6x^2 \equiv B(1-2x)+C(x-3)$. Most students used either a method of substituting values into their identity or a method of comparing coefficients with most finding the correct values for *A*, *B* and *C*.

In part (b), a minority of students struggled to make any creditable progress. These students usually evaluated $\mathrm{f}(x)$ for increasing *x*-values and concluded that the corresponding values for $\mathrm{f}(x)$ were continually decreasing. The majority used the chain rule to differentiate $\mathrm{f}(x)$. A common error at this stage was for students to differentiate $-2(1-2x)^{-1}$ to give $-2(1-2x)^{-2}$, while other students tried to apply an incorrect method of showing $\mathrm{f}''(x)<0$. Only a few of the students who obtained a correct $\mathrm{f}'(x) = -4(x-3)^{-2}-4(1-2x)^{-2}$ provided an acceptable explanation detailing why $\mathrm{f}'(x)<0$.

Question 12

This question discriminated well between students of all abilities with part (a) more accessible to students than part (b).

In part (a), most students provided a proof which progressed from one side of the identity to the other by using methodical steps and correct trigonometric identities. There was no preference seen for starting their proof on a particular side of the identity. Some students manipulated both sides of the identity to give a common expression $2\sin^2\theta$, with many providing some indication that their proof was complete.

Part (b) proved to be discriminating, with some students not realising that they could use the result from part (a). Some students substituted $1-\cos 2x$ with $\tan x \sin 2x$ and then proceeded to cancel $\tan x \sin 2x$ from both sides, while other students substituted $\tan x \sin 2x$ with $1-\cos 2x$ and then proceeded to cancel $1-\cos 2x$ from both sides. These two strategies together with applying the identity $\sec^2 x \equiv 1+\tan^2 x$ usually led to a 3-term quadratic in $\tan x$. Most students used a correct method, usually by factorising, to solve their quadratic equation in $\tan x$ but only a few stated all three correct solutions. As a result of cancelling either $\tan x \sin 2x$ or $1-\cos 2x$ from both sides of their equation, many students missed out the correct solution $x=0$. Those students who factorised out these terms were usually more successful in finding $x=0$. Some students did not give their other two solutions in the required form of the exact $x=-\frac{\pi}{4}$ and $x=1.326$ to 3 decimal places.

Question 13

This question, despite being unstructured, proved accessible, with many students scoring at least 7 out of the 10 marks available. Those students who annotated the diagram were generally more successful in formulating a correct strategy to find the area of the region *R*.

Most students differentiated $y = x\ln x$ by using the product rule to give the correct $\frac{\mathrm{d}y}{\mathrm{d}x} = 1 + \ln x$. A few students found the correct $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x}{x} + \ln x$ and simplified this incorrectly to give $\frac{\mathrm{d}y}{\mathrm{d}x} = \ln x$. Many students used a complete method to find the x-value where the normal to C at (e, e) cuts through the x-axis, with the majority finding the correct $x = 3\mathrm{e}$.

Most students integrated $y = x\ln x$ correctly by using integration by parts to give $\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2$. Many students applied the correct limits of e and 1 to obtain a correct $\frac{1}{4}\mathrm{e}^2 + \frac{1}{4}$ with some making a bracketing error when applying the limit of 1 to obtain the incorrect $\frac{1}{4}\mathrm{e}^2 - \frac{1}{4}$.

The majority of students attempted to find the area of the triangle by using the formula $\frac{1}{2}(\text{base})(\text{height})$. While some students applied the formula correctly to give $\frac{1}{2}(3\mathrm{e} - \mathrm{e})(\mathrm{e})$, there was a considerable number that applied a base length of $3\mathrm{e}$ by writing $\frac{1}{2}(3\mathrm{e})(\mathrm{e})$. A few students used the equation of the normal to find the area of the triangle by applying the correct $\int_{\mathrm{e}}^{3\mathrm{e}} \left(-\frac{1}{2}x + \frac{3}{2}\mathrm{e}\right) \mathrm{d}x$, but not many evaluated this to arrive at the correct answer of e^2.

Several incorrect strategies were seen such as trying to evaluate $\int_0^{\mathrm{e}} x\ln x \; \mathrm{d}x$, $\int_1^{3\mathrm{e}} x\ln x \; \mathrm{d}x$ or even $\int_1^{3\mathrm{e}} \left(x\ln x - \left(-\frac{1}{2}x + \frac{3}{2}\mathrm{e}\right)\right) \mathrm{d}x$. Only a minority of students applied the calculation $\frac{1}{4}\mathrm{e}^2 + \frac{1}{4} + \mathrm{e}^2$ to achieve the correct $\frac{5}{4}\mathrm{e}^2 + \frac{1}{4}$.

Question 14

This was the most demanding question on the paper. Parts (a) and (d) were accessible to many students. Most students made some creditable progress in part (b), but many students found it difficult to access the demanding material in part (c).

In part (a), most students evaluated or deduced that there were 90 mice at the start of the study.

In part (b), most students used either the chain rule or the quotient rule in their attempt to find an expression for $\frac{\mathrm{d}N}{\mathrm{d}t}$. There were many incorrect attempts at the chain rule such as $\frac{\mathrm{d}N}{\mathrm{d}t} = -6300\mathrm{e}^{-0.25t}(3+7\mathrm{e}^{-0.25t})^{-2}$ or $\frac{\mathrm{d}N}{\mathrm{d}t} = 1575(3+7\mathrm{e}^{-0.25t})^{-2}$ while the most common incorrect attempt at the quotient rule was $\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{(3+7\mathrm{e}^{-0.25t})(1)+1575\mathrm{e}^{-0.25t}}{(3+7\mathrm{e}^{-0.25t})^2}$. Although some impressive algebra was seen in manipulating a correct $\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{1575\mathrm{e}^{-0.25t}}{(3+7\mathrm{e}^{-0.25t})^2}$ to achieve $\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{N(300-N)}{1200}$, many students were unsuccessful in their attempts at eliminating t or the $\mathrm{e}^{-0.25t}$ term. Some students substituted $N = \frac{900}{3+7\mathrm{e}^{-0.25t}}$ into $\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{N(300-N)}{1200}$ and used algebra to give $\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{1575\mathrm{e}^{-0.25t}}{(3+7\mathrm{e}^{-0.25t})^2}$, and provided some indication that their proof was complete. Some students differentiated N by using implicit differentiation while other students attempted to rearrange the given equation for N to make t the subject before finding an expression for $\frac{\mathrm{d}t}{\mathrm{d}N}$. A few students were successful in applying an alternative method of solving the differential equation $\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{N(300-N)}{1200}$ to give $N = \frac{900}{3+7\mathrm{e}^{-0.25t}}$.

In part (c), most students misinterpreted the question and instead applied a method to find the time T for the maximum value of N, rather than the time T at which the rate of growth $\frac{\mathrm{d}N}{\mathrm{d}t}$ was a maximum. Many students, therefore, used $N = 300$ and manipulated $300 = \frac{900}{3+7\mathrm{e}^{-0.25T}}$ to give $\mathrm{e}^{-0.25T} = 0$. A few students differentiated $\frac{\mathrm{d}N}{\mathrm{d}t}$, with most writing $\frac{\mathrm{d}^2N}{\mathrm{d}t^2}$, which was condoned, and solved the equation $\frac{1}{1200}(300-2N)$ to give $N = 150$. It was rare to see students using the symmetry of the quadratic function $\frac{N(300-N)}{1200}$ to deduce that $N = 150$. Although some students gave their answer to part (c) as $T = 150$, most students who progressed this far solved the equation $150 = \frac{900}{3+7\mathrm{e}^{-0.25T}}$ to give the correct value for T.

In part (d), many students deduced $P = 300$ with only a few students deducing $P = 299$, which was also accepted.

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