Write your name here

Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number | Candidate Number

Further Pure Mathematics F3

Advanced/Advanced Subsidiary

Monday 27 June 2016 – Morning
Time: 1 hour 30 minutes

Paper Reference
WFM03/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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1. The curve C has equation

$$y = 9\cosh x + 3\sinh x + 7x$$

Use differentiation to find the exact x coordinate of the stationary point of C, giving your answer as a natural logarithm.

(6)

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Question 1 continued

Q1

(Total 6 marks)

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2. An ellipse has equation

$$\frac{x^2}{25} + \frac{y^2}{4} = 1$$

The point P lies on the ellipse and has coordinates $(5\cos\theta,\ 2\sin\theta)$, $0 < \theta < \frac{\pi}{2}$

The line L is a normal to the ellipse at the point P.

(a) Show that an equation for L is

$$5x\sin\theta - 2y\cos\theta = 21\sin\theta\cos\theta$$

(5)

Given that the line L crosses the y-axis at the point Q and that M is the midpoint of PQ,

(b) find the exact area of triangle OPM, where O is the origin, giving your answer as a multiple of $\sin 2\theta$

(6)

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Question 2 continued

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Question 2 continued

Q2

(Total 11 marks)

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3. Without using a calculator, find

(a) $\displaystyle\int_{-2}^{1} \frac{1}{x^2 + 4x + 13}\,dx$, giving your answer as a multiple of π,

(5)

(b) $\displaystyle\int_{-1}^{4} \frac{1}{\sqrt{4x^2 - 12x + 34}}\,dx$, giving your answer in the form $p\ln\left(q + r\sqrt{2}\right)$,

where p, q and r are rational numbers to be found.

(7)

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Question 3 continued

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Question 3 continued

Q3

(Total 12 marks)

4.

$$\mathbf{M} = \begin{pmatrix} 1 & k & 0 \\ -1 & 1 & 1 \\ 1 & k & 3 \end{pmatrix}, \text{ where } k \text{ is a constant}$$

(a) Find $\mathbf{M}^{-1}$ in terms of k.

(5)

Hence, given that $k = 0$

(b) find the matrix **N** such that

$$\mathbf{MN} = \begin{pmatrix} 3 & 5 & 6 \\ 4 & -1 & 1 \\ 3 & 2 & -3 \end{pmatrix}$$

(4)

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Question 4 continued

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Question 4 continued

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Question 4 continued

Q4

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5. Given that $y = \text{artanh}(\cos x)$

(a) show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\text{cosec } x$$

(2)

(b) Hence find the exact value of

$$\int_0^{\frac{\pi}{6}} \cos x \; \text{artanh}(\cos x)\, \mathrm{d}x$$

giving your answer in the form $a\ln\left(b + c\sqrt{3}\right) + d\pi$, where a, b, c and d are rational numbers to be found.

(5)

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Question 5 continued

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Question 5 continued

(Total 7 marks)

Q5

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6. The coordinates of the points A, B and C relative to a fixed origin O are (1, 2, 3), (–1, 3, 4) and (2, 1, 6) respectively. The plane Π contains the points A, B and C.

(a) Find a cartesian equation of the plane Π.

(5)

The point D has coordinates (k, 4, 14) where k is a positive constant.

Given that the volume of the tetrahedron $ABCD$ is 6 cubic units,

(b) find the value of k.

(4)

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Question 6 continued

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Question 6 continued

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Question 6 continued

Q6

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7. The curve C has parametric equations

$$x = 3t^4, \quad y = 4t^3, \qquad 0 \leqslant t \leqslant 1$$

The curve C is rotated through 2π radians about the x-axis. The area of the curved surface generated is S.

(a) Show that

$$S = k\pi\int_0^1 t^5(t^2+1)^{\frac{1}{2}}\,\mathrm{d}t$$

where k is a constant to be found.

(4)

(b) Use the substitution $u^2 = t^2 + 1$ to find the value of S, giving your answer in the form $p\pi\left(11\sqrt{2} - 4\right)$ where p is a rational number to be found.

(7)

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Question 7 continued

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Question 7 continued

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Question 7 continued

(Total 11 marks)

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Q7

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8.

$$I_n = \int_0^{\ln 2} \tanh^{2n} x \, dx, \quad n \geqslant 0$$

(a) Show that, for $n \geqslant 1$

$$I_n = I_{n-1} - \frac{1}{2n-1}\left(\frac{3}{5}\right)^{2n-1}$$

(5)

(b) Hence show that

$$\int_0^{\ln 2} \tanh^4 x \, dx = p + \ln 2$$

where p is a rational number to be found.

(5)

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Question 8 continued

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Question 8 continued

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Question 8 continued

Q8

(Total 10 marks)

TOTAL FOR PAPER: 75 MARKS

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