edexcel

# Mark Scheme (Results)

## Summer 2016

## Pearson Edexcel GCE Statistics
## S4

## (6686/01)

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# General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

# PEARSON EDEXCEL GCE MATHEMATICS

## General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\surd$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - $*$ The answer is printed on the paper or ag- answer given
   - $\sqsubset$ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

# June 2016
# 6686 Statistics S4
# Mark Scheme

| Question Number | Scheme | Marks |
|---|---|---|
| 1(a) | $d$ : 5 9 5 2 0 0 8 5 6 6 | M1 |
| | $\bar{d} = \frac{\sum d}{n} = 4.6$ | M1 |
| | $s^2 = \frac{296 - 10 \times 4.6^2}{9} = 9.378$ | M1 |
| | $H_0: \mu_d = 2 \qquad H_1: \mu_d > 2$ | B1 |
| | $t = \pm \frac{4.6 - 2}{\sqrt{\frac{9.378}{10}}} = \pm 2.6848$ | M1 A1 |
| | $t_9(5\%) = \pm$ 1.833… | B1 |
| | There is evidence to reject $H_0$. There is sufficient evidence to support the designers claim. | A1ft |
| | | (8) |
| (b) | The **differences** in weights are **normally** distributed. | B1 |
| | | (1) |
| | **Notes** | **Total 9** |
| (a) | M1 for attempting the $d$s | |
| | M1 for attempting $\bar{d}$ | |
| | M1 for $s_d$ or $s_d^2$ | |
| | B1 for both hypotheses correct in terms of $\mu$ or $\mu_d$ .( allow a defined symbol) | |
| | M1 for attempting the correct test statistic $\frac{\bar{d}}{s_d / \sqrt{10}}$ | |
| | A1 awrt 2.68 | |
| | B1 awrt 1.83 | |
| | A1ft for a correct comment in context | |
| (b) | B1 for a comment that mentions “differences” and “normal” distribution | |

| Question Number | Scheme | Marks |
|---|---|---|
| 2. (a) | $H_0 : \mu = 1.2$ $H_1 : \mu > 1.2$ | B1 |
| | $t_8(5\%) = 1.860$ | B1 |
| | $\bar{m} = 1.28888..$ | B1 |
| | $t = \dfrac{1.28... - 1.2}{\sqrt{\dfrac{0.031111}{9}}} = 1.511$ awrt 1.51 | M1 A1ft A1 |
| | Not significant. There is not sufficient evidence that the mean <u>weight of piglets</u> is greater than 1.2 kg | A1 |
| | | **(7)** |
| (b) | $H_0 : \sigma^2 = 0.09$ $H_1 : \sigma^2 \neq 0.09$ $[H_0 : \sigma = 0.3\ H_1 : \sigma \neq 0.3]$ | B1 |
| | $s^2 = \dfrac{15.2 - 9 \times \left(\dfrac{11.6}{9}\right)^2}{8} = 0.031111$ | B1 |
| | $[\chi^2_8(0.25) = 17.535]$ $\chi^2_8(0.975) = 2.18$ | B1 |
| | Critical region $\dfrac{(n-1)s^2}{\sigma^2} \sim \chi^2_8$ test statistic = 2.7654… awrt 2.77 | M1A1 |
| | 2.77 is not in the critical region. There is no evidence that the standard deviation of the weights of **<u>piglets</u>** is different to 0.3 | A1 |
| | | (6) |
| | **Notes** | **Total 13** |
| (a) | B1 both hypotheses<br>M1 for attempting the correct statistic<br>A1ft follow through their $s^2$<br>A1 awrt 1.51 | |
| (b) | B1 both hypotheses, must be two tail<br>B1 awrt 0.0311<br>B1 NB allow 2.733 for one tail hypotheses. (no hypotheses gains B0)<br>M1 for a correct test statistic<br>NB one tail test can get B0 B1 B1 (2.733)B0 M1 A1 A1 | |

| Question Number | Scheme | Marks |
|---|---|---|
| 3. (a) | $X$ = No of soft centres.<br>$X \sim B(20, 0.5)$<br>Critical region $X \leq 5$ or $X \geq 15$ | **B1B1**<br>(2) |
| (b) | P(Type I error) = P($X \leq 5 \mid p = 0.5$) + P($X \geq 15 \mid p = 0.5$)<br>$= 0.0207 + 0.0207 \quad = 0.0414$ | M1<br>A1<br>(2) |
| (c) | P(Type II error) = P($X < 15 \mid p = 0.25$) – P($X < 6 \mid p = 0.25$)<br>$= 1 - 0.6172 \quad = 0.3828$ | M1<br>A1<br>(2) |
| | **Notes** | **Total 6** |
| (a) | B1 $X \leq 5$<br>B1 $X \geq 15$ | |
| (b) | M1 Adding their two CR together or a correct answer<br>A1 awrt 0.0414 | |
| (c) | M1 FT their CR<br>A1 awrt 0.383 | |

| Question Number | Scheme | Marks |
|---|---|---|
| 4. (a) | Size of test $A = P(Y \leq 2)$ | |
| | $= 0.0547$ | B1 |
| | | (1) |
| (b) | Size of test $B$ = P(Rejecting $H_0 \mid p = 0.5$) | |
| | $= P(X = 0) + (1 - P(X = 0)) \times P(X = 0)$ | M1 |
| | $= 0.5^5 + (1 - 0.5^5)(0.5^5)$ | A1 |
| | $= 0.03125 + (0.96875)(0.03125)$ | |
| | $= 0.0615/0.0614$ | A1 |
| (c) | Power function of test $B$ = P(0 long screws in first 5) + P(0 long screws in second 5\| > 0 long screws in first 5) | (3) |
| | $= P(X = 0 \mid p) + [1 - P(X = 0 \mid p)] P(X = 0 \mid p)$ | M1 |
| | $= (1 - p)^5 + [1 - (1 - p)^5](1 - p)^5$ | A1 |
| | $= 2(1 - p)^5 - (1 - p)^{10}$ | |
| | | (2) |
| (d) | $r = 0.68$ | B1 |
| | | (1) |
| (e) | Test $A$ as it is more powerful for values of $p < 0.4$ | M1 A1 |
| | | (2) |
| | **Notes** | **Total 9** |
| (b) | M1 for a correct expression/selection of probabilities | |
| | A1 for a correct expression in terms of probabilities. Allow 0.0312 + (0.9688)(0.0312) | |
| (c) | M1 for a correct expression | |
| | A1 for a correct expression in terms of $p$ | |
| (e) | M1 for reason based on the power function | |
| | A1 test $A$ | |

| Question Number | Scheme | Marks |
|---|---|---|
| 5. (a) | $H_0 : \sigma^2_X = \sigma^2_Y$ $H_1 : \sigma^2_X \neq \sigma^2_Y$ | B1 |
| | $F_{8,5} = \dfrac{6.76^2}{5.42^2} = 1.556$ | M1A1 |
| | $F_{8,5}$ is 4.82 | B1 |
| | There is evidence that the variances are the same. | A1 |
| | | (5) |
| (b) | $H_0 : \mu_X = \mu_Y + 5$ $H_1 : \mu_X > \mu_Y + 5$ | B1 |
| | $s_p{}^2 = \dfrac{8\times 6.76^2 + 5\times 5.42^2}{13}, = 39.42...$ or $s_p = 6.278...$ | M1 A1 |
| | $(t_{13} =)(\pm)\dfrac{14.8-7.2-5}{s_p\sqrt{\dfrac{1}{9}+\dfrac{1}{6}}} = (\pm)0.78578...$ awrt 0.786 | M1 M1dA1 |
| | Critical value $t_{13}$ (2.5%) = 1.771 | B1 |
| | There is no evidence to Reject $H_0$ | |
| | There is evidence that the **fire brigade** in *X* does not take more than 5 minutes longer than those in *Y*. | A1cso |
| | | (8) |
| (c) | Test in part (b) requires the variances to be equal. The test in part (a) showed that the variances could be assumed to be equal. | B1 |
| | | (1) |
| | **notes** | **Total 14** |
| (a) | B1 both hypotheses | |
| | M1 Allow use of 6.76 and 5.42 instead of $6.76^2$ and $5.42^2$ | |
| | A1 awrt 1.56 | |
| (b) | B1 both hypotheses | |
| | M1 allow use of 6.76 and 5.42 instead of $6.76^2$ and $5.42^2$ | |
| | A1 awrt 39.4 or 6.28 | |
| | B1 allow p value 0.650 instead of critical value | |
| | M1 use of correct formula with their $S_p$ – condone missing 5 | |
| | M1 use of correct formula with their $S_p$ | |

| Question Number | Scheme | Marks |
|---|---|---|
| 6.(a) | $\mathrm{E}(Y) = 2\mathrm{E}\left(\bar{X}\right)$ | |
| | $= 2\times\frac{a}{2}$ | M1 |
| | $= a$ | A1cso (2) |
| (b) | $\mathrm{E}(M) = \int_0^a \frac{nm^n}{a^n}\,\mathrm{d}m$ | M1 |
| | $= \left[\frac{nm^{n+1}}{a^n(n+1)}\right]_0^a$ | |
| | $= \frac{na}{n+1}$ | A1 (2) |
| (c) | $\mathrm{Var}(M) = \int_0^a \frac{nm^{n+1}}{a^n}\,\mathrm{d}m - \left(\frac{na}{n+1}\right)^2$ | M1A1 |
| | $= \left[\frac{nm^{n+2}}{a^n(n+2)}\right]_0^a - \frac{n^2a^2}{(n+1)^2}$ | M1d |
| | $= na^2\left(\frac{(n+1)^2 - n(n+2)}{(n+1)^2(n+2)}\right)$ | |
| | $= \frac{na^2}{(n+2)(n+1)^2}$ | A1cso (4) |
| (d) | $\mathrm{E}(S) = \frac{n+1}{n}\mathrm{E}(M) = \frac{n+1}{n}\times\frac{na}{n+1} = a$ | B1 |
| | $\mathrm{Var}(S) = \left(\frac{n+1}{n}\right)^2 \frac{na^2}{(n+2)(n+1)^2} = \frac{a^2}{n(n+2)}$ | B1 |
| | $\mathrm{Var}(Y) = 4\,\mathrm{Var}\left(\bar{X}\right)$ | M1 |
| | $= 4\times\frac{a^2}{12n}$ | |
| | $= \frac{a^2}{3n}$ | A1 |
| | As $n>1$ $n(n+2) > 3n$; therefore $\mathrm{Var}(S) < \mathrm{Var}(Y)$ | M1;M1 |
| | $\therefore$ $S$ is the better estimator | A1cso (7) |
| | | **Total 15** |

| | notes | |
|---|---|---|
| (a) | M1 for $2\mathrm{E}\left(\bar{X}\right)$<br><br>A1 For $2\times\frac{a}{2}$ leading to $a$ | |
| (b) | M1 attempting to integrate correct expression | |
| (c) | M1 for attempting to integrate a correct expression for E($X^2$)<br>A1 correct E($X^2$)<br>M1d dependent on previous M mark, using correct formula for Var($M$) | |
| (d) | B1 for $\frac{n+1}{n}\mathrm{E}(M)=a$ or $\frac{n+1}{n}\times\frac{na}{n+1}=a$<br><br>M1 using 4 Var $\left(\bar{X}\right)$<br><br>NB Failure to show $S$ is unbiased gains a maximum of 5/7 lose first B1 and final A1 | |

| Question Number | Scheme | Marks |
|---|---|---|
| 7 | $\bar{x} - 2.262\frac{s}{\sqrt{10}} = 28.5$ | B1 |
| | | M1 |
| | $\bar{x} + 2.262\frac{s}{\sqrt{10}} = 48.7$ | A1 |
| | $2\bar{x} = 48.7 + 28.5$ or $2.262\frac{s}{\sqrt{10}} = \frac{1}{2}(48.7 - 28.5)$ | M1 |
| | $s = 14.1198...$ ($s^2 = 199.36$) | A1 |
| | $\left\{\frac{9\left(14.1198^2\right)}{23.589}, \frac{9\left(14.1198^2\right)}{1.735}\right\}$ | M1 |
| | | B1 B1 |
| | = ( 76.0659…, 1034.19…) | A1 (9) |
| | **notes** | **Total 9** |

B1 awrt 2.262

M1 $\bar{x} - t\ value\frac{s}{\sqrt{10}} = 28.5$

A1 both equations correct

M1 solving simultaneous leading to a value for $\bar{x}$ or $s$

A1 awrt 14.1 or awrt 199

M1 $\frac{9\left(s^2\right)}{\chi^2 value}$

B1 23.589

B1 1.735

A1 awrt 76.1 and awrt 1030

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