Mark Scheme (Results)

Summer 2016

Pearson Edexcel GCE in Further Pure Mathematics 3 (6669/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- d… or dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper or ag- answer given
- $\llcorner$ or d… The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x = ...

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x = ...

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Notes	Marks
1.	$\mathbf{A}=\begin{pmatrix}-2 & 1 & -3\\ k & 1 & 3\\ 2 & -1 & k\end{pmatrix}$		
	$\det\mathbf{A}=-2(k+3)-(k^2-6)-3(-k-2)\ row1$ or e.g. $\det\mathbf{A}=-k(k-3)+(-2k+6)-3(2-2)\ row2$ $\det\mathbf{A}=2(3+3)+(-6+3k)+k(-2-k)\ row3$ $\det\mathbf{A}=-2(k+3)-k(k-3)+2(3+3)\ col1$ $\det\mathbf{A}=-(k^2-6)+(-2k+6)+(-6+3k)\ col2$ $\det\mathbf{A}=-3(-2-k)-3(2-2)+k(-2-k)\ col3$	M1: Correct attempt at determinant (3 'elements' (may be implied if one is zero) with at least two elements correct). **Note that there are various alternatives depending on the choice of row or column.** A1: Correct determinant **in any form**	M1A1
	Note that e.g. $\det\mathbf{A}=-2\begin{vmatrix}1 & 3\\ -1 & k\end{vmatrix}-\begin{vmatrix}k & 3\\ 2 & k\end{vmatrix}-3\begin{vmatrix}k & 1\\ 2 & -1\end{vmatrix}$ scores no marks until the determinants are 'extracted'.		
	$-2(k+3)-(k^2-6)-3(-k-2)=0\Rightarrow k=...$	Sets their det**A** = 0 (= 0 may be implied) and attempts to solve a 3 term quadratic (see general guidance) as far as k = ... NB Correct quadratic is $k^2-k-6=0$	M1
	$(k+2)(k-3)=0\Rightarrow k=-2,\ 3$	Both values correct	A1
			(4)
			Total 4

Question Number	Scheme	Notes	Marks
2.	$y=\frac{x^2}{8}-\ln x,\quad 2\le x\le 3$		
	$\frac{dy}{dx}=\frac{x}{4}-\frac{1}{x}$	Correct derivative. Allow any correct equivalent e.g. $\frac{2x}{8}-\frac{1}{x}$	B1
	$L=\int\sqrt{\left(1+\left(\frac{dy}{dx}\right)^2\right)}\,dx=\int\sqrt{\left(1+\left(\frac{x}{4}-\frac{1}{x}\right)^2\right)}\,dx$	Use of a correct formula using their derivative and not the given *y*.	M1
	$=\int\sqrt{\left(1+\frac{x^2}{16}-\frac{1}{2}+\frac{1}{x^2}\right)}\,dx=\int\sqrt{\left(\frac{x^2}{16}+\frac{1}{2}+\frac{1}{x^2}\right)}\,dx=\int\sqrt{\left(\frac{x}{4}+\frac{1}{x}\right)^2}\,dx=\int\left(\frac{x}{4}+\frac{1}{x}\right)dx$ M1: Squares their derivative to obtain $ax^2+bx^{-2}+c$, where none of *a*, *b* or *c* are zero – this may be implied by e.g. $\frac{ax^4+bx^2+c}{dx^2}$ **and** adds 1 to their constant term. A1: Correct integrand $\frac{x}{4}+\frac{1}{x}$ or equivalent e.g. $\frac{x^2+4}{4x}$ (integral sign not needed)		M1 A1
	$=\frac{x^2}{8}+\ln kx$	Correct integration	A1
	$\left[\frac{x^2}{8}+\ln x\right]_2^3=\left(\frac{3^2}{8}+\ln 3\right)-\left(\frac{2^2}{8}+\ln 2\right)$	Substitutes 2 and 3 into an expression of the form $px^2+q\ln x\ (p,q\ne 0)$ and subtracts the right way round. Must be seen explicitly or may be implied by a correct exact answer for their integration. If the candidate gives the **final single answer** in decimals with no substitution shown, e.g. 1.030…this is M0.	M1
	$\frac{5}{8}+\ln\frac{3}{2}$	Cao and cso (oe e.g $0.625+\ln\frac{3}{2}$)	A1
			(7)
			Total 7

Question Number	Scheme	Notes	Marks
3(a)	$y = \text{arcoth}\, x \Rightarrow \coth y = x$ or e.g. $u = \text{arcoth}\, x \Rightarrow \coth u = x$	Changes from arcoth to coth correctly. This may be implied by e.g. $\tanh y = \frac{1}{x}$	B1
	$x = \frac{\cosh y}{\sinh y} \Rightarrow \frac{dx}{dy} = \frac{\sinh^2 y - \cosh^2 y}{\sinh^2 y}\left(= -\frac{1}{\sinh^2 y}\right)$	Uses $\coth y = \frac{\cosh y}{\sinh y}$ and attempts product or quotient rule	M1
	$\frac{dx}{dy} = -\text{cosech}^2 y = 1 - \coth^2 y$ $\Rightarrow \frac{dy}{dx} = \frac{1}{1-\coth^2 y} = \frac{1}{1-x^2}$ *	Correct completion with no errors seen and an intermediate step shown.	A1*
			(3)
	(a) Alternative 2		
	$y = \text{arcoth}\, x \Rightarrow \coth y = x$	Changes from arcoth to coth correctly. This may be implied by e.g. $\tanh y = \frac{1}{x}$	B1
	$-\text{cosech}^2 y \frac{dy}{dx} = 1$ or $-\text{cosech}^2 y = \frac{dx}{dy}$ $\left(\Rightarrow \frac{dy}{dx} = -\frac{1}{\text{cosech}^2 y}\right)$	$\pm\text{cosech}^2 y \frac{dy}{dx} = 1$ or $\pm\text{cosech}^2 y = \frac{dx}{dy}$	M1
	$\coth^2 y - 1 = \text{cosech}^2 y \Rightarrow \frac{dy}{dx} = \frac{1}{1-x^2}$ *	Correct completion with no errors seen and an intermediate step shown.	A1*
	(a) Alternative 3		
	$y = \text{arcoth}\, x \Rightarrow \coth y = x$	Changes from arcoth to coth correctly This may be implied by e.g. $\tanh y = \frac{1}{x}$	B1
	$x = \coth y = \frac{e^{2y}+1}{e^{2y}-1} \Rightarrow \frac{dx}{dy} = \frac{(e^{2y}-1)2e^{2y} - (e^{2y}+1)2e^{2y}}{(e^{2y}-1)^2}$	Expresses coth$y$ in terms of exponentials and differentiates	M1
	$\frac{dx}{dy} = \frac{-4e^{2y}}{(e^{2y}-1)^2} \Rightarrow \frac{dy}{dx} = \frac{e^{4y}-2e^{2y}+1}{-4e^{2y}} = \frac{e^{2y}-2+e^{-2y}}{-4} = -\left(\frac{e^{y}-e^{-y}}{2}\right)^2 = -\sinh^2 y = -\frac{1}{\text{cosech}^2 y}$ $= \frac{1}{1-\coth^2 y} = \frac{1}{1-x^2}$ * Completes correctly with no errors		A1*

	(a) Alternative 4		
	$y=\text{arcoth}\,x \Rightarrow \coth y = x$	Changes from arcoth to coth correctly This may be implied by e.g. $\tanh y = \frac{1}{x}$	B1
	$x=\coth y=\frac{e^y+e^{-y}}{e^y-e^{-y}} \Rightarrow \frac{dx}{dy}=\frac{\left(e^y-e^{-y}\right)^2-\left(e^y+e^{-y}\right)^2}{\left(e^y-e^{-y}\right)^2}$	Expresses coth*y* in terms of exponentials and differentiates	M1
	$\frac{dx}{dy}=\frac{-4}{\left(e^y-e^{-y}\right)^2} \Rightarrow \frac{dy}{dx}=-\frac{1}{\text{cosech}^2 y}$		
	$\coth^2 y - 1 = \text{cosech}^2 y \Rightarrow \frac{dy}{dx}=\frac{1}{1-x^2}$ *	Correct completion with no errors seen and an intermediate step shown.	A1*

	(a) Alternative 5		
	$y=\text{arcoth}\,x=\frac{1}{2}\ln\left(\frac{1+x}{x-1}\right)$	**Correct** ln form for arcoth	B1
	$\frac{dy}{dx}=\frac{1}{2}\left[\frac{x-1}{x+1}\times\frac{(x-1)-(x+1)}{(x-1)^2}\right]$ or $\frac{1}{2}\ln\left(\frac{1+x}{x-1}\right)=\frac{1}{2}\ln(1+x)-\frac{1}{2}\ln(x-1)$ $\Rightarrow \frac{dy}{dx}=\frac{1}{2(x+1)}-\frac{1}{2(x-1)}$	Attempts to differentiate using the chain rule and quotient rule or writes as two logarithms and differentiates.	M1
	$\frac{dy}{dx}=\frac{1}{1-x^2}$	Correct completion with no errors seen.	A1
	Note that use of $\text{arcoth}\,x=\frac{1}{\text{artanh}x}\left(=\frac{1}{\frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)}\right)$ **scores no marks**		

	(a) Alternative 6		
	$y=\text{arcoth}\,x \Rightarrow \coth y = x$	Changes from arcoth to coth correctly This may be implied by e.g. $\tanh y=\frac{1}{x}$	B1
	$\tanh y=\frac{1}{x} \Rightarrow -\frac{1}{x^2}=\text{sech}^2\,y\frac{\mathrm{d}y}{\mathrm{d}x}$	$\pm\frac{1}{x^2}=\pm\text{sech}^2\,y\frac{\mathrm{d}x}{\mathrm{d}y}$	M1
	$-\frac{1}{x^2}=\left(1-\frac{1}{x^2}\right)\frac{\mathrm{d}y}{\mathrm{d}x} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{1-x^2}$	Correct completion with no errors seen.	A1*
	(a) Alternative 7		
	$y=\text{arcoth}\,x=\text{artanh}\left(\frac{1}{x}\right)$	Expresses arcoth in terms of artanh correctly	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{1-\left(\frac{1}{x}\right)^2}\times -x^{-2}$	Differentiates using the chain rule	M1
	$=\frac{-1}{x^2-1}=\frac{1}{1-x^2}$	Correct completion with no errors seen.	A1*

(b)	$y=(\text{arcoth}\,x)^2 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=2(\text{arcoth}\,x)\times\frac{1}{1-x^2}$	Correct first derivative	B1
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{2}{1-x^2}(1-x^2)^{-1}+4x\text{arcoth}\,x\times(1-x^2)^{-2}$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{2(1-x^2)\times\frac{1}{1-x^2}+2\text{arcoth}\,x\times 2x}{(1-x^2)^2}\left(=\frac{4x\text{arcoth}x+2}{(1-x^2)^2}\right)$		M1A1
	M1: Attempts product or quotient rule on an expression of the form $\frac{k\text{arcoth}\,x}{1-x^2}$ Product rule requires $\pm P(1-x^2)^{-2}\pm Qx\,\text{arcoth}x\,(1-x^2)^{-2}$ oe Quotient rule requires $\frac{\pm P\pm Qx\,\text{arcoth}x}{(1-x^2)^2}$ oe		
	$(1-x^2)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-2x\frac{\mathrm{d}y}{\mathrm{d}x}=(1-x^2)\left(\frac{4x\text{arcoth}x+2}{(1-x^2)^2}\right)-2x\times\left(\frac{2\text{arcoth}\,x}{1-x^2}\right)$ or $(1-x^2)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{2}{1-x^2}+2x\times\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)$		M1
	M1: Substitutes their first and second derivatives into the lhs of the differential equation **or** multiplies through by $(1-x^2)$ and replaces $2(\text{arcoth}\,x)\times\frac{1}{1-x^2}$ by $\frac{\mathrm{d}y}{\mathrm{d}x}$		
	$(1-x^2)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-2x\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2}{1-x^2}$	Correct conclusion with no errors	A1cso
			(5)

	(b) Alternative 1		
	$y=(\text{arcoth}\,x)^2 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=2(\text{arcoth}\,x)\times\frac{1}{1-x^2}$	Correct first derivative	B1
	$(1-x^2)\frac{\mathrm{d}y}{\mathrm{d}x}=2\text{arcoth}\,x\Rightarrow(1-x^2)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-2x\frac{\mathrm{d}y}{\mathrm{d}x}=...$	M1: Multiplies through by $1-x^2$ and attempts product rule on $(1-x^2)\frac{\mathrm{d}y}{\mathrm{d}x}$. Requires $(1-x^2)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\pm Px\frac{\mathrm{d}y}{\mathrm{d}x}$ oe A1: Correct differentiation	M1A1
	$\frac{\mathrm{d}(2\text{arcoth}\,x)}{\mathrm{d}x}=\frac{2}{1-x^2}$	Differentiates rhs using the result from part (a)	M1
	$(1-x^2)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-2x\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2}{1-x^2}$	Correct conclusion with no errors	A1cso

	(b) Alternative 2		
	$y=(\text{arcoth}\,x)^2 \Rightarrow y^{\frac{1}{2}}=\text{arcoth}\,x \Rightarrow \frac{1}{2}y^{-\frac{1}{2}}\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{1-x^2}$	Correct differentiation	B1
	$\frac{1}{2}y^{-\frac{1}{2}}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-\frac{1}{4}y^{-\frac{3}{2}}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=\frac{2x}{(1-x^2)^2}$	M1: Correct use of product rule to give $py^{-\frac{1}{2}}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-qy^{-\frac{3}{2}}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ A1: $\frac{1}{2}y^{-\frac{1}{2}}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-\frac{1}{4}y^{-\frac{3}{2}}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=\frac{2x}{(1-x^2)^2}$	M1A1
	Then substitute as before to obtain $\frac{2}{1-x^2}$		M1A1cso
			Total 8

Question Number	Scheme	Notes	Marks
4(i)	$15+2x-x^2=16-(x-1)^2$	Correct completion of the square. Allow e.g. $15+2x-x^2=-\left[(x-1)^2-16\right]$ Allow 4^2 for 16	B1
	$\int \frac{1}{\sqrt{16-(x-1)^2}}\,dx=\arcsin\left(\frac{x-1}{4}\right)$	M1: $k\arcsin(f(x))$ A1: Correct integration	M1A1
	$\left[\arcsin\left(\frac{x-1}{4}\right)\right]_3^5=\arcsin 1-\arcsin\frac{1}{2}$	Correct use of correct limits	**d**M1
	$=\frac{\pi}{3}$		A1
	May see:		
	$15+2x-x^2=16-(1-x)^2$	Correct completion of the square. Allow e.g. $15+2x-x^2=-\left[(1-x)^2-16\right]$ Allow 4^2 for 16	B1
	$\int \frac{1}{\sqrt{16-(1-x)^2}}\,dx=-\arcsin\left(\frac{1-x}{4}\right)$	M1: $k\arcsin(f(x))$ A1: Correct integration	M1A1
	$\left[-\arcsin\left(\frac{1-x}{4}\right)\right]_3^5=-\arcsin(-1)+\arcsin\left(-\frac{1}{2}\right)$	Correct use of correct limits	**d**M1
	$=\frac{\pi}{3}$		A1
	By substitution 1:		
	$15+2x-x^2=16-(x-1)^2$	Correct completion of the square. Allow e.g. $15+2x-x^2=-\left[(1-x)^2-16\right]$ Allow 4^2 for 16	B1
	$x-1=4\sin\theta \Rightarrow \int \frac{1}{\sqrt{16-(x-1)^2}}\,dx=\int \frac{1}{\sqrt{16-(4\sin\theta)^2}}4\cos\theta\,d\theta$		
	$=\int d\theta=\theta$	M1: A full substitution leading to $k\theta$ or $k\times$ their variable A1: Correct integration	M1A1
	$[\theta]_{\frac{\pi}{6}}^{\frac{\pi}{2}}=\frac{\pi}{2}-\frac{\pi}{6}$	Correct use of correct limits	**d**M1
	$=\frac{\pi}{3}$		A1

	By substitution 2:		
	$15+2x-x^2=16-(x-1)^2$	Correct completion of the square. Allow e.g. $15+2x-x^2=-\left[(1-x)^2-16\right]$ Allow 4^2 for 16	B1
	$x-1=u\Rightarrow\int\frac{1}{\sqrt{16-(x-1)^2}}\mathrm{d}x=\int\frac{1}{\sqrt{16-u^2}}\mathrm{d}u$		
	$\int\frac{1}{\sqrt{16-u^2}}\mathrm{d}x=\arcsin\left(\frac{u}{4}\right)$	M1: $k\arcsin(f(u))$ A1: Correct integration	M1A1
	$\left[\arcsin\left(\frac{u}{4}\right)\right]_2^4=\arcsin 1-\arcsin\frac{1}{2}$	Correct use of correct limits	**d**M1
	$=\frac{\pi}{3}$		A1
	By substitution 3:		
	$15+2x-x^2=16-(x-1)^2$	Correct completion of the square. Allow e.g. $15+2x-x^2=-\left[(1-x)^2-16\right]$ Allow 4^2 for 16	B1
	$x-1=4\cos\theta\Rightarrow\int\frac{1}{\sqrt{16-(x-1)^2}}\mathrm{d}x=\int\frac{1}{\sqrt{16-(4\cos\theta)^2}}-4\sin\theta\,\mathrm{d}\theta$		
	$=\int-\mathrm{d}\theta=-\theta$	M1: A full substitution leading to $k\theta$ or $k\times$ their variable A1: Correct integration	M1A1
	$[-\theta]_{\frac{\pi}{3}}^{0}=0+\frac{\pi}{3}$	Correct use of correct limits	**d**M1
	$=\frac{\pi}{3}$		A1
			(5)
(ii)(a)	$5\cosh x-4\sinh x=5\left(\frac{e^x+e^{-x}}{2}\right)-4\left(\frac{e^x-e^{-x}}{2}\right)$	Substitutes correct exponential forms	B1
	$=\frac{e^x+9e^{-x}}{2}$ or $\frac{e^x}{2}+\frac{9e^{-x}}{2}$	Expands and collects terms in e^x and e^{-x}	M1
	$=\frac{e^{2x}+9}{2e^x}$ *	Correct completion with no errors	A1*
	More working may be shown but allow e.g. $\frac{e^x+9e^{-x}}{2}=\frac{e^{2x}+9}{2e^x}$ or $\frac{e^x}{2}+\frac{9e^{-x}}{2}=\frac{e^{2x}+9}{2e^x}$		
			(3)

(b)	$u = e^x \Rightarrow \frac{du}{dx} = e^x$	Correct derivative. Allow equivalents e.g. $\frac{dx}{du} = \frac{1}{u}$, $du = e^x dx$	B1
	$\int \frac{2e^x}{e^{2x}+9}\,dx = \int \frac{2u}{u^2+9}.\frac{du}{u}$	Complete substitution into $\int \frac{2e^x}{e^{2x}+9}\,dx$. Condone omission of "d$u$" provided the substitution is otherwise complete apart from this. May be implied by e.g. $\int \frac{2}{u^2+9}\,du$	M1
	$= \frac{2}{3}\arctan\left(\frac{u}{3}\right)(+c)$	$k$arctan(f(u)) only. **Dependent on the first method mark.**	dM1
	$= \frac{2}{3}\arctan\left(\frac{e^x}{3}\right)(+c)$	Cao (+c not required)	A1
			(4)
			Total 12

Question Number	Scheme	Notes	Marks
5	$\dfrac{x^2}{16}-\dfrac{y^2}{9}=1 \quad P(4\sec\theta, 3\tan\theta)$		
(a)	$\dfrac{dy}{dx}=\dfrac{3\sec^2\theta}{4\sec\theta\tan\theta}\left(=\dfrac{3}{4\sin\theta}\right)$ or $\dfrac{2x}{16}-\dfrac{2y}{9}\dfrac{dy}{dx}=0\Rightarrow\dfrac{dy}{dx}=\dfrac{8\sec\theta}{16}\times\dfrac{9}{6\tan\theta}$ or $y=3\left(\dfrac{x^2}{16}-1\right)^{\frac{1}{2}}\Rightarrow\dfrac{dy}{dx}=\dfrac{3}{2}\left(\dfrac{x^2}{16}-1\right)^{-\frac{1}{2}}\dfrac{x}{8}$ $=\dfrac{3}{2}\left(\dfrac{(4\sec\theta)^2}{16}-1\right)^{-\frac{1}{2}}\dfrac{4\sec\theta}{8}$	M1: Correct gradient method. Finds $\dfrac{dy}{d\theta}=p\sec^2\theta$ and $\dfrac{dx}{d\theta}=q\sec\theta\tan\theta$ and uses $\dfrac{dy}{dx}=\dfrac{dy}{d\theta}\times\dfrac{d\theta}{dx}$ or differentiates implicitly to give $px+qy\dfrac{dy}{dx}=0$ and substitutes for *y* and *x* to find $\dfrac{dy}{dx}$ or differentiates explicitly to give $\dfrac{dy}{dx}=px\left(qx^2-1\right)^{-\frac{1}{2}}$ and substitutes for *x* A1: Correct derivative in terms of trig. functions, e.g. $\dfrac{3\sec^2\theta}{4\sec\theta\tan\theta}, \dfrac{8\sec\theta}{16}\times\dfrac{9}{6\tan\theta}$ Does not need to be simplified.	M1 A1
	Normal gradient $-\dfrac{4\sin\theta}{3}$	Correct perpendicular gradient rule. Does not need to be simplified.	M1
	$y-3\tan\theta=-\dfrac{4\sin\theta}{3}(x-4\sec\theta)$	Correct straight line method using a gradient (does not need to be simplified) in terms of θ that has come from calculus and is not the tangent gradient. If they use $y = mx + c$ then they must reach as far as finding *c*.	M1
	$3y+4x\sin\theta=25\tan\theta$*	Correct proof with no errors and one intermediate step from the previous line. Allow $25\tan\theta=3y+4x\sin\theta$	A1*
			(5)

(b)	$b^2 = a^2\left(e^2 - 1\right) \Rightarrow 9 = 16\left(e^2 - 1\right) \Rightarrow e = \frac{5}{4}$	M1: Use of the correct eccentricity formula to obtain a value for *e* A1: Correct value for *e*. Ignore ±	M1A1
	$x = \frac{a}{e} \Rightarrow x = \frac{16}{5}$ or $\frac{4}{5/4}$ etc.	Correct value for $\frac{a}{e}$ Ignore ±	A1
	$\theta = \frac{\pi}{4},\ x = \frac{16}{5} \Rightarrow 3y + 2\sqrt{2} \times \frac{16}{5} = 25$	Substitutes $\theta = \frac{\pi}{4}$ into the given normal equation and uses their **positive** directrix equation to obtain an equation in *y* or in *y* and e only.	M1
	$y = \frac{25}{3} - \frac{32}{15}\sqrt{2}$	B1: $a = \frac{25}{3}$ oe or $b = -\frac{32}{15}$ oe B1: $a = \frac{25}{3}$ oe and $b = -\frac{32}{15}$ oe	B1B1 (A marks on EPEN)
	Special Case: If the correct form of the answer is never seen but it appears correctly as a single fraction, allow B1B0 e.g. $y = \frac{125 - 32\sqrt{2}}{15}$		
			(6)
			Total 11

Question Number	Scheme	Notes	Marks
6(a)	$\begin{pmatrix} p & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & q \end{pmatrix}\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} = \lambda\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} p-\lambda & -2 & 0 \\ -2 & 6-\lambda & -2 \\ 0 & -2 & q-\lambda \end{pmatrix}\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$	This statement is sufficient for this mark. May be implied by one correct equation e.g. $2p+4=2\lambda$, $-4-12-2=-2\lambda$, $4+q=\lambda$	M1
	$-4-12-2=-2\lambda \Rightarrow \lambda = 9$	M1: Compares *y*-components to obtain a value for λ. Note that $-4-12-2=-2\lambda$ leading to a value for λ scores both method marks. If working is not clear, at least 2 terms of $"-4-12-2"$ should be correct. A1: Correct eigenvalue	M1A1
			(3)
(b)	$\lambda = 9 \Rightarrow 2p+4=18 \Rightarrow p=7$ $\lambda = 9 \Rightarrow 4+q=9 \Rightarrow q=5$	M1: Uses their eigenvalue to form an equation in *p* or *q* A1: Either *p* = 7 or *q* = 5 A1: Both *p* = 7 and *q* = 5	M1A1A1
			(3)
(c)	$\begin{pmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 6\begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \begin{aligned} 7x-2y&=6x \\ -2x+6y-2z&=6y \\ -2y+5z&=6z \end{aligned}$		M1
	Uses the eigenvalue 6 and their value of *p* or *q* correctly to obtain at least 2 equations.		
	$\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ or e.g. $\begin{pmatrix} 1 \\ \frac{1}{2} \\ -1 \end{pmatrix}$	This vector or any multiple of this vector.	A1
	Note that an eigenvector can be found from the cross product of any 2 rows of $\mathbf{M}-6\mathbf{I}$ e.g. $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 0 & -2 \\ 0 & -2 & -1 \end{vmatrix} = \begin{pmatrix} -4 \\ -2 \\ 4 \end{pmatrix}$		
			(2)

(d)	$\mathbf{P} = \begin{pmatrix} 2 & "2" & 1 \\ -2 & "1" & 2 \\ 1 & "-2" & 2 \end{pmatrix}$	Correct ft **P**. This should be a matrix of eigenvectors two of which are given in the question together with their eigenvector found from part (c). If an attempt is made to normalise the eigenvectors then allow the ft if slips are made when normalising.	B1ft
	$\mathbf{D} = \begin{pmatrix} "9" & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 3 \end{pmatrix}$	Forms the matrix **D** by writing the eigenvalues 6, 3 and their λ on the leading diagonal and zeros elsewhere **or** attempts to calculate $\mathbf{P}^{\mathrm{T}}\mathbf{MP}$ to obtain a single 3 by 3 matrix. Consistency not needed for this mark.	M1
	$\left(\mathbf{P} = \frac{1}{3}\begin{pmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 9 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 3 \end{pmatrix}\right)$ or $\left(\mathbf{P} = \begin{pmatrix} 2 & -2 & 1 \\ -2 & -1 & 2 \\ 1 & 2 & 2 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 81 & 0 & 0 \\ 0 & 54 & 0 \\ 0 & 0 & 27 \end{pmatrix}\right)$ Fully correct and consistent matrices		A1
	Note that the answers to part (d) may be implied e.g. $\mathbf{D} = \mathbf{P}^{\mathrm{T}}\mathbf{MP} = \begin{pmatrix} 2 & -2 & 1 \\ -2 & -1 & 2 \\ 1 & 2 & 2 \end{pmatrix}\begin{pmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{pmatrix}\begin{pmatrix} 2 & -2 & 1 \\ -2 & -1 & 2 \\ 1 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 81 & 0 & 0 \\ 0 & 54 & 0 \\ 0 & 0 & 27 \end{pmatrix}$ Would score all 3 marks by implication.		
			(3)
			Total 11

Question Number	Scheme	Notes	Marks
7(a)	$\dfrac{\sin nx}{\sin x}-\dfrac{\sin(n-2)x}{\sin x}=\dfrac{\sin nx-\sin nx\cos 2x+\cos nx\sin 2x}{\sin x}$ Expands $\sin(n-2)x$ correctly		M1
	$=\dfrac{\sin nx-\sin nx\left(1-2\sin^2 x\right)+2\sin x\cos x\cos nx}{\sin x}$ Replaces $\cos 2x$ and $\sin 2x$ by the correct trigonometric identities		M1
	$=2\sin nx\sin x+2\cos nx\cos x$		
	$=2\cos(n-1)x$		
	$(\therefore I_n-I_{n-2})=\int 2\cos(n-1)x\,\mathrm{d}x$ *	Correct completion with no errors. The I_n-I_{n-2} does not need to be seen explicitly but $\int 2\cos(n-1)x\,\mathrm{d}x$ must seen, including the integral sign.	A1*
			(3)
	(a) Way 2 (factor formula)		
	$\dfrac{\sin nx}{\sin x}-\dfrac{\sin(n-2)x}{\sin x}=\dfrac{2\cos\left(\frac{nx+nx-2x}{2}\right)\sin\left(\frac{nx-nx+2x}{2}\right)}{\sin x}$ Use of the correct factor formula		M1
	$=\dfrac{2\cos(nx-x)\sin x}{\sin x}$	Attempts to replaces $nx+nx-2x$ with $2nx-2x$ and attempts to replace $nx-nx+2x$ with $2x$	M1
	$=2\cos(n-1)x$		
	$(I_n-I_{n-2})=\int 2\cos(n-1)x\,\mathrm{d}x$ *	Correct completion with no errors. The I_n-I_{n-2} does not need to be seen explicitly but $\int 2\cos(n-1)x\,\mathrm{d}x$ must seen, including the integral sign.	A1*
	(a) Way 3		
	$I_n=\int\dfrac{\sin((n-1)x+x)}{\sin x}\,\mathrm{d}x$	Uses $\sin nx=\sin((n-1)x+x)$	M1
	$=\int\dfrac{\sin(n-1)x\cos x+\sin x\cos(n-1)x}{\sin x}\,\mathrm{d}x$	Expands $\sin((n-1)x+x)$ correctly	M1
	$=\dfrac{1}{2}\int\dfrac{\sin nx+\sin(n-2)x}{\sin x}\,\mathrm{d}x+\int\cos(n-1)x\,\mathrm{d}x$		
	$=\dfrac{1}{2}I_n+\dfrac{1}{2}I_{n-2}+\int\cos(n-1)x\,\mathrm{d}x$		
	$\therefore I_n-I_{n-2}=\int 2\cos(n-1)x\,\mathrm{d}x$ *	Correct completion with no errors.	A1*

	(a) Way 4		
	$\frac{\sin nx}{\sin x} = \frac{\sin((n-2)x+2x)}{\sin x}$	Uses $\sin nx = \sin((n-2)x+2x)$	M1
	$= \frac{\sin(n-2)x(1-2\sin^2 x)+2\sin x\cos x\cos(n-2)x}{\sin x}$ Replaces $\cos 2x$ and $\sin 2x$ by the correct trigonometric identities		M1
	$= \frac{\sin(n-2)x}{\sin x} - 2\sin x\sin(n-2)x + 2\cos x\cos(n-2)x$		
	$= \frac{\sin(n-2)x}{\sin x} + 2\cos((n-2)x+x)$		
	$I_n = I_{n-2} + 2\int \cos(n-1)x\,\mathrm{d}x$		
	$\therefore I_n - I_{n-2} = \int 2\cos(n-1)x\,\mathrm{d}x$ *	Correct completion with no errors.	A1*
	(a) Way 5		
	$\sin nx = \sin((n-1)x+x)$ and $\sin(n-2)x = \sin((n-1)x-x)$		M1
	$\frac{\sin nx}{\sin x} - \frac{\sin(n-2)x}{\sin x} = \frac{\sin(n-1)x\cos x+\cos(n-1)x\sin x-(\sin(n-1)x\cos x-\sin x\cos(n-1)x)}{\sin x}$ Replaces $\sin((n-1)x+x)$ with $\sin(n-1)x\cos x+\cos(n-1)x\sin x$ and Replaces $\sin((n-1)x-x)$ with $\sin(n-1)x\cos x-\cos(n-1)x\sin x$		M1
	$\frac{\sin nx}{\sin x} - \frac{\sin(n-2)x}{\sin x} = \frac{2\sin x\cos(n-1)x}{\sin x}$		
	$(\therefore I_n - I_{n-2}) = \int 2\cos(n-1)x\,\mathrm{d}x$ *	Correct completion with no errors. The $I_n - I_{n-2}$ does not need to be seen explicitly but $\int 2\cos(n-1)x\,\mathrm{d}x$ must seen, including the integral sign.	A1

(b)	$\int \cos 4x\,\mathrm{d}x = k\sin 4x$ or $\int \cos 2x\,\mathrm{d}x = k\sin 2x$	$\cos 4x$ integrated to $\pm k\sin 4x$ or $\cos 2x$ integrated to $\pm k\sin 2x$	M1
	$2\int \cos 4x\,\mathrm{d}x = \frac{1}{2}\sin 4x$ and $2\int \cos 2x\,\mathrm{d}x = \sin 2x$	Both $2\cos 4x$ and $2\cos 2x$ integrated correctly with the correct (possibly un-simplified) coefficients	A1
	$\int \frac{\sin 5x}{\sin x}\mathrm{d}x = \frac{2\sin(4x)}{4} + I_3$ or $\int \frac{\sin 3x}{\sin x}\mathrm{d}x = \frac{2\sin(2x)}{2} + I_1$	One application of reduction formula. This may appear in any form and there does not need to be any integration e.g. $I_5 = \int 2\cos 4x\,\mathrm{d}x + I_3$ **or** e.g. $I_3 = \int 2\cos 2x\,\mathrm{d}x + I_1$	M1
	$\int \frac{\sin 5x}{\sin x}\mathrm{d}x = \frac{2\sin(4x)}{4} + I_3$ and $\int \frac{\sin 3x}{\sin x}\mathrm{d}x = \frac{2\sin(2x)}{2} + I_1$	Two applications of reduction formula. This may appear in any form and there does not need to be any integration e.g. $I_5 = \int 2\cos 4x\,\mathrm{d}x + I_3$ and e.g. $I_3 = \int 2\cos 2x\,\mathrm{d}x + I_1$ **Note that** $\int \frac{\sin 3x}{\sin x}\mathrm{d}x$ **may be attempted using trig. Identities and can score full marks as long as use of the reduction formula is seen at least once.**	M1
	$I_1 = \frac{\pi}{12}$	(Could be implied by their final answer)	B1
	$\left[\frac{2\sin(4x)}{4} + \frac{2\sin(2x)}{2}\right]_{\frac{\pi}{12}}^{\frac{\pi}{6}} = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{4} - \frac{1}{2}$	Correct use of the given limits at least once on an expression of the form $\pm k\sin 4x$ or $\pm k\sin 2x$	M1
	$\int_{\frac{\pi}{12}}^{\frac{\pi}{6}} \frac{\sin 5x}{\sin x}\mathrm{d}x = \frac{1}{12}\left(\pi + 6\sqrt{3} - 6\right)$	cao	A1
			(7)
	Note that correct work leading to $\left[\frac{2\sin(4x)}{4} + \frac{2\sin(2x)}{2} + x\right]$ or equivalent could score the first 4 marks		
			Total 10

Question Number	Scheme	Notes	Marks
8			
(a)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 4 & 3 \\ 2 & -1 & 1 \end{vmatrix} = \begin{pmatrix} 7 \\ 5 \\ -9 \end{pmatrix}$	M1: Attempt cross product between direction vectors or any 2 vectors **in the plane**. If working is not shown or is unclear, 2 elements should be correct for their vectors for this mark. A1: Correct vector	M1A1
	$\begin{pmatrix} 1 \\ -5 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 7 \\ 5 \\ -9 \end{pmatrix} (= 7 - 25 + 18)$	Attempts $\begin{pmatrix} 1 \\ -5 \\ -2 \end{pmatrix} \bullet$ their vector product	M1
	$\begin{pmatrix} 1 \\ -5 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 7 \\ 5 \\ -9 \end{pmatrix} = 7 - 25 + 18 = 0 \therefore$ perpendicular	Correctly obtains = 0 and gives a conclusion.	A1
	Note: $\begin{pmatrix} 1 \\ -5 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 7 \\ 5 \\ -9 \end{pmatrix} = 0 \quad \therefore$ perpendicular scores M1A0 here. However $\begin{pmatrix} 1 \\ -5 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 7 \\ 5 \\ -9 \end{pmatrix} = 7 - 25 + 18 = 0 \quad \therefore$ perpendicular scores M1A1 **BUT** If $\begin{pmatrix} 7 \\ 5 \\ -9 \end{pmatrix}$ is incorrect then $\begin{pmatrix} 1 \\ -5 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} a \\ b \\ c \end{pmatrix} = a - 5b - 2c$ needs to be seen to score the M mark		
			(4)
(b)	$\begin{pmatrix} 7 \\ 5 \\ -9 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = 8 \Rightarrow 7x + 5y - 9z = 8$	M1: Uses $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and their vector product to find the cartesian equation of Π_2. You may need to check their "8" if no working is shown but it must be clear that $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ (or a point on the plane) is being used. A1: Correct equation (any multiple or equivalent equation)	M1A1
	Note that part (b) is possible without part (a): e.g. $x = 1 + \lambda + 2\mu, \quad y = 2 + 4\lambda - \mu, \quad z = 1 + 3\lambda + \mu$. $\Rightarrow y + z = 3 + 7\lambda$ and $x + 2y = 5 + 9\lambda \Rightarrow 9(y + z) - 7(x + 2y) = -8$ $\therefore 7x + 5y - 9z = 8$ Score as M1: Full method leading to a Cartesian equation, A1: Correct equation		
			(2)

(c)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & 5 & -9 \\ 1 & -5 & -2 \end{vmatrix} = \begin{pmatrix} -55 \\ 5 \\ -40 \end{pmatrix}$	M1: Attempt cross product of normal vectors. A1: $k(11\mathbf{i} - \mathbf{j} + 8\mathbf{k})$	M1A1
	$x = 0 : (0,\ -\frac{1}{5},\ -1),\quad y = 0 : (-\frac{11}{5},\ 0,\ -\frac{13}{5}),\quad z = 0 : (\frac{11}{8},\ -\frac{13}{40},\ 0)$ Note that points on the line satisfy $(11t,\ -\frac{1}{5} - t,\ -1 + 8t)$		M1A1
	M1: Attempt point on the line (x, y and z). A1: Correct coordinates		
	$(\mathbf{r} - (-\frac{1}{5}\mathbf{j} - \mathbf{k})) \times (11\mathbf{i} - \mathbf{j} + 8\mathbf{k}) = \mathbf{0}$	**dd**M1: ($\mathbf{r}$ – their point) × their direction "= 0" not required for this mark. **Dependent on both previous method marks.** A1: Correct equation (oe)	**dd**M1A1
			(6)
			12 marks

	Alternatives for part (c) by simultaneous equations		
	Case 1: Eliminates *y* then obtains f(*x*) = g(*y*) = *z*		
	$x-5y-2z=3,\ 7x+5y-9z=8\ \Rightarrow 8x-11z=11$		
	$z=\dfrac{8x-11}{11}, x=\dfrac{11+11z}{8}\Rightarrow\dfrac{11+11z}{8}-5y-2z=3\Rightarrow z=\dfrac{-40y-13}{5}$		
	$\dfrac{8x-11}{11}=\dfrac{-40y-13}{5}=z$	M1: Obtains f(*x*) = g(*y*) = *z* A1: Correct expressions	M1A1
	$\dfrac{x-\frac{11}{8}}{\frac{11}{8}}=\dfrac{y+\frac{13}{40}}{-\frac{1}{8}}=\dfrac{z(-0)}{(1)}$	M1: Correct processing on at least one expression (not *z*) to enable identification of position and direction. A1: Correct equations	M1A1
	$(\mathbf{r}-(\frac{11}{8}\mathbf{i}-\frac{13}{40}\mathbf{j}))\times(\frac{11}{8}\mathbf{i}-\frac{1}{8}\mathbf{j}+\mathbf{k})=\mathbf{0}$	**dd**M1: (**r** – their point) × their direction "= 0" not required for this mark. **Dependent on both previous method marks.** A1: Correct equation (oe)	**dd**M1A1
	Case 2: Eliminates *x* then obtains f(*x*) = *y* = g(*z*)		
	$x-5y-2z=3,\ 7x+5y-9z=8\Rightarrow 40y+5z=-13$		
	$y=\dfrac{-13-5z}{40}, z=\dfrac{-13-40y}{5}\Rightarrow x-5y+2\left(\dfrac{13+40y}{5}\right)=3\Rightarrow y=\dfrac{-5x-11}{55}$		
	$\dfrac{-5x-11}{55}=y=\dfrac{-13-5z}{40}$	M1: Obtains f(*x*) = *y* = g(*z*) A1: Correct expressions	M1A1
	$\dfrac{x+\frac{11}{5}}{-11}=\dfrac{y(-0)}{(1)}=\dfrac{z+\frac{13}{5}}{-8}$	M1: Correct processing on at least one expression (not *y*) to enable identification of position and direction. A1: Correct equations	M1A1
	$(\mathbf{r}-(-\frac{11}{5}\mathbf{i}-\frac{13}{5}\mathbf{k}))\times(-11\mathbf{i}+\mathbf{j}-8\mathbf{k})=\mathbf{0}$	**dd**M1: (**r** – their point) × their direction "= 0" not required for this mark. **Dependent on both previous method marks.** A1: Correct equation (oe)	**dd**M1A1
	Case 3: Eliminates *z* then obtains *x* = f(*y*) = g(*z*)		
	$x-5y-2z=3, 7x+5y-9z=8\ \Rightarrow 5x+55y=-11$		
	$x=\dfrac{-55y-11}{5}, y=\dfrac{-11-5x}{55}\Rightarrow x+5\left(\dfrac{11+5x}{55}\right)-2x=3\Rightarrow x=\dfrac{11z+11}{8}$		
	$x=\dfrac{-55y-11}{5}=\dfrac{11z+11}{8}$	M1: Obtains *x* = f(*y*) = g(*z*) A1: Correct expressions	M1A1
	$\dfrac{x(-0)}{(1)}=\dfrac{y+\frac{1}{5}}{-\frac{1}{11}}=\dfrac{z+1}{\frac{8}{11}}$	M1: Correct processing on at least one expression (not *z*) to enable identification of position and direction. A1: Correct equations	M1A1
	$(\mathbf{r}-(-\frac{1}{5}\mathbf{j}-\mathbf{k}))\times(\mathbf{i}-\frac{1}{11}\mathbf{j}+\frac{8}{11}\mathbf{k})=\mathbf{0}$	**dd**M1: (**r** – their point) × their direction "= 0" not required for this mark. **Dependent on both previous method marks.** A1: Correct equation (oe)	**dd**M1A1

	Alternatives for part (c) by parameters		
	Case 1: Eliminates *x*		
	$x-5y-2z=3,\ 7x+5y-9z=8\ \Rightarrow 8x-11z=11$		
	$x=t\Rightarrow z=-1+\frac{8}{11}t,\ y=-\frac{1}{5}-\frac{1}{11}t$	M1: Obtains *x*, *y* and *z* in terms of λ A1: Correct expressions	M1A1
	$Pos:-\frac{1}{5}\mathbf{j}-\mathbf{k}\quad Dir:\mathbf{i}-\frac{1}{11}\mathbf{j}+\frac{8}{11}\mathbf{k}$	M1: Uses their equations to obtain position and direction A1: Correct position and direction	M1A1
	$(\mathbf{r}-(-\frac{1}{5}\mathbf{j}-\mathbf{k}))\times(\mathbf{i}-\frac{1}{11}\mathbf{j}+\frac{8}{11}\mathbf{k})=\mathbf{0}$	**dd**M1: (**r** – their point) × their direction "= 0" not required for this mark. **Dependent on both previous method marks.** A1: Correct equation (oe)	**dd**M1A1
	Case 2: Eliminates *y*		
	$x-5y-2z=3,\ 7x+5y-9z=8\ \Rightarrow 40y+15z=-13$		
	$y=t\Rightarrow z=-\frac{13}{5}-8t,\ y=-\frac{1}{5}-11t$	M1: Obtains *x*, *y* and *z* in terms of λ A1: Correct expressions	M1A1
	$Pos:-\frac{11}{5}\mathbf{i}-\frac{13}{5}\mathbf{k}\quad Dir:-\frac{11}{5}\mathbf{i}+\mathbf{j}-8\mathbf{k}$	M1: Uses their equations to obtain position and direction A1: Correct position and direction	M1A1
	$(\mathbf{r}-(-\frac{11}{5}\mathbf{i}-\frac{13}{5}\mathbf{k}))\times(-11\mathbf{i}+\mathbf{j}-8\mathbf{k})=\mathbf{0}$	**dd**M1: (**r** – their point) × their direction "= 0" not required for this mark. **Dependent on both previous method marks.** A1: Correct equation (oe)	ddM1A1
	Case 3: Eliminates *z*		
	$x-5y-2z=3,\ 7x+5y-9z=8\ \Rightarrow 8x-11z=11$		
	$z=t\Rightarrow x=\frac{11}{8}+\frac{11}{8}t,\ y=-\frac{13}{40}-\frac{1}{8}t$	M1: Obtains *x*, *y* and *z* in terms of λ A1: Correct expressions	M1A1
	$Pos:\frac{11}{8}\mathbf{i}-\frac{13}{40}\mathbf{j}\quad Dir:\frac{11}{8}\mathbf{i}-\frac{1}{8}\mathbf{j}+\mathbf{k}$	M1: Uses their equations to obtain position and direction A1: Correct position and direction	M1A1
	$(\mathbf{r}-(\frac{11}{8}\mathbf{i}-\frac{13}{40}\mathbf{j}))\times(\frac{11}{8}\mathbf{i}-\frac{1}{8}\mathbf{j}+\mathbf{k})=\mathbf{0}$	**dd**M1: (**r** – their point) × their direction "= 0" not required for this mark. **Dependent on both previous method marks.** A1: Correct equation (oe)	**dd**M1A1

	Alternative for part (c) by finding 2 points on the line		
	$x=0:(0,\ -\frac{1}{5},\ -1)$, $y=0:(-\frac{11}{5},\ 0,\ -\frac{13}{5})$, $z=0:(\frac{11}{8},\ -\frac{13}{40},\ 0)$ M1: Attempts two points on the line A1: Two correct coordinates		M1A1
	Dir: $-\frac{1}{5}\mathbf{j}-\mathbf{k}-\left(-\frac{11}{5}\mathbf{i}-\frac{13}{5}\mathbf{k}\right)=\frac{11}{5}\mathbf{i}-\frac{1}{5}\mathbf{j}+\frac{8}{5}\mathbf{k}$	M1: Subtracts to obtain direction A1: Correct direction	M1A1
	$(\mathbf{r}-(-\frac{1}{5}\mathbf{j}-\mathbf{k}))\times(\frac{11}{5}\mathbf{i}-\frac{1}{5}\mathbf{j}+\frac{8}{5}\mathbf{k})=\mathbf{0}$	**dd**M1: (**r** – their point) × their direction "= 0" not required for this mark. **Dependent on both previous method marks.** A1: Correct equation (oe)	**dd**M1A1

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