Mark Scheme (Results)

Summer 2015

Pearson Edexcel GCE in Mechanics 5 (6681/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

<u>'M' marks</u>
These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.
e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.
The following criteria are usually applied to the equation.

To earn the M mark, the equation
(i) should have the correct number of terms
(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct
e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.
For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.
e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

<u>'A' marks</u>
These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

<u>'B' marks</u>
These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.

3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- $\sqsubset$ The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra g in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.
- Use of g = 9.81 should be penalised once per (complete) question.

  N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

- Marks must be entered in the same order as they appear on the mark scheme.

- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),……then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft
- Mechanics Abbreviations

  M(A) Taking moments about A.

  N2L Newton's Second Law (Equation of Motion)

  NEL Newton's Experimental Law (Newton's Law of Impact)

  HL Hooke's Law

  SHM Simple harmonic motion

  PCLM Principle of conservation of linear momentum

  RHS, LHS Right hand side, left hand side.

Question Number	Scheme	Marks
1.	$\mathbf{AB} = (-3\mathbf{i} + (a-4)\mathbf{j} + (-1 - a)\mathbf{k})$	B1
	Work done $= 3 = (2\mathbf{i} + a\mathbf{j} - 3\mathbf{k}).(-3\mathbf{i} + (a-4)\mathbf{j} + (-1-a)\mathbf{k})$	M1
	$3 = -6 + a(a-4) - 3(-1-a)$	A1
	$0 = a^2 - a - 6$	A1
	$0 = (a-3)(a+2)$	M1
	$a = 3$ since $a > 0$.	A1 **6**
	Notes	
	B1 for correct **AB** in any form. First M1 for $3 = (2\mathbf{i} + a\mathbf{j} - 3\mathbf{k})$.their **AB** (allow **BA**) Need an attempt. First A1 for any correct equation Second A1 for $0 = a^2 - a - 6$ Second M1 for solving a quadratic (2 solutions) Third A1 for $a = 3$	

Question Number	Scheme	Marks
2	$\text{IF} = e^{\int -\tan t \, dt} = \cos t$	M1 A1
	$\frac{d}{dt}(\mathbf{r}\cos t) = \sin t \cos t \, \mathbf{i}$	
	$\mathbf{r}\cos t = \int \sin t \cos t \, \mathbf{i} \, dt$	M1
	$\mathbf{r}\cos t = \frac{1}{2}\sin^2 t \, \mathbf{i} \, (+\mathbf{C})$	A1
	$t = 0, \mathbf{r} = -\frac{1}{2}\mathbf{i} \Rightarrow \mathbf{C} = -\frac{1}{2}\mathbf{i}$	M1 A1
	$\mathbf{r}\cos t = \frac{1}{2}\sin^2 t \, \mathbf{i} - \frac{1}{2}\mathbf{i}$	
	$\mathbf{r} = -\frac{1}{2}\cos t \, \mathbf{i}$ oe	**DM1** A1
		8
	Notes	
	First M1 for $e^{\int -\tan t \, dt}$ (allow if – sign omitted) First A1 for $\cos t$ Second M1 see scheme (multiply both sides by IF and integrate) Second A1 for a correct equation (without **C**) ($-\frac{1}{2}\cos^2 t$ or $-\frac{1}{4}\cos 2t$) Third M1 for use of initial conditions Third A1 for a correct **C** Fourth M1 **dependent** on second M1 for producing an expression for **r** Fourth A1 for **r =** any equivalent form (does not need to be simplified)	

Question Number	Scheme	Marks
3(a)	$(2\mathbf{j}+\mathbf{k}) + (-2\mathbf{i}-\mathbf{j}) + \mathbf{F}_3 = \mathbf{0} \quad => \quad \mathbf{F}_3 = (2\mathbf{i}-\mathbf{j}-\mathbf{k})$ N Magnitude $= \sqrt{2^2+(-1)^2+(-1)^2} = \sqrt{6}$ N	M1 A1 M1 A1 (4)
3(b)	$(\mathbf{i}+2\mathbf{j}+2\mathbf{k}) \times (2\mathbf{j}+\mathbf{k}) + (-\mathbf{i}-\mathbf{j}+\mathbf{k}) \times (-2\mathbf{i}-\mathbf{j}) + (x\mathbf{i}+y\mathbf{j}+z\mathbf{k}) \times (2\mathbf{i}-\mathbf{j}-\mathbf{k})$ $= -2\mathbf{i}-\mathbf{j}+2\mathbf{k} + \mathbf{i}-2\mathbf{j}-\mathbf{k} + (-y+z)\mathbf{i}+(2z+x)\mathbf{j}+(-x-2y)\mathbf{k}$ $-1-y+z=0$ $-3+2z+x=0$ $1-x-2y=0$ $x=1, y=0, z=1$ is a solution $\mathbf{r} = (\mathbf{i}+\mathbf{k})+t(2\mathbf{i}-\mathbf{j}-\mathbf{k})$	M1 A3 M1 A1 M1 A1 (8) **12**
3(b) Alt Concurrency Principle	$\mathbf{r} = (\mathbf{i}+2\mathbf{j}+2\mathbf{k})+\lambda(2\mathbf{j}+\mathbf{k})$ $\mathbf{r} = (-\mathbf{i}-\mathbf{j}+\mathbf{k})+\mu(-2\mathbf{i}-\mathbf{j})$ $1+0 = -1-2\mu$ $2+2\lambda = -1-\mu$ $2+\lambda = 1$ $=> \lambda = \mu = -1$ so point has pv $(\mathbf{i}+\mathbf{k})$ $\mathbf{r} = (\mathbf{i}+\mathbf{k})+t(2\mathbf{i}-\mathbf{j}-\mathbf{k})$	M1 A3 M1 A1 M1 A1
	Notes	
3(a)	First M1 for $\Sigma\mathbf{F}_i = \mathbf{0}$ First A1 for $\mathbf{F}_3 = (2\mathbf{i}-\mathbf{j}-\mathbf{k})$ Second M1 for $\lvert\mathbf{F}_3\rvert = \sqrt{2^2+(-1)^2+(-1)^2}$ Second A1 for $\sqrt{6}$ or 2 sf or better.	
3(b)	First M1 for consistent $\Sigma\, \mathbf{r} \times \mathbf{F}$ or $\Sigma\, \mathbf{F} \times \mathbf{r}$ using their $\mathbf{F}_3$ First A3 for correct vector products (for either of above) -1 for each incorrect product Second M1 for equating all 3 components to zero Fourth A1 for 3 correct equations Third M1 for trying to find a point and getting an equation in correct form (or any other complete method) Fifth A1 for answer (non-unique) **N.B.** They could take moments about another point e.g. $\mathbf{r}_1$ or $\mathbf{r}_2$	
3(b) Alt	First M1 for finding equations of lines of action (and later equating) First A3 for correct equations -1 each error Second M1 for equating all 3 components Fourth A1 for 3 correct equations Third M1 for trying to find a point and getting an equation in correct form (or any other complete method) Fifth A1 for answer (non-unique)	

Question Number	Scheme	Marks
4(a)	$(m+\delta m)(v+\delta v)-mv=-mg\delta t$	M1 A1
	$m\delta v+v\delta m=-mg\delta t$	**DM1**
	$\frac{\mathrm{d}v}{\mathrm{d}t}+\frac{v}{m}\frac{\mathrm{d}m}{\mathrm{d}t}=-g$	
	$m=m_0e^{kt} \Rightarrow \frac{\mathrm{d}m}{\mathrm{d}t}=m_0ke^{kt}(=km)$	M1 A1
	$\frac{\mathrm{d}v}{\mathrm{d}t}+\frac{v}{m}km=-g$ i.e. $kv+\frac{\mathrm{d}v}{\mathrm{d}t}=-g$ PRINTED	A1 (6)
(b)	$\int_{\frac{g}{k}}^{v}\frac{\mathrm{d}v}{kv+g}=-\int_0^T\mathrm{d}t$	M1
	$\frac{1}{k}\left[\ln(kv+g)\right]_v^{\frac{g}{k}}=T$	M1 A1
	$v=0 \Rightarrow \frac{1}{k}\ln 2=T$	**DM1**
	$m=m_0e^{kT}=2m_0$	M1 A1
		(6)
		12
4(b)	**OR** $\int\frac{\mathrm{d}v}{g+kv}=-\int\mathrm{d}t$	M1
	$\frac{1}{k}\ln(g+kv)=-t+(C)$	A1
	$t=0, v=\frac{g}{k}\Rightarrow C=(\frac{1}{k}\ln 2g)$	M1
	$\frac{1}{k}\ln(g+kv)=-t+\frac{1}{k}\ln 2g$	
	Put $v=0,\ t=\frac{1}{k}\ln 2$	**DM1**
	$m=m_0\mathrm{e}^{kt}=2m_0$	M1 A1 (6)

4(b)	$\frac{dv}{dt}+kv=-g$ $\text{IF}=e^{kt}$ $\frac{d(ve^{kt})}{dt}=-ge^{kt}$ $ve^{kt}=\int -ge^{kt}\,dt$ **OR** $ve^{kt}=-\frac{g}{k}e^{kt}+C$ $t=0, v=\frac{g}{k}\Rightarrow C=(\frac{2g}{k})$ $ve^{kt}=-\frac{g}{k}e^{kt}+\frac{2g}{k}$ Put $v=0,\ e^{kt}=2$ $m=m_0e^{kt}=2m_0$	 M1 A1 M1 **DM1** M1 A1 (6)
	Notes	
4(a)	First M1 for momentum equation (correct number of terms, excluding any $\delta m\delta v$ terms) First A1 for a correct equation Second M1, dependent on first M1, for simplifying and dividing by δt and taking limits Third M1 for differentiating the mass equation Second A1 for $\frac{dm}{dt}=km$ Third A1 for PRINTED ANSWER	
4(b)	First M1 for separating and integrating First A1 correct equation (without C) Second M1 for using limits or conditions Third M1, **dependent on first M1**, for putting $v=0$ to give an equation in k and t only Fourth M1 for solving for m Second A1 for correct answer N.B. If they put v = 0 in DE and use dv/dt = -g, NO MARKS	

Question Number	Scheme	Marks
5(a)	$I_T = \frac{1}{4}ma^2 + ma^2$ $= \frac{5}{4}ma^2$ $mga\cos\frac{\pi}{3} = \frac{5}{4}ma^2\ddot{\theta}$ $\frac{2g}{5a} = \ddot{\theta}$	M1 A1 M1 **A1ft** A1 (5)
(b)	$mg\cos\frac{\pi}{3} \pm X = ma\ddot{\theta}$ $\lvert X \rvert = \frac{1}{2}mg - \frac{2}{5}mg$ $= \frac{1}{10}mg$	M1 A1 A1 A1 (4) **9**
	Notes	
5(a)	First M1 for use of perp <u>and</u> parallel axes theorem First A1 $5ma^2/4$ Second M1 for moments about the axis or differentiate a general energy equation Second A1ft on their I for correct equation Third A1 for answer	
5(b)	First M1 for resolving along the rod First A1 A1 for a correct equation (A1 for each side) Need $\pi/3$ Third A1 for $mg/10$ (<u>must be positive</u>)	

Question Number	Scheme	Marks
6(a)	$I_L = \frac{1}{3}3m(2a)^2 + 2m(2a)^2 = 12ma^2$ $M(L),\quad -3mga\sin\theta - 2mg.2a\sin\theta = 12ma^2\ddot{\theta}$ $-\dfrac{7g\sin\theta}{12a} = \ddot{\theta}$ For small θ, $\sin\theta \approx \theta$, $\quad -\dfrac{7g\theta}{12a} = \ddot{\theta}$ so SHM with $\omega = \sqrt{\dfrac{7g}{12a}}$ $so,\ \ T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{12a}{7g}}$	M1 A1 M1 **A2 ft** M1 M1 A1 (8)
(b)	$3mga(1-\cos 60^\circ) + 2mg2a(1-\cos 60^\circ) = \frac{1}{2}12ma^2\omega^2$ $J.2a = 12ma^2\omega$ $J = 6ma\sqrt{\dfrac{7g}{12a}} = 6m\sqrt{\dfrac{7ga}{12}} = m\sqrt{21ag}$	M1 A1 **A1ft** A1 (3rd **DM**) M1 A1 **A1f**t A1 (8) **16**
	Notes	
6(a)	First M1 for finding MI First A1 for $12ma^2$ Second M1 for moments about the axis Second and Third A1 **ft**, on their *I*, for correct equation (A1 for each side) They may use CM of rod + particle Third M1 for small angle approx. and comparison with standard SHM to give an ω value (need – sign in their DE) Fourth M1 for use of $2\pi/\omega$ Fourth A1 cao for any equivalent answer	
6(b)	First M1 for energy equation First A1 **ft** for KE term Second A1 for PE terms Third A1 is now 3rd **DM** mark, dependent on both previous M marks, for solving for *J* and precedes final A mark Second M1 for imp-momentum equation Fourth A1 for LHS on scheme Fifth A1 **ft** for RHS on scheme Sixth A1 cao for any equivalent answer	

Question Number	Scheme	Marks
7(a)	$\delta V = \pi y^2 \delta x$	M1
	$\delta m = \pi y^2 \delta x \dfrac{3m}{2\pi a^3}$	M1
	$\delta I = \frac{1}{4}\delta m y^2 + \delta m x^2$	M1 M1 A1
	$= \frac{1}{4}\delta m(y^2 + 4x^2)$	
	$= \frac{1}{4}\pi(a^2 - x^2)\delta x \dfrac{3m}{2\pi a^3}(a^2 - x^2 + 4x^2)$	M1
	$= \frac{1}{4}\pi(a^2 - x^2)(a^2 + 3x^2)\delta x \dfrac{3m}{2\pi a^3}$	
	$= \dfrac{3m}{8a^3}(a^4 + 2a^2x^2 - 3x^4)\delta x$	A1
	$I = \dfrac{3m}{8a^3}\displaystyle\int_0^a (a^4 + 2a^2x^2 - 3x^4)\,\mathrm{d}x$	M1
	$= \dfrac{3m}{8a^3}\left[a^4x + \dfrac{2a^2x^3}{3} - \dfrac{3x^5}{5}\right]_0^a$	A1
	$= \dfrac{2ma^2}{5}$	A1 (10)
(b)	$I = 2 \text{ x } 2 \text{ x } (\frac{1}{2}M)\dfrac{a^2}{5}$	M1
	$= \dfrac{2Ma^2}{5}$	A1 (2)
		12
	Notes	
7(a)	First M1 for vol. element Second M1 for their δV x correct density Third M1 for ¼ $\delta m y^2$ Fourth M1 for use of parallel axes First A1 for a correct expression in terms of x, y and δm Fifth M1 for sub for δm and y Second A1 for a correct δI in terms of x only Sixth M1 for integrating with correct limits Second A1 for correct integral Third A1 for the answer	
7(b)	M1 for use of additive rule <u>with adjusted mass</u> A1 for correct answer N.B. No marks for non-hence method	

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