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Mark Scheme (Results)

Summer 2014

Pearson Edexcel GCE in Further Pure Mathematics FP2R (6668/01R)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⎣ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x = …

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x = …

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0,\ \ q \neq 0$, leading to x = …

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

<u>Method mark</u> for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Question Number	Scheme	Marks
1. **(a)**	$\frac{2}{4r^2-1}=\frac{A}{2r+1}+\frac{B}{2r-1}$ $2=A(2r-1)+B(2r+1)\ \Rightarrow A=-1,\ \ B=1$ $\frac{2}{4r^2-1}=\frac{1}{2r-1}-\frac{1}{2r+1}$	M1A1 (2)
(b)	$(2)\sum_{r=1}^{n}\frac{1}{4r^2-1}=\sum_{r=1}^{n}\left(\frac{1}{2r-1}-\frac{1}{2r+1}\right)$ $=1-\frac{1}{3}+\frac{1}{3}-\frac{1}{5}+\frac{1}{5}-\frac{1}{7}+...+\frac{1}{2n-1}-\frac{1}{2n+1}=1-\frac{1}{2n+1}$ $=\frac{2n+1-1}{2n+1}$ $\sum_{r=1}^{n}\frac{1}{4r^2-1}=\frac{n}{2n+1}$ *	M1A1ft A1 (3) **[5]**
	Notes for Question 1	
(a)	M1 complete method for finding PFs A1 both PFs correct Award M1A1 for **both** PFs seen correct w/o working. M0A0 otherwise	
(b)	M1 showing fractions with their PFs. Min 2 at start and 1 at end. Must start at 1 and end at *n*. Required sum may be used or 2 x sum A1ft Identify 2 non-cancelling fractions, follow through their PFs - sum or 2 x sum A1cso correct final answer	

Question Number	Scheme	Marks
2	$3x-5-\frac{2}{x}=0$ (or <) or mult through by x^2	
	$\frac{3x^2-5x-2}{x}=0$ (or <)	
	$\frac{(3x+1)(x-2)}{x}=0$ or $x(3x+1)(x-2)=0$	M1
	CVs $x=-\frac{1}{3}, 2$	A1
	$x = 0$	B1
	$x<-\frac{1}{3}, \ 0<x<2$ or in set language (with curved brackets for A1)	M1A1 (5)
	Special case If $\leqslant$ used deduct final mark only.	**[5]**
	Notes for Question 2	
	M1 obtaining two non-zero cvs by any valid method (not calculator) A1 non-zero cvs correct B1 $x = 0$ M1 deducing one appropriate range from their cvs A1 both ranges correct First 3 marks – award with inequalities or = M1A0 if strict inequality **not** used **ALT: If multiplied through by *x*:** $x > 0$: $3x^2-5x<2 \quad (3x+1)(x-2)<0$	
	cvs $x=-\frac{1}{3}, x=2$	M1(solve quad)
	$\therefore\ 0,<x<2$	B1,A1
	$x < 0$	
	$3x^2-5x-2>0$	M1
	cvs $x=-\frac{1}{3}, x=2$	
	$\therefore\ x<-\frac{1}{3}$	A1

Question Number	Scheme	Marks
3.		
(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}+2y\tan x=\mathrm{e}^{4x}\cos^2 x$	
	$\mathrm{e}^{2\int \tan x\,\mathrm{d}x}=\mathrm{e}^{2\ln\sec x}=\sec^2 x$ or $\frac{1}{\cos^2 x}$	M1A1
	$\sec^2 x\frac{\mathrm{d}y}{\mathrm{d}x}+2y\tan x\sec^2 x=\mathrm{e}^{4x}\cos^2 x\sec^2 x$	dM1
	$\frac{\mathrm{d}}{\mathrm{d}x}\left(y\sec^2 x\right)=\mathrm{e}^{4x}$	B1ft($y\sec^2 x$)
	$y\sec^2 x=\frac{1}{4}\mathrm{e}^{4x}\quad(+c)$	M1
	$y=\left(\frac{1}{4}\mathrm{e}^{4x}+c\right)\cos^2 x$ oe	A1
		(6)
(b)	$y=1,\quad x=0\quad 1=\left(\frac{1}{4}+c\right)$	M1
	$c=\frac{3}{4}$	
	$y=\frac{1}{4}\left(\mathrm{e}^{4x}+3\right)\cos^2 x$ oe	A1
		(2) **[8]**
	Notes for Question 3	
(a)	M1 attempting the integrating factor, including integration of (2)tan x $\ln\cos$ or $\ln\sec$ seen A1 correct integrating factor $\sec^2 x$ or $\frac{1}{\cos^2 x}$ M1 multiplying the equation by the integrating factor – may be implied by the next line. B1ft $y\times$ their IF M1 attempting a complete integration of rhs Must include $k\mathrm{e}^{4x}$ but $4\mathrm{e}^{4x}$ would imply differentiation. Constant not needed (Incorrect IF may lead to integration by parts, so integration must be complete) A1 correct solution in form $y = ...$ constant must be included	
(b)	M1 using given initial conditions to obtain a value for c A1 fully correct final answer May be in the form $y\sec^2 x=...$ or $4y\sec^2 x=...$	

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>4.</td><td>$(y=)r\sin\theta = 2\cos 2\theta \sin\theta$
$\frac{\mathrm{d}y}{\mathrm{d}\theta} = -4\sin 2\theta\sin\theta + 2\cos 2\theta\cos\theta$
$2\sin 2\theta\sin\theta - \cos 2\theta\cos\theta = 0$
$4\sin^2\cos\theta - \left(1-2\sin^2\theta\right)\cos\theta = 0$
$\left(6\sin^2\theta - 1\right)\cos\theta = 0$
$\left(\cos\theta = 0 \quad \text{no solutions in range}\right)$
$\therefore \sin\theta = \frac{1}{\sqrt{6}}$</td><td>M1
M1A1

dM1

ddM1A1</td></tr>
<tr><td></td><td>ALT for last 3 marks above:
$\cos 2\theta\cos\theta = 2\sin 2\theta\sin\theta \Rightarrow \tan 2\theta\tan\theta = 1/2$
$\frac{2\tan^2\theta}{1-\tan\theta} = \frac{1}{2}$
$5\tan^2\theta = 1 \quad \tan\theta = 1/\sqrt{5}$
$(\sin\theta = 1/\sqrt{6} \quad \cos\theta = \sqrt{5/6}$</td><td>dM1(double angle formula)
ddM1A1</td></tr>
<tr><td></td><td>$r\sin\theta = 2\cos 2\theta\sin\theta$
$\sin\theta = \frac{1}{\sqrt{6}} \Rightarrow \cos 2\theta = 1-2\sin^2\theta = 1-2\times\frac{1}{6} = \frac{2}{3}$
Eqn. l: $r\sin\theta = 2\times\frac{2}{3}\times\frac{1}{\sqrt{6}} = \frac{4}{3\sqrt{6}}$
$r = \frac{2\sqrt{6}}{9}\operatorname{cosec}\theta$ oe $\left(0<\theta<\pi\right)$ Must be seen in exact form</td><td>M1
M1
A1
[9]</td></tr>
<tr><td colspan="3">Notes for Question 4</td></tr>
<tr><td></td><td>M1 Using $y = r\sin\theta = 2\cos 2\theta\sin\theta$
M1 differentiate $r\sin\theta$ or $r\cos\theta$ using product rule or $\cos 2\theta = 1-2\sin^2\theta$ and chain rule
A1 correct differentiation of $r\sin\theta$
dM1 equate their derivative to 0 and use $\cos 2\theta = 1-2\sin^2\theta$ if not used prior to differentiation, or an appropriate double angle formula for their derivative. Depends on second M mark
ddM1 solve the resulting equation. Depends on second and third M mark
A1 correct value for $\sin\theta$ or $\tan\theta$ or $\cos\theta$ depending on the equation solved
M1 use their value for a trig function to obtain an exact value for $\cos 2\theta$ and $\sin\theta$ if needed now. May be implied by the next stage.
M1 use their values for $\sin\theta$ and $\cos 2\theta$ in $r\sin\theta = 2\cos 2\theta\sin\theta$
NB: These two M marks require
$0 \le \sin\theta \le 1/\sqrt{2}, \quad 1/\sqrt{2} \le \cos\theta \le 1, \quad 0 \le \tan\theta \le 1$
A1 correct equation in form $r = ..\left(0<\theta<\pi \quad \text{not needed}\right)$</td><td></td></tr>
</table>

Question Number	Scheme	Marks
5.		
(a)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\frac{2}{y}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2-2$	
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ seen	B1
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=-\frac{4}{y}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\frac{2}{y^2}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^3$	M1 ($\div$ and diff) A1A1
	Alt: $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x}\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)+y\frac{\mathrm{d}^3y}{\mathrm{d}x^3}+4\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)+2\frac{\mathrm{d}y}{\mathrm{d}x}=0$	M1
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\frac{1}{y}\left(-5\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-2\right)\frac{\mathrm{d}y}{\mathrm{d}x}$	A1 A1
		(4)
(b)	At $x=0$ $\quad\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{1}{2}\left(-2\times\left(\frac{1}{2}\right)^2-4\right)=-\frac{9}{4}\quad(\text{or } -2.25)$	M1A1
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\frac{1}{2}\left(-5\times\frac{1}{2}\times-\frac{9}{4}-2\times\frac{1}{2}\right)=\frac{37}{16}\quad(\text{or } 2.325)$	A1
	$y=2+\frac{1}{2}x+\left(-\frac{9}{4}\right)\frac{x^2}{2!}+\left(\frac{37}{16}\right)\frac{x^3}{3!}+\ldots$	M1(2! or 2, 3! or 6)
	$y=2+\frac{1}{2}x-\frac{9}{8}x^2+\frac{37}{96}x^3+\ldots$	A1
	0.5 1.125 0.3854 3 sf or better	(5) **[9]**

Notes for Question 5		
(a)	B1 $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ seen in the differentiation M1 divide equation by y and differentiate wrt x chain and product/quotient rules needed A1A1 -1 for each error. Ignore any simplification following the differentiation and obtaining $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\ldots$ ALT: B1 as above M1 differentiating before dividing A1A1 rearrange to a correct expression for $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$, -1 each error	
(b)	M1 using values for x and $\frac{\mathrm{d}y}{\mathrm{d}x}$ to obtain a value for $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ A1 correct value for $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ A1 correct value for $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$ M1 Taylor's series formed using their values for the differentials, accept 2! or 2 and 3! or 6. A1 correct series, must start $y =$ (or end $=y$)	

Question Number	Scheme	Marks
6.		
(a)	$w=(u-\mathrm{i}=)\dfrac{x+\mathrm{i}y}{\mathrm{i}x-y+1}$	M1
	$w=(u-\mathrm{i}=)\dfrac{x+\mathrm{i}y}{\mathrm{i}x-y+1}\times\dfrac{(1-y-\mathrm{i}x)}{(1-y-\mathrm{i}x)}$	M1
	$w=(u-\mathrm{i}=)\dfrac{x-xy+xy+\mathrm{i}\left(y-y^2-x^2\right)}{(1-y)^2+x^2}$	A1
	$-1=\dfrac{y-y^2-x^2}{(1-y)^2+x^2}$	M1
	$-1+2y-y^2-x^2=y-y^2-x^2$	
	$y=1$	A1 (5)
	Alternative 1: $z=\dfrac{w}{1-w\mathrm{i}}$	M1
	$x+\mathrm{i}y=\dfrac{u-\mathrm{i}}{1-(u-\mathrm{i})\mathrm{i}}$	M1
	$=\dfrac{u-\mathrm{i}}{-u\mathrm{i}}$	A1
	$=\mathrm{i}+\dfrac{1}{u}$	M1
	$y=1$	A1 (5)
	Alternative 2:	
	$\lvert w+2\mathrm{i}\rvert=\lvert w\rvert$	M1
	$\left\lvert\dfrac{z}{\mathrm{i}z+1}+2\mathrm{i}\right\rvert=\left\lvert\dfrac{z}{\mathrm{i}z+1}\right\rvert$	M1
	$\left\lvert\dfrac{z-2z+2\mathrm{i}}{\mathrm{i}z+1}\right\rvert=\left\lvert\dfrac{z}{\mathrm{i}z+1}\right\rvert$	A1
	$\lvert z-2\mathrm{i}\rvert=\lvert z\rvert$	M1
	$\Rightarrow y=1$	A1 (5)

Question Number	Scheme	Marks
(b)	$u+\mathrm{i}v=\dfrac{x+\frac{1}{2}\mathrm{i}}{\mathrm{i}\left(x+\frac{1}{2}\mathrm{i}\right)+1}=\dfrac{x+\frac{1}{2}\mathrm{i}}{\mathrm{i}x+\frac{1}{2}}$	M1
	$u+\mathrm{i}v=\dfrac{x+\frac{1}{2}\mathrm{i}}{\mathrm{i}x+\frac{1}{2}}\times\dfrac{\frac{1}{2}-x\mathrm{i}}{\frac{1}{2}-x\mathrm{i}}$	M1
	$u+\mathrm{i}v=\dfrac{x+\mathrm{i}\left(\frac{1}{4}-x^2\right)}{\frac{1}{4}+x^2}$	A1
	$u=\dfrac{x}{\frac{1}{4}+x^2} \qquad v=\dfrac{\frac{1}{4}-x^2}{\frac{1}{4}+x^2}$	M1
	$u^2+v^2=\dfrac{x^2+\left(\frac{1}{4}-x^2\right)^2}{\left(\frac{1}{4}+x^2\right)^2}=\dfrac{\frac{1}{16}-\frac{1}{2}x^2+x^2+x^4}{\left(\frac{1}{4}+x^2\right)^2}=1$	M1
	$u^2+v^2=1$, Centre O *	M1,A1 (6)
	Alternative 1: $w=u+\mathrm{i}v=\dfrac{x+\frac{1}{2}\mathrm{i}}{\mathrm{i}x+\frac{1}{2}}$, $\left(=r\mathrm{e}^{\mathrm{i}\theta}\right)$ or $\dfrac{x+\mathrm{i}y}{\mathrm{i}x+y}$	M1
	$\lvert w\rvert=\dfrac{\sqrt{x^2+\frac{1}{4}}}{\sqrt{x^2+\frac{1}{4}}}$, $=1$	M1M1,A1
	$u^2+v^2=1\ \therefore$	M1
	Centre O *	A1 **[11]**

Question Number	Scheme	Marks
	Alternative 2: $\lvert z-\mathrm{i}\rvert=\lvert z\rvert$ $\left\lvert\dfrac{w}{1-\mathrm{i}w}-\dfrac{\mathrm{i}(1-\mathrm{i}w)}{1-\mathrm{i}w}\right\rvert=\left\lvert\dfrac{w}{1-\mathrm{i}w}\right\rvert$ $\lvert w-\mathrm{i}+\mathrm{i}^2w\rvert=\lvert w\rvert$ $\Rightarrow \lvert w\rvert=1$ $u^2+v^2=1$ Centre O	M1 M1 A1 M1 M1 A1 (6)
	Alternative 3: $w=\dfrac{z}{\mathrm{i}z+1}$ $z=\dfrac{w}{1-\mathrm{i}w}=\dfrac{u+\mathrm{i}v}{1-(u+\mathrm{i}v)\mathrm{i}}$ Realise the denominator Correct result Set imaginary part $=\dfrac{1}{2}$ and simplify expression $u^2+v^2=1$ Centre O	 M1 M1 A1 M1 M1 A1 (6)

Notes for Question 6		
(a)	M1 substitute $z = x + \mathrm{i}y$ M1 multiply the numerator and the denominator by conjugate of the denominator A1 correct equation with real denominator on rhs M1 use $w = u - \mathrm{i}$ and equate imaginary part in (their) equation to -1 A1 deducing $y = 1$ **Alternative 1** M1 re-arrange to $z = ...$ M1 replace w with $u - \mathrm{i}$ or replace with $u + \mathrm{i}v$ and realise the denominator A1 correct rhs May still have v M1 separate real and imaginary parts in (their) above equation. This may be implied by a correct answer. A1 deducing $y = 1$ **Alternative 2:** M1 M1 A1 M1 A1 **Alternative 3:** M1 M1 A1 M1 A1	
(b)	M1 substitute $z = x + \mathrm{i}1/2$ or work with $x + \mathrm{i}y$ M1 multiply the numerator and the denominator by conjugate of the denominator A1 correct equation with real denominator on rhs M1 use their u, v and find u^2+v^2 M1 simplify and cancel including use of $y=\frac{1}{2}$ A1cso $u^2+v^2=1$ Centre O **Alternative 1**: M1 as above M1 find $\|w\|$ M1 substitute $y=\frac{1}{2}$ A1 $\|w\|=1$ M1 deduce circle centre O A1cso $u^2+v^2=1$	

	Alternative 2: M1 M1 A1 M1 M1 A1 **Alternative 3:** M1 M1 A1 M1 A1	

Question Number	Scheme	Marks
7. (a)	$\left(\cos\theta+\mathrm{i}\sin\theta\right)^5=\cos 5\theta+\mathrm{i}\sin 5\theta$	B1
	$=\cos^5\theta+5\cos^4\left(\mathrm{i}\sin\theta\right)+\frac{5\times4}{2!}\cos^3\theta\left(\mathrm{i}\sin\theta\right)^2$ $+\frac{5\times4\times3}{3!}\cos^2\theta\left(\mathrm{i}\sin\theta\right)^3+\frac{5\times4\times3\times2}{4!}\cos\theta\left(\mathrm{i}\sin\theta\right)^4+\left(\mathrm{i}\sin\theta\right)^5$	M1
	$=\cos^5\theta+5\mathrm{i}\cos^4\theta\sin\theta-10\cos^3\theta\sin^2\theta$ $-10\mathrm{i}\cos^2\theta\sin^3\theta+5\cos\theta\sin^4\theta+\mathrm{i}\sin^5\theta$	A1
	$\sin 5\theta=5\cos^4\theta\sin\theta-10\cos^2\theta\sin^3\theta+\sin^5\theta$	
	$=5\left(1-\sin^2\theta\right)^2\sin\theta-10\left(1-\sin^2\theta\right)\sin^3\theta+\sin^5\theta$	M1
	$\sin 5\theta=16\sin^5\theta-20\sin^3\theta+5\sin\theta$ *	A1 (5)
(b)	Let $x=\sin\theta$ $\quad 16x^5-20x^3+5x=-\frac{1}{2}\Rightarrow\sin 5\theta=-\frac{1}{2}$	M1
	$5\theta=210,\ 330,\ 570,\ 690,\ 930,\ 1050,\ 1290$ (or in radians) Or 210, 570, 930, 1290, 1650	A1, A1
	$\theta=42,\ 66,\ (114),\ (138),\ 186,\ 210,\ 258$ (or in radians) Or 42, 114, 186, 258, 330	dM1(at least 2 values)
	$\sin\theta=0.669,\ 0.914,\ -0.105,\ -0.5,\ -0.978$	A1 (5)
(c)	$\int_0^{\frac{\pi}{4}}\left(4\sin^5\theta-5\sin^3\theta\right)\mathrm{d}\theta=\frac{1}{4}\int_0^{\frac{\pi}{4}}\left(\sin 5\theta-5\sin\theta\right)\mathrm{d}\theta$	M1
	$=\frac{1}{4}\left[-\frac{1}{5}\cos 5\theta+5\cos\theta\right]_0^{\frac{\pi}{4}}$	A1
	$\frac{1}{4}\left[-\frac{1}{5}\cos\frac{5\pi}{4}+5\cos\frac{\pi}{4}-\left(-\frac{1}{5}+5\right)\right]$	
	$=\frac{1}{4}\left[\frac{1}{5}\times\frac{1}{\sqrt{2}}+\frac{5}{\sqrt{2}}-4\frac{4}{5}\right]$	M1
	$=\frac{13\sqrt{2}}{20}-\frac{6}{5}$	A1 (4)
		[14]

Notes for Question 7		
(a)	B1 applies de Moivre correctly M1 uses binomial theorem to expand $(\cos\theta + \mathrm{i}\sin\theta)^5$ May only show imaginary parts - ignore errors in real part A1 simplifies coefficients to obtain a simplified result with all imaginary terms correct M1 equates imaginary parts and obtains an expression for $\sin 5\theta$ in terms of powers of $\sin\theta$ A1 cso correct result	
(b)	M1 uses substitution $x = \sin\theta$ deduces that $\sin 5\theta = \pm\frac{1}{2}$ A1A1 gives a set of results for 5θ - A1 for 3 useable results A1 for the remaining 2 useable results (no repeats in the set of 5) M1 at least 2 values for θ A1 for the 5 **different** values of *x*	
(c)	M1 uses previous work to change the integrand A1 correct result after integrating - limits can be ignored M1 substitute given limits and use numerical values for trig functions A1 final answer correct (oe provided in the given form)	

Question Number	Scheme	Marks
8. **(a)**	$x = e^z$	
	$\frac{dx}{dy} = e^z \frac{dz}{dy}$	M1
	$\frac{dy}{dx} = e^{-z} \frac{dy}{dz}$	A1
	$\frac{d^2y}{dx^2} = -e^{-z}\frac{dz}{dx}\times\frac{dy}{dz} + e^{-z}\frac{d^2y}{dz^2}\times\frac{dz}{dx} = \frac{1}{x^2}\left(-\frac{dy}{dz}+\frac{d^2y}{dz^2}\right)$	M1A1A1
	$x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 2y = 3\ln x$	
	$x^2\left(-\frac{1}{x^2}\frac{dy}{dz}+\frac{1}{x^2}\frac{d^2y}{dz^2}\right) + 2x\times\frac{1}{x}\frac{dy}{dz} - 2y = 3z$	M1
	$\frac{d^2y}{dz^2} + \frac{dy}{dz} - 2y = 3z$	A1 (7)
	Alt: $z = \ln x$	
	$\frac{dy}{dx} = \frac{dy}{dz}\times\frac{dz}{dx} = \frac{1}{x}\frac{dy}{dz}$	M1A1
	$\frac{d^2y}{dx^2} = -\frac{1}{x^2}\frac{dy}{dz} + \frac{1}{x}\frac{d^2y}{dz^2}\times\frac{dz}{dx} = -\frac{1}{x^2}\frac{dy}{dz} + \frac{1}{x^2}\frac{d^2y}{dz^2}$	M1A1A1
	$x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 2y = 3\ln x$	
	$x^2\left(-\frac{1}{x^2}\frac{dy}{dz}+\frac{1}{x^2}\frac{d^2y}{dz^2}\right) + 2x\times\frac{1}{x}\frac{dy}{dz} - 2y = 3z$	M1
	$\frac{d^2y}{dz^2} + \frac{dy}{dz} - 2y = 3z$	A1 (7)

Question Number	Scheme	Marks
(b)	Aux eqn: $m^2+m-2=0$	
	$(m+2)(m-1)=0$	
	$m=-2, 1$	M1A1
	CF: $y=Ae^{-2z}+Be^{z}$	A1
	PI: Try $y=az+b$	
	$\frac{dy}{dz}=a \quad \frac{d^2y}{dz^2}=0$	
	$a-2(az+b)=3z$	
	$a=-\frac{3}{2}, \quad b=-\frac{3}{4}$	M1
	Complete soln: $y=Ae^{-2z}+Be^{z}-\frac{3}{2}z-\frac{3}{4}$	A1A1 (6)
(c)	$y=Ax^{-2}+Bx-\frac{3}{2}\ln x-\frac{3}{4}$	B1 ft (1)
		[14]

Notes for Question 8

(a)	M1 differentiates $x=e^{z}$ wrt y; chain rule must be used A1 correct differentiation M1 differentiates again to obtain $\frac{d^2y}{dx^2}$ A1A1 one mark for each correct term M1 substitutes in the given equation A1cso obtains the required equation ALT: Works with $z=\ln x$; marks awarded as above	
(b)	M1 forms and solves the auxiliary equation A1 both values for m correct - may be implied by their CF A1 correct CF M1 tries a suitable expression for the PF and obtains values for the constants in the PF A1A1 shows the complete solution; one mark for each correct term in the PF	
(c)	B1ft reverses the substitution to obtain the solution in the form $y = ...$ Follow through their complete solution from (b)	

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