Mark Scheme (Results)

January 2014

Pearson Edexcel International Advanced Level

Mechanics 3 (WME03/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- ⎾ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Notes From Chief Examiner

- Usual rules for M marks: correct no. of terms; dim correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra g in a resolution is accuracy error not method error.
- Omission of mass from a resolution is method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.
- Use of g = 9.81 should be penalised once per (complete) question.
- N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *ONCE* per complete question.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),……then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft.

Question Number	Scheme	Marks
1.	$v=\sqrt{\left(8x^{\frac{3}{2}}-4\right)}$ $v^2=\left(8x^{\frac{3}{2}}-4\right)$ $2v\frac{dv}{dx}=12x^{\frac{1}{2}}$ $F=0.5\times 6x^{\frac{1}{2}}=3x^{\frac{1}{2}}$ $x=4\Rightarrow F=6$	M1 A1 A1 M1dep A1 **5**
Notes		
	M1 for attempting to differentiate the expression for v^2 - chain rule must be used on lhs. A1 for correct $x^{\frac{1}{2}}$ A1 for 6 Award both only if work fully correct M1dep for using NL2 with $m=0.5$ to obtain an expression for F in terms of x A1cso for $F=6$ *Alternatives*: for the first 3 marks	

$\frac{dv}{dx}=\frac{1}{2}\left(8x^{\frac{3}{2}}-4\right)^{-\frac{1}{2}}\times 12x^{\frac{1}{2}}$ $\frac{dv}{dx}=\frac{1}{2v}\times 12x^{\frac{1}{2}}$ $\quad v\frac{dv}{dx}=6x^{\frac{1}{2}}$	M1 Must be a complete method to obtain accel in terms of x A1rhs A1lhs
$\frac{dv}{dt}=\frac{1}{2}\left(8x^{\frac{3}{2}}-4\right)^{-\frac{1}{2}}\times 12x^{\frac{1}{2}}\times\frac{dx}{dt}$ $\frac{dv}{dt}=\frac{1}{2}\left(8x^{\frac{3}{2}}-4\right)^{-\frac{1}{2}}\times 12x^{\frac{1}{2}}\times\left(8x^{\frac{3}{2}}-4\right)^{\frac{1}{2}}=6x^{\frac{1}{2}}$	M1A1A1 Award as above

Question Number	Scheme	Marks
2	$\frac{2mg}{2l}\left(\left(\frac{1}{2}l\right)^2 - x^2\right) = \frac{1}{4}mg\left(\frac{1}{2}l + x\right)$ $8x^2 + 2lx - l^2 = 0$ $(4x - l)(2x + l) = 0$ $x = \frac{1}{4}l$ or $-\frac{1}{2}l$ distance $= \frac{1}{2}l + \frac{1}{4}l = \frac{3}{4}l$	M1A1;M1 A 1 M1 A1 M1dep A1 A1 **9**

Notes

M1 for the difference of 2 elastic energy terms, not nec in a complete energy equation.

A1 for a correct difference

M1 for a work energy equation, loss of EPE = work done against friction(not dep on previous mark)

A1 for a fully correct equation

M1dep for re-arranging to a three term quadratic, dependent on the second M mark, or use the difference of 2 squares to get a linear equation

A1 for a correct 3 term quadratic, terms in any order

M1dep for solving the resulting quadratic, usual rules. Dependent on all second and third M marks

A1 for $x = \frac{1}{4}l$ $x = -\frac{1}{2}l$ need not be shown

A1cao and cso distance $= \frac{3}{4}l$

Question Number	Scheme	Marks
3	$\frac{9}{8}mg - mg = \frac{mu^2}{2a}$	M1 A1
	$u^2 = \frac{ag}{4}$	A1
	$\frac{1}{2}m\left(\frac{ag}{4}\right) - \frac{1}{2}m\left(\frac{ag}{20}\right) = mg2a\left(1-\cos\theta\right)$	M1 A1 A1
	$\theta = 18°$ nearest degree	M1dep A1 **8**

Notes

M1	for NL2 along the radius at the bottom or top. Must have 2 forces and an acceleration
A1	for a fully correct equation ie $\frac{9}{8}mg - mg = \frac{mu^2}{2a}$ oe Must be at the bottom
A1	for obtaining $u^2 = \frac{ag}{4}$
M1	for an energy equation from the bottom or top to the point where the speed is $\sqrt{\frac{ag}{20}}$ (this may be *v* here and for the 2A marks). Must have a difference of KE terms and a GPE term.
A1ft	for correct difference of KE terms or correct PE term (from bottom) Follow through their *u*.
A1	for a completely correct equation
M1dep	for substituting $v = \sqrt{\frac{ag}{20}}$ and solving for θ Dependent on both previous M marks
A1cao	$\theta = 18°$ **must be nearest degree.**
	If candidates do the energy equation first, give those 3 marks for an equation with *u* (speed at bottom) and $\sqrt{\frac{ag}{20}}$. The final M mark will then be for substituting $u^2 = \frac{ag}{4}$ and solving for θ. If the radius is *a* throughout, treat as mis-read. If sometimes *a* and sometimes 2*a* mark each equation on it own merit.

Question Number	Scheme	Marks
4(a)		
	$\pi\int_0^1 e^{-2x}dx = \frac{\pi}{-2}\left[e^{-2x}\right]_0^1$	M1 A1
	$= \frac{\pi}{2}\left(1-e^{-2}\right)$ PRINTED ANSWER	A1cso 3
(b)		
	$\pi\int_0^1 xe^{-2x}dx = \pi\left[\frac{-1}{2}xe^{-2x}\right]_0^1 - \pi\int_0^1 \frac{-1}{2}e^{-2x}dx$	M1 A1
	$= \pi\left(-\frac{1}{2}e^{-2} + \frac{1}{2}\left[-\frac{1}{2}e^{-2x}\right]_0^1\right)$	M1dep A1ft
	$= \pi\left(-\frac{1}{2}e^{-2} - \frac{1}{4}\left(e^{-2}-1\right)\right)$	
	$= \pi\left(\frac{1}{4} - \frac{3}{4}e^{-2}\right)$	A1cao
	$\bar{x} = \frac{\pi\left(\frac{1}{4}-\frac{3}{4}e^{-2}\right)}{\frac{\pi}{2}\left(1-e^{-2}\right)} = \frac{1}{2}\frac{\left(e^2-3\right)}{\left(e^2-1\right)}$	M1 A1
		(7) **10**

Notes for Question 4

A note about π: (a) is a "show that" so π must be included throughout (unless a put in at the end of (a), with a convincing argument for doing so). No answer given in (b), so allow the first 5 marks (as earned) without π **provided** either no πs or both πs appear for the final 2 marks. If the final fraction has the denominator π only, the last 3 marks will be lost

(a)

M1 for using $V = \pi\int y^2\mathrm{d}x = \pi\int \mathrm{e}^{-2x}\mathrm{d}x$ and attempting the integration. limits not needed for this mark

A1 for correct integration, correct limits must be shown

A1cso for $V = \frac{\pi}{2}\left(1-\mathrm{e}^{-2}\right)$ * Must be seen in this form

(b)

M1 for attempting the integration of $\pi\int x\mathrm{e}^{-2x}\mathrm{d}x$ by parts - limits not needed yet. Allow if intention to integrate $\pi\int xy^2\mathrm{d}x$ is shown.

A1 for a correct result with or w/o limits (check signs carefully)

M1dep for attempting the next integral, limits not needed

A1 ft for substituting the correct limits in their integral

A1cao for $\pi\left(\frac{1}{4}-\frac{3}{4}\mathrm{e}^{-2}\right)$ oe

M1 for using $\bar{x} = \frac{(\pi)\int xy^2\mathrm{d}x}{(\pi)\int y^2\mathrm{d}x}$ with their integrals, must be the correct way up.

A1 for $\bar{x} = \frac{\left(\mathrm{e}^2-3\right)}{2\left(\mathrm{e}^2-1\right)}$ oe **must be in terms of e**. Must have only 2 terms in each of the numerator and denominator and no fractions in either.

Question Number	Scheme	Marks
5(a)	$3k\frac{2}{3}\pi r^3 \quad k\pi r^2 3r \quad 3k\frac{2}{3}\pi r^3 + k\pi r^2 3r$	B1
	$(2) \quad (3) \quad (5)$	
	$\left(\frac{3r}{8}+3r\right) \quad \frac{3r}{2} \quad \bar{x}$	B1
	$\left(\frac{3r}{8}+3r\right).2 + \frac{3r}{2}.3 = 5\bar{x}$	M1 A1ft
	$\frac{9r}{4} = \bar{x}$ PRINTED ANSWER	A1 (5)
(b)	$R = W$; $F = P$	B1
	$P.2r\sin\alpha = W(\frac{9r}{4}\sin\alpha - r\cos\alpha)$	M1 A1 A1
	$P = W(\frac{9}{8} - \frac{1}{2}\cot\alpha)$	A1
	$F = \mu R$	
	$\frac{1}{8}(9 - 4\cot\alpha) = \mu$ PRINTED ANSWER	M1depA1cso (7) **12**

Notes

(a) B1 for a correct ratio of masses

B1 for correct distances of the c of ms of the two components, hopefully from *O*, but can be from another point

M1 for a moments equation about *O* or their chosen point. Must have three terms and be dimensionally correct

A1ft for a correct equation, follow through their ratio of masses and distances, but **not** 1:3:4 (from mass/unit vol)

A1cso for $\bar{x} = \frac{9r}{4}$ *

Special case: Using volumes: max B0B1M1A1A1

(b)B1 for the two shown equations

M1 for a moments equation about the point of contact

A1A1 Award A2 if eqn fully correct; A1A0 if one error

A1 for re-arranging to obtain $P = W\left(\frac{9}{8} - \frac{1}{2}\cot\alpha\right)$

M1dep for using $F = \mu R$ together with the expression for *P* and the first two equations to obtain an expression for μ

A1cso for $\mu = \frac{1}{8}(9 - 4\cot\alpha)$ * must be this form

Question Number	Scheme	Marks
6(a)	$(6a)^2 + (8a)^2 = (10a)^2$	M1
	by Pythag (converse), APB = 90 ^{o} PRINTED ANSWER	A1 (2)
(b)	$T_1 \sin\alpha + T_2 \cos\alpha = mr\omega^2$	M1 A2
	$T_1 \cos\alpha - T_2 \sin\alpha = mg$	M1 A1
	$r = 8a\sin\alpha$	M1 A1
	$\sin\alpha = \frac{3}{5}$ or $\cos\alpha = \frac{4}{5}$	B1
	solving, $T_2 = \dfrac{3m}{25}(32a\omega^2 - 5g)$	M1
	$T_2 \geq 0 \Rightarrow \omega = \sqrt{\dfrac{5g}{32a}}$	M1 A1
	max time $= \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{32a}{5g}}$ PRINTED ANSWER	M1A1 (13) **15**

Notes for Question 6

(a)

M1 for squaring the sides and showing they fit Pythagoras' theorem or ratio of sides 3:4:5 or use the cosine rule

A1cso for stating that (the converse of) Pythagoras' theorem shows that $APB = 90°$ * or appropriate conclusion for their method

(b)

M1 for NL2 horizontally. There must be two tensions, both resolved, but may be the same, and an acceleration (either form accepted here) Sine/cos interchange is an accuracy error.

A1 for any two correct terms

A1 for the third correct term. Acceleration must be in the form $mr\omega^2$ and tensions must be different for both these marks to be awarded

M1 for resolving vertically. Again, two tensions, both resolved but may be the same, and sine/cos interchange is an accuracy error.

A1 for a fully correct equation with different tensions.

M1 for finding the radius as $r = 8a\sin\alpha$ or $8a\cos\alpha$

A1 for $r = 8a\sin\alpha$ May not be shown explicitly

B1 for a correct value for $\sin\alpha$ or $\cos\alpha$

M1dep for solving to obtain an expression for T_2 in terms of m, g, a, ω. Dependent on all M marks above **and** two different tensions. Or making $T_2 = 0$ in the above equations and solving for ω

M1dep for using $T_2 \geqslant 0$ in *their* expression for T_1 to obtain an expression for ω in terms of g and a Dependent on the previous M mark $T_2 < 0$ gets M0

A1 for $\omega_{\min} = \sqrt{\frac{5g}{32a}}$ oe

M1 for using $\frac{2\pi}{\omega}$ with their ω to obtain the maximum time

A1cso for max time = $2\pi\sqrt{\frac{32a}{5g}}$

Question Number	Scheme	Marks
7 (a)	$\frac{8mge}{l} = mg$	M1
	$e = \frac{1}{8}l$	A1 (2)
(b)	$-mg - T = m\ddot{x}$	M1 A1
	$-mg - \frac{8mg}{l}(x - \frac{1}{8}l) = m\ddot{x}$	M1dep A1
	$-\frac{8g}{l}x = \ddot{x}$	A1
	SHM, period $2\pi\sqrt{\frac{l}{8g}}$ PRINTED ANSWER	A1cso (6)
(c)	$a = \frac{1}{2}l - \frac{1}{8}l = \frac{3}{8}l$	B1
	$u^2 = \frac{8g}{l}((\frac{3}{8}l)^2 - (\frac{-1}{8}l)^2)$	M1 A1
	$u = \sqrt{gl}$	A1 (4)
(d)	$x = -a\cos\omega t$	
	$\dot{x} = a\omega\sin\omega t$	
	$\sqrt{\frac{9gl}{32}} = \frac{3l}{8}\sqrt{\frac{8g}{l}}\sin\sqrt{\frac{8g}{l}}t$	M1 A1
	$\frac{1}{2} = \sin\sqrt{\frac{8g}{l}}t$	
	$t = \frac{\pi}{6}\sqrt{\frac{l}{8g}}$	M1dep A1 (4)
		16

Notes for Question 7

(a)

M1 for Hooke's law and equating tension to weight

A1cao for $e = \frac{1}{8}l$

(b)

M1 for NL2 vertically, weight and tension needed, $\ddot{x}$ or a for acceleration here

A1 for a correct equation with $\ddot{x}$ or a

M1dep for using HL to replace the tension with an expression in terms of x Dependent on the previous M mark Must have $\ddot{x}$ now

A1 for this equation correct

A1 for re-arranging to get $-\frac{8g}{l}x = \ddot{x}$ oe

A1cso for the conclusion SHM and the period $2\pi\sqrt{\frac{l}{8g}}$ *

(c)

B1 for using the information in the question to obtain amp $= \frac{3}{8}l$

M1 for using $v^2 = \omega^2\left(a^2 - x^2\right)$ with *their* ω and a

A1 for a correct, unsimplified expression for u^2 in terms of l and g

A1cao for $u = \sqrt{gl}$

By energy: B1 for EPE, M1 equation, A1 correct equation, A1 answer

(d)

M1 for using $\dot{x} = a\omega\sin\omega t$ (or v instead of $\dot{x}$) with *their* a and ω and the given speed

A1 for a fully correct equation

M1dep for solving *their* equation **must use radians**

A1cao for $t = \frac{\pi}{6}\sqrt{\frac{l}{8g}}$ or $0.5235...\sqrt{\frac{l}{8g}}$ oe. (if sub for g seen, must be 2 or 3 sf)

Alternative for (d):

Use $v^2 = \omega^2\left(a^2 - x^2\right)$ with *their* ω and a and the given speed M1

$x = \frac{3l}{16}\sqrt{3}$ or $x^2 = \frac{27l^2}{256}$ oe A1

Use $x = a\cos\omega t$ with *their* x, ω and a and solve in radians M1dep

$t = \frac{\pi}{6}\sqrt{\frac{l}{8g}}$ or $0.5235...\sqrt{\frac{l}{8g}}$ oe. (if sub for g seen, must be 2 or 3 sf)

A1cao

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