

Mark Scheme (Results)

Summer 2010

GCE

GCE Decision Mathematics D2 (6690/01)

Edexcel is one of the leading examining and awarding bodies in the UK and throughout the world. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers.

Through a network of UK and overseas offices, Edexcel's centres receive the support they need to help them deliver their education and training programmes to learners.

For further information, please call our GCE line on 0844 576 0025, our GCSE team on 0844 576 0027, or visit our website at www.edexcel.com.

If you have any subject specific questions about the content of this Mark Scheme that require the help of a subject specialist, you may find our **Ask The Expert** email service helpful.

Ask The Expert can be accessed online at the following link:

<http://www.edexcel.com/Aboutus/contact-us/>

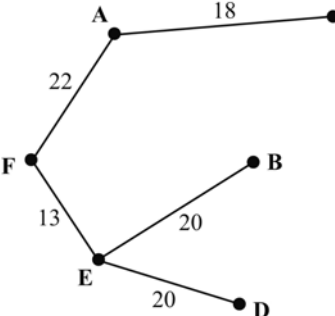
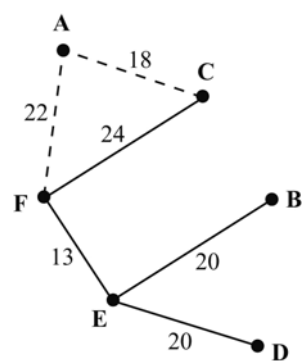
Summer 2010

Publications Code UA023714

All the material in this publication is copyright

© Edexcel Ltd 2010

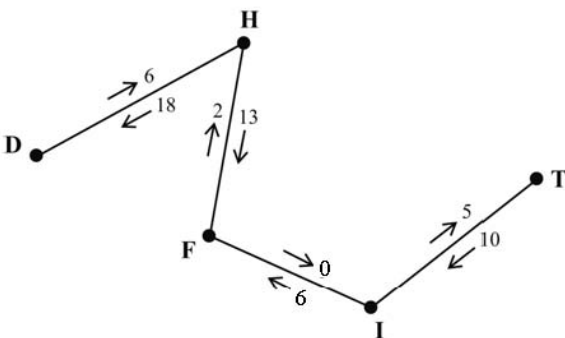
Summer 2010
Decision Mathematics D2 6690
Mark Scheme

Question Number	Scheme	Marks
Q1 (a)	 <p>(b) Minimum Spanning tree length 93, so upper bound is £186</p> <p>(c) A C F E B D A 18 24 13 20 22 28 Length 125 A C F E D B A 18 24 13 20 22 36 Length 133</p> <p>(d) Best upper bound is £125</p> <p>(e) Delete A</p>  <p>RMST weight = 77 Lower bound = $77 + 18 + 22 = £117$</p>	<p>M1 A1 (2)</p> <p>B1ft (1)</p> <p>M1 A1 (3)</p> <p>B1ft (1)</p> <p>M1 A1</p> <p>M1 A1 (4)</p> <p>[11]</p>

Question Number	Scheme	Marks																
Q2 (a)	<p>Since maximising, subtract all elements from some $n \geq 27$</p> $\begin{bmatrix} 12 & 6 & 8 & 13 \\ 10 & 5 & 11 & 60 \\ 5 & 6 & 3 & 8 \\ 11 & 4 & 7 & 16 \end{bmatrix}$ <p>Reduce rows $\begin{bmatrix} 6 & 0 & 2 & 7 \\ 5 & 0 & 6 & 55 \\ 2 & 3 & 0 & 5 \\ 7 & 0 & 3 & 12 \end{bmatrix}$ then columns $\begin{bmatrix} 4 & 0 & 2 & 2 \\ 3 & 0 & 6 & 50 \\ 0 & 3 & 0 & 0 \\ 5 & 0 & 3 & 7 \end{bmatrix}$</p> $\begin{bmatrix} 2 & 0 & 0 & 0 \\ 1 & 0 & 4 & 48 \\ 0 & 5 & 0 & 0 \\ 3 & 0 & 1 & 5 \end{bmatrix}$ $\begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 0 & 3 & 47 \\ 0 & 6 & 0 & 0 \\ 2 & 0 & 0 & 4 \end{bmatrix}$ <p>(b) Three optimal allocations:</p> <table border="1" data-bbox="657 1249 898 1406"> <tbody> <tr> <td>Harry</td> <td>3</td> <td>4</td> <td>4</td> </tr> <tr> <td>Jess</td> <td>1</td> <td>1</td> <td>2</td> </tr> <tr> <td>Louis</td> <td>4</td> <td>3</td> <td>1</td> </tr> <tr> <td>Saul</td> <td>2</td> <td>2</td> <td>3</td> </tr> </tbody> </table> <p>Total amount earned by team: £90</p>	Harry	3	4	4	Jess	1	1	2	Louis	4	3	1	Saul	2	2	3	<p>1M1 2M1</p> <p>3M1 A1</p> <p>4M1 A1ft</p> <p>5M1A1 (8)</p> <p>M1</p> <p>A1 (2) [10]</p>
Harry	3	4	4															
Jess	1	1	2															
Louis	4	3	1															
Saul	2	2	3															

Question Number	Scheme	Marks																																																																																																																																																																																																																																	
Q3 (a)	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td></td><td>A</td><td>B</td><td>C</td><td>D</td></tr> <tr><td>X</td><td>18</td><td>31</td><td>4</td><td></td></tr> <tr><td>Y</td><td></td><td></td><td>18</td><td>29</td></tr> </table>		A	B	C	D	X	18	31	4		Y			18	29	B1 (1)																																																																																																																																																																																																																		
	A	B	C	D																																																																																																																																																																																																																															
X	18	31	4																																																																																																																																																																																																																																
Y			18	29																																																																																																																																																																																																																															
(b)	<p>e.g.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td></td><td></td><td>28</td><td>20</td><td>19</td><td>22</td></tr> <tr><td></td><td></td><td>A</td><td>B</td><td>C</td><td>D</td></tr> <tr><td>0</td><td>X</td><td>x</td><td>x</td><td>x</td><td>-6</td></tr> <tr><td>-5</td><td>Y</td><td>-8</td><td>-3</td><td>x</td><td>x</td></tr> </table> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td></td><td>A</td><td>B</td><td>C</td><td>D</td></tr> <tr><td>X</td><td>18 - θ</td><td>31</td><td>4 + θ</td><td></td></tr> <tr><td>Y</td><td>θ</td><td></td><td>18 - θ</td><td>29</td></tr> </table> <p style="text-align: center;">Entering cell: AY $\theta = 18$</p> <div style="display: flex; justify-content: space-around;"> <div style="width: 45%;"> <p style="text-align: center;">Either Exiting cell: XA</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td></td><td>A</td><td>B</td><td>C</td><td>D</td></tr> <tr><td>X</td><td></td><td>31</td><td>22</td><td></td></tr> <tr><td>Y</td><td>18</td><td></td><td>0</td><td>29</td></tr> </table> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td></td><td></td><td>20</td><td>20</td><td>19</td><td>22</td></tr> <tr><td></td><td></td><td>A</td><td>B</td><td>C</td><td>D</td></tr> <tr><td>0</td><td>X</td><td>8</td><td>x</td><td>x</td><td>-6</td></tr> <tr><td>-5</td><td>Y</td><td>x</td><td>-3</td><td>x</td><td>x</td></tr> </table> <p style="text-align: center;">Entering cell: XD</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td></td><td>A</td><td>B</td><td>C</td><td>D</td></tr> <tr><td>X</td><td></td><td>31</td><td>22 - θ</td><td>θ</td></tr> <tr><td>Y</td><td>18</td><td></td><td>0 + θ</td><td>29 - θ</td></tr> </table> <p style="text-align: center;">Exiting cell: XC $\theta = 22$</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td></td><td>A</td><td>B</td><td>C</td><td>D</td></tr> <tr><td>X</td><td></td><td>31</td><td></td><td>22</td></tr> <tr><td>Y</td><td>18</td><td></td><td>22</td><td>7</td></tr> </table> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td></td><td></td><td>14</td><td>20</td><td>13</td><td>16</td></tr> <tr><td></td><td></td><td>A</td><td>B</td><td>C</td><td>D</td></tr> <tr><td>0</td><td>X</td><td>14</td><td>x</td><td>6</td><td>x</td></tr> <tr><td>1</td><td>Y</td><td>x</td><td>-9</td><td>x</td><td>x</td></tr> </table> </div> <div style="width: 45%;"> <p style="text-align: center;">Or Exiting cell: YC</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td></td><td>A</td><td>B</td><td>C</td><td>D</td></tr> <tr><td>X</td><td>0</td><td>31</td><td>22</td><td></td></tr> <tr><td>Y</td><td>18</td><td></td><td></td><td>29</td></tr> </table> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td></td><td></td><td>28</td><td>20</td><td>19</td><td>30</td></tr> <tr><td></td><td></td><td>A</td><td>B</td><td>C</td><td>D</td></tr> <tr><td>0</td><td>X</td><td>x</td><td>x</td><td>x</td><td>-14</td></tr> <tr><td>-13</td><td>Y</td><td>x</td><td>5</td><td>8</td><td>x</td></tr> </table> <p style="text-align: center;">Entering cell: XD</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td></td><td>A</td><td>B</td><td>C</td><td>D</td></tr> <tr><td>X</td><td>0 - θ</td><td>31</td><td>22</td><td>θ</td></tr> <tr><td>Y</td><td>18 + θ</td><td></td><td></td><td>29 - θ</td></tr> </table> <p style="text-align: center;">Exiting cell: XA $\theta = 0$</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td></td><td>A</td><td>B</td><td>C</td><td>D</td></tr> <tr><td>X</td><td></td><td>31</td><td>22</td><td>0</td></tr> <tr><td>Y</td><td>18</td><td></td><td></td><td>29</td></tr> </table> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td></td><td></td><td>14</td><td>20</td><td>19</td><td>16</td></tr> <tr><td></td><td></td><td>A</td><td>B</td><td>C</td><td>D</td></tr> <tr><td>0</td><td>X</td><td>14</td><td>x</td><td>x</td><td>x</td></tr> <tr><td>1</td><td>Y</td><td>x</td><td>-9</td><td>-6</td><td>x</td></tr> </table> </div> </div>			28	20	19	22			A	B	C	D	0	X	x	x	x	-6	-5	Y	-8	-3	x	x		A	B	C	D	X	18 - θ	31	4 + θ		Y	θ		18 - θ	29		A	B	C	D	X		31	22		Y	18		0	29			20	20	19	22			A	B	C	D	0	X	8	x	x	-6	-5	Y	x	-3	x	x		A	B	C	D	X		31	22 - θ	θ	Y	18		0 + θ	29 - θ		A	B	C	D	X		31		22	Y	18		22	7			14	20	13	16			A	B	C	D	0	X	14	x	6	x	1	Y	x	-9	x	x		A	B	C	D	X	0	31	22		Y	18			29			28	20	19	30			A	B	C	D	0	X	x	x	x	-14	-13	Y	x	5	8	x		A	B	C	D	X	0 - θ	31	22	θ	Y	18 + θ			29 - θ		A	B	C	D	X		31	22	0	Y	18			29			14	20	19	16			A	B	C	D	0	X	14	x	x	x	1	Y	x	-9	-6	x	M1 A1 2M1 2A1ft 3M1 3A1 4M1 4A1ft 5A1 (9)
		28	20	19	22																																																																																																																																																																																																																														
		A	B	C	D																																																																																																																																																																																																																														
0	X	x	x	x	-6																																																																																																																																																																																																																														
-5	Y	-8	-3	x	x																																																																																																																																																																																																																														
	A	B	C	D																																																																																																																																																																																																																															
X	18 - θ	31	4 + θ																																																																																																																																																																																																																																
Y	θ		18 - θ	29																																																																																																																																																																																																																															
	A	B	C	D																																																																																																																																																																																																																															
X		31	22																																																																																																																																																																																																																																
Y	18		0	29																																																																																																																																																																																																																															
		20	20	19	22																																																																																																																																																																																																																														
		A	B	C	D																																																																																																																																																																																																																														
0	X	8	x	x	-6																																																																																																																																																																																																																														
-5	Y	x	-3	x	x																																																																																																																																																																																																																														
	A	B	C	D																																																																																																																																																																																																																															
X		31	22 - θ	θ																																																																																																																																																																																																																															
Y	18		0 + θ	29 - θ																																																																																																																																																																																																																															
	A	B	C	D																																																																																																																																																																																																																															
X		31		22																																																																																																																																																																																																																															
Y	18		22	7																																																																																																																																																																																																																															
		14	20	13	16																																																																																																																																																																																																																														
		A	B	C	D																																																																																																																																																																																																																														
0	X	14	x	6	x																																																																																																																																																																																																																														
1	Y	x	-9	x	x																																																																																																																																																																																																																														
	A	B	C	D																																																																																																																																																																																																																															
X	0	31	22																																																																																																																																																																																																																																
Y	18			29																																																																																																																																																																																																																															
		28	20	19	30																																																																																																																																																																																																																														
		A	B	C	D																																																																																																																																																																																																																														
0	X	x	x	x	-14																																																																																																																																																																																																																														
-13	Y	x	5	8	x																																																																																																																																																																																																																														
	A	B	C	D																																																																																																																																																																																																																															
X	0 - θ	31	22	θ																																																																																																																																																																																																																															
Y	18 + θ			29 - θ																																																																																																																																																																																																																															
	A	B	C	D																																																																																																																																																																																																																															
X		31	22	0																																																																																																																																																																																																																															
Y	18			29																																																																																																																																																																																																																															
		14	20	19	16																																																																																																																																																																																																																														
		A	B	C	D																																																																																																																																																																																																																														
0	X	14	x	x	x																																																																																																																																																																																																																														
1	Y	x	-9	-6	x																																																																																																																																																																																																																														
(c)	Negative improvement index so not optimal	B1ft (1) [11]																																																																																																																																																																																																																																	
Q4 (a)	Minimax route																																																																																																																																																																																																																																		

Question Number	Scheme	Marks																																																																										
	<table border="1" data-bbox="440 253 1117 920"> <thead> <tr> <th>Stage</th> <th>State</th> <th>Action</th> <th>Dest.</th> <th>Value</th> </tr> </thead> <tbody> <tr> <td></td> <td>G</td> <td>GT</td> <td>T</td> <td>17*</td> </tr> <tr> <td rowspan="2">1</td> <td>H</td> <td>HT</td> <td>T</td> <td>21*</td> </tr> <tr> <td>I</td> <td>IT</td> <td>T</td> <td>29*</td> </tr> <tr> <td rowspan="2">2</td> <td rowspan="2">D</td> <td>DG</td> <td>G</td> <td>$\max(22, 17) = 22^*$</td> </tr> <tr> <td>DH</td> <td>H</td> <td>$\max(31, 21) = 31$</td> </tr> <tr> <td rowspan="2"></td> <td rowspan="2">E</td> <td>EH</td> <td>H</td> <td>$\max(34, 21) = 34^*$</td> </tr> <tr> <td>EI</td> <td>I</td> <td>$\max(39, 29) = 39$</td> </tr> <tr> <td></td> <td>F</td> <td>FI</td> <td>I</td> <td>$\max(52, 29) = 52^*$</td> </tr> <tr> <td rowspan="2">3</td> <td rowspan="2">A</td> <td>AD</td> <td>D</td> <td>$\max(41, 22) = 41$</td> </tr> <tr> <td>AE</td> <td>E</td> <td>$\max(38, 34) = 38^*$</td> </tr> <tr> <td></td> <td>B</td> <td>BE</td> <td>E</td> <td>$\max(44, 34) = 44^*$</td> </tr> <tr> <td></td> <td>C</td> <td>CE</td> <td>E</td> <td>$\max(36, 34) = 36^*$</td> </tr> <tr> <td></td> <td></td> <td>CF</td> <td>F</td> <td>$\max(35, 52) = 52$</td> </tr> <tr> <td rowspan="3">4</td> <td rowspan="3">S</td> <td>SA</td> <td>A</td> <td>$\max(37, 38) = 38^*$</td> </tr> <tr> <td>SB</td> <td>B</td> <td>$\max(39, 44) = 44$</td> </tr> <tr> <td>SC</td> <td>C</td> <td>$\max(41, 36) = 41$</td> </tr> </tbody> </table>	Stage	State	Action	Dest.	Value		G	GT	T	17*	1	H	HT	T	21*	I	IT	T	29*	2	D	DG	G	$\max(22, 17) = 22^*$	DH	H	$\max(31, 21) = 31$		E	EH	H	$\max(34, 21) = 34^*$	EI	I	$\max(39, 29) = 39$		F	FI	I	$\max(52, 29) = 52^*$	3	A	AD	D	$\max(41, 22) = 41$	AE	E	$\max(38, 34) = 38^*$		B	BE	E	$\max(44, 34) = 44^*$		C	CE	E	$\max(36, 34) = 36^*$			CF	F	$\max(35, 52) = 52$	4	S	SA	A	$\max(37, 38) = 38^*$	SB	B	$\max(39, 44) = 44$	SC	C	$\max(41, 36) = 41$	<p>1M1 A1</p> <p>2M1 A1 A1</p> <p>3M1 A1ft A1ft</p> <p>A1ft</p> <p>(9)</p>
Stage	State	Action	Dest.	Value																																																																								
	G	GT	T	17*																																																																								
1	H	HT	T	21*																																																																								
	I	IT	T	29*																																																																								
2	D	DG	G	$\max(22, 17) = 22^*$																																																																								
		DH	H	$\max(31, 21) = 31$																																																																								
	E	EH	H	$\max(34, 21) = 34^*$																																																																								
		EI	I	$\max(39, 29) = 39$																																																																								
	F	FI	I	$\max(52, 29) = 52^*$																																																																								
3	A	AD	D	$\max(41, 22) = 41$																																																																								
		AE	E	$\max(38, 34) = 38^*$																																																																								
	B	BE	E	$\max(44, 34) = 44^*$																																																																								
	C	CE	E	$\max(36, 34) = 36^*$																																																																								
		CF	F	$\max(35, 52) = 52$																																																																								
4	S	SA	A	$\max(37, 38) = 38^*$																																																																								
		SB	B	$\max(39, 44) = 44$																																																																								
		SC	C	$\max(41, 36) = 41$																																																																								
(b)	Route: SAEHT Greatest annual cost: £38 000	M1 A1ft (2)																																																																										
(c)	Average expenditure $\frac{37+38+34+21}{4} = \frac{130}{4} = \text{£}32\,500$	M1A1 (2) [13]																																																																										

Question Number	Scheme	Marks
Q5 (a) (b) (c) (d) (e)	Initial flow = 41	B1 (1)
	Capacity of $C_1 = 69$	B1
	Capacity of $C_2 = 64$	B1
		(2)
		M1 A1 (2)
e.g. SBADHT – 2 SCGEDHT – 2	M1 A1 A1 (3)	
maximum flow = minimum cut e.g. cut through SA, SB, CE, GE, GI or HT, FI, GI	DM1 A1 (2) [10]	
<p>Notes:</p> (a) 1B1: cao (b) 1B1: cao (permit B1 if 2 correct answers, but transposed) 2B1: cao (c) 1M1: Two numbers on each arc 1A1: cao (d) 1M1: One valid flow augmenting route, S to T, found and value (≤ 4) stated. 1A1: Flow increased by at least 2 2A1: Flow increased by 4 (e) 1DM1: Must have attempted (d) and made an attempt at a cut. 1A1: cut correct – may be drawn. Refer to max flow-min cut theorem three words out of four.		

Question Number	Scheme	Marks																																																																																																																																		
Q6 (a)	$P - x - 2y - 6z = 0$	B1 (1)																																																																																																																																		
(b)	<table border="1" style="margin-bottom: 10px;"> <thead> <tr><th>b.v</th><th>x</th><th>y</th><th>z</th><th>r</th><th>s</th><th>t</th><th>Value</th></tr> </thead> <tbody> <tr><td>r</td><td>0</td><td>1</td><td>2</td><td>1</td><td>0</td><td>0</td><td>24</td></tr> <tr><td>s</td><td>2</td><td>1</td><td>4</td><td>0</td><td>1</td><td>0</td><td>28</td></tr> <tr><td>t</td><td>-1</td><td>$\frac{1}{2}$</td><td>3</td><td>0</td><td>0</td><td>1</td><td>22</td></tr> <tr><td>P</td><td>-1</td><td>-2</td><td>-6</td><td>0</td><td>0</td><td>0</td><td>0</td></tr> </tbody> </table> <table border="1" style="margin-bottom: 10px;"> <thead> <tr><th>b.v.</th><th>x</th><th>y</th><th>z</th><th>r</th><th>s</th><th>t</th><th>Value</th><th>Row Ops.</th></tr> </thead> <tbody> <tr><td>r</td><td>-1</td><td>$\frac{1}{2}$</td><td>0</td><td>1</td><td>$-\frac{1}{2}$</td><td>0</td><td>10</td><td>$R_1 - 2R_2$</td></tr> <tr><td>z</td><td>$\frac{1}{2}$</td><td>$\frac{1}{4}$</td><td>1</td><td>0</td><td>$\frac{1}{4}$</td><td>0</td><td>7</td><td>$R_2 \div 4$</td></tr> <tr><td>t</td><td>$-\frac{5}{2}$</td><td>$-\frac{1}{4}$</td><td>0</td><td>0</td><td>$-\frac{3}{4}$</td><td>1</td><td>1</td><td>$R_3 - 3R_2$</td></tr> <tr><td>P</td><td>2</td><td>$-\frac{1}{2}$</td><td>0</td><td>0</td><td>$\frac{3}{2}$</td><td>0</td><td>42</td><td>$R_4 + 6R_2$</td></tr> </tbody> </table> <table border="1"> <thead> <tr><th>b.v.</th><th>x</th><th>y</th><th>z</th><th>r</th><th>s</th><th>t</th><th>Value</th><th>Row Ops.</th></tr> </thead> <tbody> <tr><td>y</td><td>-2</td><td>1</td><td>0</td><td>2</td><td>-1</td><td>0</td><td>20</td><td>$R_1 \div \frac{1}{2}$</td></tr> <tr><td>z</td><td>1</td><td>0</td><td>1</td><td>$-\frac{1}{2}$</td><td>$\frac{1}{2}$</td><td>0</td><td>2</td><td>$R_2 - \frac{1}{4}R_1$</td></tr> <tr><td>t</td><td>-3</td><td>0</td><td>0</td><td>$\frac{1}{2}$</td><td>-1</td><td>1</td><td>6</td><td>$R_3 + \frac{1}{4}R_1$</td></tr> <tr><td>P</td><td>1</td><td>0</td><td>0</td><td>1</td><td>1</td><td>0</td><td>52</td><td>$R_4 + \frac{1}{2}R_1$</td></tr> </tbody> </table>	b.v	x	y	z	r	s	t	Value	r	0	1	2	1	0	0	24	s	2	1	4	0	1	0	28	t	-1	$\frac{1}{2}$	3	0	0	1	22	P	-1	-2	-6	0	0	0	0	b.v.	x	y	z	r	s	t	Value	Row Ops.	r	-1	$\frac{1}{2}$	0	1	$-\frac{1}{2}$	0	10	$R_1 - 2R_2$	z	$\frac{1}{2}$	$\frac{1}{4}$	1	0	$\frac{1}{4}$	0	7	$R_2 \div 4$	t	$-\frac{5}{2}$	$-\frac{1}{4}$	0	0	$-\frac{3}{4}$	1	1	$R_3 - 3R_2$	P	2	$-\frac{1}{2}$	0	0	$\frac{3}{2}$	0	42	$R_4 + 6R_2$	b.v.	x	y	z	r	s	t	Value	Row Ops.	y	-2	1	0	2	-1	0	20	$R_1 \div \frac{1}{2}$	z	1	0	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	2	$R_2 - \frac{1}{4}R_1$	t	-3	0	0	$\frac{1}{2}$	-1	1	6	$R_3 + \frac{1}{4}R_1$	P	1	0	0	1	1	0	52	$R_4 + \frac{1}{2}R_1$	<p>M1 A1</p> <p>M1 A1ft A1 (5)</p> <p>M1 A1ft</p> <p>M1 A1 (4)</p>
b.v	x	y	z	r	s	t	Value																																																																																																																													
r	0	1	2	1	0	0	24																																																																																																																													
s	2	1	4	0	1	0	28																																																																																																																													
t	-1	$\frac{1}{2}$	3	0	0	1	22																																																																																																																													
P	-1	-2	-6	0	0	0	0																																																																																																																													
b.v.	x	y	z	r	s	t	Value	Row Ops.																																																																																																																												
r	-1	$\frac{1}{2}$	0	1	$-\frac{1}{2}$	0	10	$R_1 - 2R_2$																																																																																																																												
z	$\frac{1}{2}$	$\frac{1}{4}$	1	0	$\frac{1}{4}$	0	7	$R_2 \div 4$																																																																																																																												
t	$-\frac{5}{2}$	$-\frac{1}{4}$	0	0	$-\frac{3}{4}$	1	1	$R_3 - 3R_2$																																																																																																																												
P	2	$-\frac{1}{2}$	0	0	$\frac{3}{2}$	0	42	$R_4 + 6R_2$																																																																																																																												
b.v.	x	y	z	r	s	t	Value	Row Ops.																																																																																																																												
y	-2	1	0	2	-1	0	20	$R_1 \div \frac{1}{2}$																																																																																																																												
z	1	0	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	2	$R_2 - \frac{1}{4}R_1$																																																																																																																												
t	-3	0	0	$\frac{1}{2}$	-1	1	6	$R_3 + \frac{1}{4}R_1$																																																																																																																												
P	1	0	0	1	1	0	52	$R_4 + \frac{1}{2}R_1$																																																																																																																												
(c)	<p>$P = 52 \quad x = 0 \quad y = 20 \quad z = 2 \quad r = 0 \quad s = 0 \quad t = 6$</p> <p>Notes: (a) 1B1: cao (b) 1M1: correct pivot located, attempt to divide row 1A1: pivot row correct including change of b.v. 2M1: (ft) Correct row operations used at least once or stated correctly. 1A1ft: Looking at non zero-and-one columns, one column ft correct 2A1: cao. 3M1: (ft)Correct pivot identified – negative pivot gets M0 M0 1A1: ft pivot row correct including change of bv – but don't penalise b.v. twice. 4M1: (ft) Correct row operations used at least once or stated correctly. 1A1: cao (c) 1M1: At least 4 values stated. No negative. Reading off bottom row gets M0. 1A1ft: At least 4 values correct. 2A1: cao</p>	<p>M1 A1ft A1 (3) [13]</p>																																																																																																																																		

Question Number	Scheme	Marks
Q7	<div style="display: flex; justify-content: space-between;"> <div style="width: 45%;"> <p> $\begin{bmatrix} -4 & 5 & 1 \\ 3 & -1 & -2 \\ -3 & 0 & 2 \end{bmatrix} \rightarrow \text{add 5 to all entries}$ </p> <p> Either Define variables e.g. let p_1, p_2 and p_3 be the probability that A plays rows 1, 2 and 3 respectively. </p> <p> Maximise $P = V$ </p> <p> Subject to: $V - p_1 - 8p_2 - 2p_3 \leq 0$ $V - 10p_1 - 4p_2 - 5p_3 \leq 0$ $V - 6p_1 - 3p_2 - 7p_3 \leq 0$ $p_1 + p_2 + p_3 \leq 1$ $p_1, p_2, p_3 \geq 0$ </p> <p> Notes: 1M1: Adding $n (\geq 4)$ to all entries 1B1: Defining variables 1B1: Objective correct 2M1: At least 3 constraints, using columns, one of correct form 1A1ft: one correct constraint – excluding non-negativity constraint 2A1ft: two correct constraints – excluding non-negativity constraint 3A1: cao including non-negativity constraint </p> </div> <div style="width: 45%; border-left: 1px solid black; padding-left: 10px;"> <p> $\begin{bmatrix} 1 & 10 & 6 \\ 8 & 4 & 3 \\ 2 & 5 & 7 \end{bmatrix}$ </p> <p> Or Define variables e.g. let p_1, p_2 and p_3 be the probability that A plays rows 1, 2 and 3 respectively. </p> <p> Let $x_i = \frac{p_i}{V}$ </p> <p> Minimise $P = x_1 + x_2 + x_3$ </p> <p> Subject to $x_1 + 8x_2 + 2x_3 \geq 1$ $10x_1 + 4x_2 + 5x_3 \geq 1$ $6x_1 + 3x_2 + 7x_3 \geq 1$ $x_1, x_2, x_3 \geq 0$ </p> </div> </div>	<p>M1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>[7]</p>

Notes for Question 1

- (a) 1M1: Spanning tree found. Allow $1 \times 2 \times 43$ across top of table or 93
1A1: CAO must see tree or list of arcs
- (b) 1B1ft: 186 their 93×2
- (c) 1M1: One Nearest Neighbour each vertex visited at least once (condone lack of return to start)
1A1: One correct route and length CAO – must return to start.
2A1: Second correct route and length CAO – must return to start.
- (d) 1B1ft: ft but only on three different values.
- (e) 1M1: Finding correct RMST (maybe implicit) 77 sufficient, or correct numbers. 4 arcs.
1A1: CAO tree or 77.
2M1: Adding 2 least arcs to A, 18 and 22 or 40 only
2A1: CAO 117

Notes for Question 2

- (a) 1M1: Subtracting from some $n \geq 27$, condone up to two errors
 2M1: Dealing with (Jess, 4) entry.
 3M1: Reducing rows then columns
 1A1: cao (pick up (J,4) value here)
 4M1: Double covered +e; one uncovered – e; and one single covered unchanged.
 2 lines needed to 3 lines needed.
 2A1ft: ft correct - no errors
 5M1: Double covered +e; one uncovered – e; and one single covered unchanged.
 3 line to 4 line solution.
 3A1: correct - no errors
- (b) 1M1: A complete, correct solution.
 1A1: cao

Q2 Special case (Minimises)

$$\begin{bmatrix} 18 & 24 & 22 & 17 \\ 20 & 25 & 19 & 60 \\ 25 & 24 & 27 & 22 \\ 19 & 26 & 23 & 14 \end{bmatrix} \xrightarrow{\text{row reduction}} \begin{bmatrix} 1 & 7 & 5 & 0 \\ 1 & 6 & 0 & 41 \\ 3 & 2 & 5 & 0 \\ 5 & 12 & 9 & 0 \end{bmatrix}$$

M0
M1

$$\xrightarrow{\text{column reductions}} \begin{bmatrix} 0^* & 5 & 5 & 0 \\ 0 & 4 & 0^* & 41 \\ 2 & 0^* & 5 & 0 \\ 4 & 10 & 9 & 0^* \end{bmatrix}$$

M1

A1

M0
M0

Solution:

Harry	- 1	M1
Jess	- 3	
Louis	- 2	
Saul	- 4	

Total £75		A1
-----------	--	----

Maximum 5 marks

Notes for Question 3

- (a) 1B1: Cao
- (b) 1M1: 6 shadow costs and precisely 3 improvement indices stated. (no extra zeros)
1A1: cao.
2M1: A valid route, negative II chosen, only one empty square used, θ 's balance.
2A1ft: improved solution (no extra zeros)
3M1ft: 6 shadow costs and precisely 3 improvement indices stated (no extra zeros)
3A1: cao.
4M1ft: A valid route, negative II chosen, only one empty square used, θ 's balance.
4A1ft: improved solution (no extra zeros)
5A1=5M1: 6 shadow costs and precisely 3 improvement indices, (or 1 negative improvement index), stated (no extra zeros).
- (c) 1B1ft=1A1ft: cao for conclusion, but must follow from at least one negative in a third 'set' of IIs.

Misreads for Q3b Not choosing most negative.

	A	B	C	D
X	18	31	4	
Y			18	29

		28	20	19	22
		A	B	C	D
0	X	x	x	x	-6
-5	Y	-8	-3	x	x

<p>Either</p> <p>Entering cell: XD</p> <table border="1" style="margin: 0 auto;"> <tr> <td></td> <td>A</td> <td>B</td> <td>C</td> <td>D</td> </tr> <tr> <td>X</td> <td>18</td> <td>31</td> <td>$4 - \theta$</td> <td>θ</td> </tr> <tr> <td>Y</td> <td></td> <td></td> <td>$18 + \theta$</td> <td>$29 - \theta$</td> </tr> </table> <p>Exiting cell: XC $\theta = 4$</p> <table border="1" style="margin: 10px auto;"> <tr> <td></td> <td>A</td> <td>B</td> <td>C</td> <td>D</td> </tr> <tr> <td>X</td> <td>18</td> <td>31</td> <td></td> <td>4</td> </tr> <tr> <td>Y</td> <td></td> <td></td> <td>22</td> <td>25</td> </tr> </table> <table border="1" style="margin: 10px auto;"> <tr> <td></td> <td></td> <td>28</td> <td>20</td> <td>13</td> <td>16</td> </tr> <tr> <td></td> <td></td> <td>A</td> <td>B</td> <td>C</td> <td>D</td> </tr> <tr> <td>0</td> <td>X</td> <td>x</td> <td>x</td> <td>6</td> <td>x</td> </tr> <tr> <td>1</td> <td>Y</td> <td>-14</td> <td>-9</td> <td>x</td> <td>x</td> </tr> </table>		A	B	C	D	X	18	31	$4 - \theta$	θ	Y			$18 + \theta$	$29 - \theta$		A	B	C	D	X	18	31		4	Y			22	25			28	20	13	16			A	B	C	D	0	X	x	x	6	x	1	Y	-14	-9	x	x	<p>Or</p> <p>Entering cell: YB</p> <table border="1" style="margin: 0 auto;"> <tr> <td></td> <td>A</td> <td>B</td> <td>C</td> <td>D</td> </tr> <tr> <td>X</td> <td>18</td> <td>$31 - \theta$</td> <td>$4 + \theta$</td> <td></td> </tr> <tr> <td>Y</td> <td></td> <td>θ</td> <td>$18 - \theta$</td> <td>29</td> </tr> </table> <p>Exiting cell: YC $\theta = 18$</p> <table border="1" style="margin: 10px auto;"> <tr> <td></td> <td>A</td> <td>B</td> <td>C</td> <td>D</td> </tr> <tr> <td>X</td> <td>18</td> <td>13</td> <td>22</td> <td></td> </tr> <tr> <td>Y</td> <td></td> <td>18</td> <td></td> <td>29</td> </tr> </table> <table border="1" style="margin: 10px auto;"> <tr> <td></td> <td></td> <td>28</td> <td>20</td> <td>19</td> <td>25</td> </tr> <tr> <td></td> <td></td> <td>A</td> <td>B</td> <td>C</td> <td>D</td> </tr> <tr> <td>0</td> <td>X</td> <td>x</td> <td>x</td> <td>x</td> <td>-9</td> </tr> <tr> <td>-8</td> <td>Y</td> <td>-5</td> <td>x</td> <td>3</td> <td>x</td> </tr> </table>		A	B	C	D	X	18	$31 - \theta$	$4 + \theta$		Y		θ	$18 - \theta$	29		A	B	C	D	X	18	13	22		Y		18		29			28	20	19	25			A	B	C	D	0	X	x	x	x	-9	-8	Y	-5	x	3	x
	A	B	C	D																																																																																																									
X	18	31	$4 - \theta$	θ																																																																																																									
Y			$18 + \theta$	$29 - \theta$																																																																																																									
	A	B	C	D																																																																																																									
X	18	31		4																																																																																																									
Y			22	25																																																																																																									
		28	20	13	16																																																																																																								
		A	B	C	D																																																																																																								
0	X	x	x	6	x																																																																																																								
1	Y	-14	-9	x	x																																																																																																								
	A	B	C	D																																																																																																									
X	18	$31 - \theta$	$4 + \theta$																																																																																																										
Y		θ	$18 - \theta$	29																																																																																																									
	A	B	C	D																																																																																																									
X	18	13	22																																																																																																										
Y		18		29																																																																																																									
		28	20	19	25																																																																																																								
		A	B	C	D																																																																																																								
0	X	x	x	x	-9																																																																																																								
-8	Y	-5	x	3	x																																																																																																								

Candidates can get

2M1 2A1 for first route and the improved solution

3M1 3A0 – 6 shadow costs and 3 IIs

4M1 for finding a valid route and 4A1 if their route leads to an improved solution

[A0 – 6 shadow costs and 3 IIs but it is CAO]

Notes for Question 4

Throughout section (a):

- Condone lack of destination column and/or reversed stage numbers throughout.
- Only penalise incorrect result in Value - ie ignore working values.
- Penalise absence of state or action column with first two A marks earned only
- Penalise empty/errors in stage column with first A mark earned only.

- (a) 1M1: First, T, stage complete and working backwards.
 1A1: CAO (condone lack of *)
 2M1: Second stage completed. Penalise reversed states here and in (b). Bod if something in each column.
 2A1: Any 2 states correct. Penalise * errors, with an A mark, only once in the question).
 3A1: All 3 states correct. (Penalise * errors only once in the question).
 3M1: 3rd and 4th stages completed. Bod if something in each column.
 4A1ft: Any 2 states correct. (Penalise * errors only once in the question). A, B or C
 5A1ft: All 3 states correct. (Penalise * errors only once in the question). A, B and C.
 6A1ft: Final, S, state correct. (Penalise * errors only once in the question).
- (b) 1M1: Route (S to T or vv.) and cost stated
 1A1ft: CAO (Penalise reversed states here)
- (c) 1M1: Sum of four arcs /4 (do not isw here if they 'add' to this method)
 1A1: CAO (32 500 gets both marks)

Special cases (and misreads)

SC1 Maximin: treat as misread.

MAX 11/13

SC2 Maximum: 1M1,1A1; 2M0; 3M1,4A1ft,5A0,6A1ft, M1A1ft M1A1ft **MAX 9/13**

SC3 Minimum: Marks awarded as above SC2

SC4 Maximax: 1M1,1A1; 2M0; 3M1,4A0,5A0,6A0,M1A1ft M1A1ft **MAX 7/13**

SC5 Minimin: Marks awarded as above SC4

SC6 Working forwards:

1M1,1A0; 2M0; 3M1,4A0,5A0,6A0,M1A1ft M1A1ft **MAX6/13**

Anything else annotate and send to review.

Q4 Misreads

SC 1 Maximin

Stage	State	Action	Dest	Value
1	G	GT	T	17*
	H	HT	T	21*
	I	IT	T	29*
2	D	DG	G	$\min(22, 17) = 17$
		DH	H	$\min(31, 21) = 21^*$
	E	EH	H	$\min(34, 21) = 21$
		EI	I	$\min(39, 29) = 29^*$
	F	FI	I	$\min(52, 29) = 29^*$
3	A	AD	D	$\min(41, 21) = 21$
		AE	E	$\min(38, 29) = 29^*$
	B	BE	E	$\min(44, 29) = 29^*$
	C	CE	E	$\min(36, 29) = 29^*$
		CF	F	$\min(35, 29) = 29^*$
4	S	SA	A	$\min(37, 29) = 29^*$
		SB	B	$\min(39, 29) = 29^*$
		SC	C	$\min(41, 29) = 29^*$

SC 2 Maximum route

Stage	State	Action	Dest	Value
1	G	GT	T	17*
	H	HT	T	21*
	I	IT	T	29*
2	D	DG	G	$22 + 17 = 39$
		DH	H	$31 + 21 = 52^*$
	E	EH	H	$34 + 21 = 55$
		EI	I	$39 + 29 = 68^*$
	F	FI	I	$52 + 29 = 81^*$
3	A	AD	D	$41 + 52 = 93$
		AE	E	$38 + 68 = 106^*$
	B	BE	E	$44 + 68 = 112^*$
	C	CE	E	$36 + 68 = 104$
		CF	F	$35 + 81 = 116^*$
4	S	SA	A	$37 + 106 = 143$
		SB	B	$39 + 112 = 151$
		SC	C	$41 + 116 = 157^*$

Route: SCFIT

SC3 Minimum route

Stage	State	Action	Dest	Value
1	G	GT	T	17*
	H	HT	T	21*
	I	IT	T	29*
2	D	DG	G	$22 + 17 = 39^*$
		DH	H	$31 + 21 = 52$
	E	EH	H	$34 + 21 = 55^*$
		EI	I	$39 + 29 = 68$
	F	FI	I	$52 + 29 = 81^*$
3	A	AD	D	$41 + 39 = 80^*$
		AE	E	$38 + 55 = 93$
	B	BE	E	$44 + 55 = 99^*$
		C	CE	E
		CF	F	$35 + 81 = 116$
4	S	SA	A	$37 + 80 = 117^*$
		SB	B	$39 + 99 = 138$
		SC	C	$41 + 91 = 132$

Route: SADGT

SC 4 Maximax route

Stage	State	Action	Dest.	Value
1	G	GT	T	17*
	H	HT	T	21*
	I	IT	T	29*
2	D	DG	G	$\max(22, 17) = 22$
		DH	H	$\max(31, 21) = 31^*$
	E	EH	H	$\max(34, 21) = 34$
		EI	I	$\max(39, 29) = 39^*$
	F	FI	I	$\max(52, 29) = 52^*$
3	A	AD	D	$\max(41, 31) = 41$
		AE	E	$\max(38, 39) = 39^*$
	B	BE	E	$\max(44, 39) = 44^*$
		C	CE	E
		CF	F	$\max(35, 52) = 52^*$
4	S	SA	A	$\max(37, 39) = 39$
		SB	B	$\max(39, 44) = 44$
		SC	C	$\max(41, 52) = 52^*$

Route SCFIT

SC 5 Minimin

Stage	State	Action	Dest	Value
1	G	GT	T	17*
	H	HT	T	21*
	I	IT	T	29*
2	D	DG	G	$\min(22, 17) = 17^*$
		DH	H	$\min(31, 21) = 21$
	E	EH	H	$\min(34, 21) = 21^*$
		EI	I	$\min(39, 29) = 29$
	F	FI	I	$\min(52, 29) = 29^*$
3	A	AD	D	$\min(41, 17) = 17^*$
		AE	E	$\min(38, 21) = 21$
	B	BE	E	$\min(44, 21) = 21^*$
	C	CE	E	$\min(36, 21) = 21^*$
		CF	F	$\min(35, 29) = 29$
4	S	SA	A	$\min(37, 17) = 17^*$
		SB	B	$\min(39, 21) = 21$
		SC	C	$\min(41, 21) = 21$

Route SADGT

SC 6 Working forwards S to T

Stage	State	Action	Dest	Value
1	A	AS	S	37*
	B	BS	S	39*
	C	CS	S	41*
	D	DA	A	$\max(41, 37) = 41^*$
	E	EA	A	$\max(38, 37) = 38^*$
		EB	B	$\max(44, 39) = 44$
		EC	C	$\max(36, 41) = 41$
	F	FC	C	$\max(35, 41) = 41^*$
3	G	GD	D	$\max(22, 41) = 41^*$
	H	HD	D	$\max(31, 41) = 41$
		HE	E	$\max(34, 38) = 38^*$
	I	IE	E	$\max(39, 38) = 39^*$
		IF	F	$\max(52, 41) = 52$
4	T	TG	G	$\max(17, 41) = 41$
		TH	H	$\max(21, 38) = 38^*$
		TI	I	$\max(29, 39) = 39$

Route SAEHT

Q6b Misreads Alternative 1

Increasing x first,

b.v.	x	y	z	r	s	t	value	row ops
r	0	1	2	1	0	0	24	R_1 no change
x	1	$\frac{1}{2}$	2	0	$\frac{1}{2}$	0	14	$R_2 \div 2$
t	0	1	5	0	$\frac{1}{2}$	1	36	$R_3 + R_2$
P	0	$-\frac{3}{2}$	-4	0	$\frac{1}{2}$	0	14	$R_4 + R_2$

then y next

b.v.	x	y	z	r	s	t	value	row ops
y	0	1	2	1	0	0	24	$R_1 \div 1$
x	1	0	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	2	$R_2 - \frac{1}{2}R_1$
t	0	0	3	-1	$\frac{1}{2}$	1	12	$R_3 - R_1$
P	0	0	-1	$\frac{3}{2}$	$\frac{1}{2}$	1	50	$R_4 + \frac{3}{2}R_1$

then z .

b.v.	x	y	z	r	s	t	value	row ops
y	-2	1	0	2	-1	0	20	$R_1 - 2R_2$
z	1	0	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	2	$R_2 \div 2$
t	-3	0	0	$\frac{1}{2}$	-1	1	6	$R_3 - 3R_2$
P	0	0	0	1	1	1	52	$R_4 + R_2$

Q6b Misreads Alternative 2

Increasing x first

b.v.	x	y	z	r	s	t	value	row ops
r	0	1	2	1	0	0	24	R_1 no change
x	1	$\frac{1}{2}$	2	0	$\frac{1}{2}$	0	14	$R_2 \div 2$
t	0	1	5	0	$\frac{1}{2}$	1	36	$R_3 + R_2$
P	0	$-\frac{3}{2}$	-4	0	$\frac{1}{2}$	0	14	$R_4 + R_2$

Increasing z next

b.v.	x	y	z	r	s	t	value	row ops
r	-1	$\frac{1}{2}$	0	1	$-\frac{1}{2}$	0	10	$R_1 - 2R_2$
z	$\frac{1}{2}$	$\frac{1}{4}$	1	0	$\frac{1}{4}$	0	7	$R_2 \div 2$
t	$-\frac{5}{2}$	$-\frac{1}{4}$	0	0	$\frac{3}{4}$	1	1	$R_3 - 5R_2$
P	2	$-\frac{1}{2}$	0	0	$\frac{3}{2}$	0	42	$R_4 + 4R_2$

then increasing y

b.v.	x	y	z	r	s	t	value	row ops
y	-2	1	0	2	-1	0	20	$R_1 \div \frac{1}{2}$
z	1	0	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	2	$R_2 - \frac{1}{4}R_1$
t	-3	0	0	$\frac{1}{2}$	-1	1	6	$R_3 + \frac{1}{4}R_1$
P	1	0	0	1	1	0	52	$R_4 + \frac{1}{2}R_1$

Q6b Misreads Alternative 3

Increasing y first

b.v.	x	y	z	r	s	t	value	row ops
y	0	1	2	1	0	0	24	$R_1 \div 1$
s	2	0	2	-1	1	0	4	$R_2 - R_1$
t	-1	0	2	$-\frac{1}{2}$	0	1	10	$R_3 - \frac{1}{2}R_1$
P	-1	0	-2	2	0	0	48	$R_4 + 2R_1$

Increasing x next

b.v.	x	y	z	r	s	t	value	row ops
y	0	1	2	1	0	0	24	R_1 no changw
x	1	0	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	2	$R_2 \div 2$
t	0	0	3	-1	$\frac{1}{2}$	1	12	$R_3 - 3R_2$
P	0	0	-1	$\frac{3}{2}$	$\frac{1}{2}$	0	50	$R_4 + R_2$

then increasing z

b.v.	x	y	z	r	s	t	value	row ops
y	-2	1	0	2	-1	0	20	$R_1 - 2R_2$
z	1	0	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	2	$R_2 \div 1$
t	-3	0	0	$\frac{1}{2}$	-1	1	6	$R_3 + R_2$
P	1	0	0	1	1	0	52	$R_4 + R_2$

Q6b Misreads Alternative 4

Increasing y first

b.v.	x	y	z	r	s	t	value	row ops
y	0	1	2	1	0	0	24	$R_1 \div 1$
s	2	0	2	-1	1	0	4	$R_2 - R_1$
t	-1	0	2	$-\frac{1}{2}$	0	1	10	$R_3 - \frac{1}{2}R_1$
P	-1	0	-2	2	0	0	48	$R_4 + 2R_1$

Increasing z next

b.v.	x	y	z	r	s	t	value	row ops
y	-2	1	0	2	-1	0	20	$R_1 - 2R_2$
z	1	0	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	2	$R_2 \div 2$
t	-3	0	0	$\frac{1}{2}$	-1	1	6	$R_3 - 2R_2$
P	1	0	0	1	1	0	52	$R_4 + 2R_2$

Further copies of this publication are available from
Edexcel Publications, Adamsway, Mansfield, Notts, NG18 4FN

Telephone 01623 467467
Fax 01623 450481

Email publications@linneydirect.com

Order Code UA023714 Summer 2010

For more information on Edexcel qualifications, please visit www.edexcel.com/quals

Edexcel Limited. Registered in England and Wales no.4496750
Registered Office: One90 High Holborn, London, WC1V 7BH