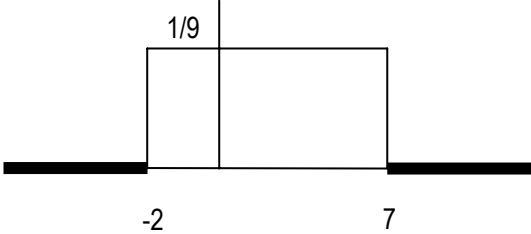
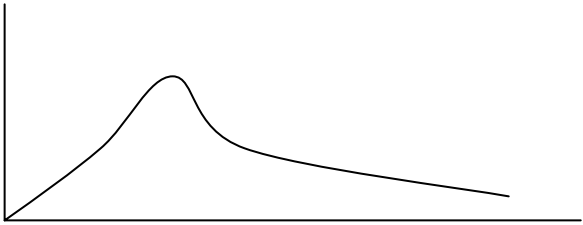


**January 2009  
6684 Statistics S2  
Mark Scheme**

Question Number	Scheme	Marks
<b>1</b>	The random variable $X$ is the number of daisies in a square. Poisson(3)	B1
(a)	$1 - P(X \leq 2) = 1 - 0.4232 \quad 1 - e^{-3}\left(1 + 3 + \frac{3^2}{2!}\right)$ $= 0.5768$	M1 A1 (3)
(b)	$P(X \leq 6) - P(X \leq 4) = 0.9665 - 0.8153 \quad e^{-3}\left(\frac{3^5}{5!} + \frac{3^6}{6!}\right)$ $= 0.1512$	M1 A1 (2)
(c)	$\mu = 3.69$ $\text{Var}(X) = \frac{1386}{80} - \left(\frac{295}{80}\right)^2$ $= 3.73/3.72/3.71 \quad \text{accept } s^2 = 3.77$	B1 M1 A1 (3)
(d)	For a Poisson model, Mean = Variance; For these data $3.69 \approx 3.73$ $\Rightarrow$ Poisson model	B1 (1)
(e)	$\frac{e^{-3.6875} 3.6875^4}{4!} = 0.193$ <p style="text-align: right;">allow their mean or var Awrt 0.193 or 0.194</p>	M1 A1 ft (2)

Question Number	Scheme	Marks
2	<p>(a) <math>f(x) = \begin{cases} \frac{1}{9} &amp; -2 \leq x \leq 7 \\ 0 &amp; \text{otherwise} \end{cases}</math></p> <p>(b) </p> <p>(c) <math>E(X) = \underline{2.5}</math> <math>\text{Var}(X) = \frac{1}{12}(7+2)^2</math> or <math>\underline{6.75}</math> both</p> <p><math>E(X^2) = \text{Var}(X) + E(X)^2</math></p> <p><math>= 6.75 + 2.5^2</math></p> <p><math>= 13</math></p> <p><b>alternative</b></p> <p><math>\int_{-2}^7 x^2 f(x) dx = \left[ \frac{x^3}{27} \right]_{-2}^7</math> attempt to integrate and use limits of -2 and 7</p> <p><math>= 13</math></p> <p>(d) <math>P(-0.2 &lt; X &lt; 0.6) = \frac{1}{9} \times 0.8</math></p> <p><math>= \frac{4}{45}</math> or 0.0889 Or equiv awrt 0.089</p>	<p>B1 B1 (2)</p> <p>B1 B1 (2)</p> <p>B1  M1 A1 (3)</p> <p>B1 M1 A1</p> <p>M1 A1 (2)</p>

Question Number	Scheme	Marks
3	<p>(a) <math>X \sim B(20, 0.3)</math></p> <p><math>P(X \leq 2) = 0.0355</math></p> <p><math>P(X \geq 11) = 1 - 0.9829 = 0.0171</math></p> <p>Critical region is <math>(X \leq 2) \cup (X \geq 11)</math></p> <p>(b) Significance level = <math>0.0355 + 0.0171, = 0.0526</math> or 5.26%</p> <p>(c) Insufficient evidence to reject <math>H_0</math> <b>Or</b> sufficient evidence to accept <math>H_0</math> /not significant  <math>x = 3</math> ( or the value) is not in the critical region or <math>0.1071 &gt; 0.025</math></p> <p>Do not allow inconsistent comments</p>	<p>M1</p> <p>A1 A1 (3)</p> <p>M1 A1 (2)</p> <p>B1 ft</p> <p>B1 ft (2)</p>

Question Number	Scheme	Marks
4	(a) $\int_0^{10} kt dt = 1$ or Area of triangle = 1 $\left[ \frac{kt^2}{2} \right]_0^{10} = 1$ or $10 \times 0.5 \times 10k = 1$ or linear equation in k $50k = 1$ $k = \frac{1}{50}$ cso	M1 M1 A1 (3)
	(b) $\int_6^{10} kt dt = \left[ \frac{kt^2}{2} \right]_6^{10}$ $= \frac{16}{25}$	M1 A1 (2)
	(c) $E(T) = \int_0^{10} kt^2 dt = \left[ \frac{kt^3}{3} \right]_0^{10}$ $= 6\frac{2}{3}$	M1 A1
	$\text{Var}(T) = \int_0^{10} kt^3 dt - \left(6\frac{2}{3}\right)^2 = \left[ \frac{kt^4}{4} \right]_0^{10} - \left(6\frac{2}{3}\right)^2$ $= 50 - \left(6\frac{2}{3}\right)^2$ $= 5\frac{5}{9}$	M1;M1dep A1 (5)
	(d) 10	B1 (1)
(e) 	B1  (1)	

Question Number	Scheme	Marks
5	(a) $X$ represents the number of defective components.  $P(X = 1) = (0.99)^9 (0.01) \times 10 = 0.0914$	M1A1  (2)
	(b) $P(X \geq 2) = 1 - P(X \leq 1)$ $= 1 - (p)^{10} - (a)$ $= 0.0043$	M1 A1✓ A1  (3)
	(c) $X \sim \text{Po}(2.5)$  $P(1 \leq X \leq 4) = P(X \leq 4) - P(X = 0)$ $= 0.8912 - 0.0821$ $= 0.809$	B1B1  M1  A1
	<p>Normal distribution used. B1 for mean only</p> <hr/> <p>Special case for parts a and b            If they use 0.1 do not treat as misread as it makes it easier.            (a) M1 A0 if they have 0.3874            (b) M1 A1ft A0 they will get 0.2639            (c) Could get B1 B0 M1 A0</p> <hr/> <p>For any other values of <math>p</math> which are in the table do not use misread. Check using the tables. They could get (a) M1 A0 (b) M1 A1ft A0 (c) B1 B0 M1 A0</p>	(4)

Question Number	Scheme	Marks
6 (a)(i)	$H_0 : \lambda = 7 \quad H_1 : \lambda > 7$ <p><math>X = \text{number of visits. } X \sim \text{Po}(7)</math></p> $P(X \geq 10) = 1 - P(X \leq 9) = 0.1695$ $1 - P(X \leq 10) = 0.0985$ $1 - P(X \leq 9) = 0.1695$ $\text{CR } X \geq 11$ <p><math>0.1695 &gt; 0.10</math>, <math>\text{CR } X \geq 11</math>            Not significant or it is not in the critical region or do not reject <math>H_0</math>            The rate of visits on a Saturday is not greater/ is unchanged</p>	B1 B1 M1 A1 M1 A1 no ft
(ii)	$X = 11$	B1 (7)
(b)	(The visits occur) randomly/ independently or singly or constant rate	B1 (1)
(c)	$[H_0 : \lambda = 7 \quad H_1 : \lambda > 7 \quad (\text{or } H_0 : \lambda = 14 \quad H_1 : \lambda > 14)]$ <p><math>X \sim N;(14,14)</math></p> $P(X \geq 20) = P\left(z \geq \frac{19.5 - 14}{\sqrt{14}}\right) \quad \text{+/- 0.5, stand}$ $= P(z \geq 1.47)$ $= 0.0708 \quad \text{or } z = 1.2816$ <p><math>0.0708 &lt; 0.10</math> therefore significant. The rate of visits is greater on a Saturday</p>	B1;B1 M1 M1 A1dep both M A1dep 2 <sup>nd</sup> M (6)

Question Number	Scheme	Marks
7	<p>(a) <math>F(x_0) = \int_1^x -\frac{2}{9}x + \frac{8}{9} dx = \left[-\frac{1}{9}x^2 + \frac{8}{9}x\right]_1^x</math>  <math>= \left[-\frac{1}{9}x^2 + \frac{8}{9}x\right] - \left[-\frac{1}{9} + \frac{8}{9}\right]</math>  <math>= -\frac{1}{9}x^2 + \frac{8}{9}x - \frac{7}{9}</math></p>	M1A1 A1 (3)
	<p>(b) <math>F(x) = \begin{cases} 0 &amp; x &lt; 1 \\ -\frac{1}{9}x^2 + \frac{8}{9}x - \frac{7}{9} &amp; 1 \leq x \leq 4 \\ 1 &amp; x &gt; 4 \end{cases}</math></p>	B1B1✓ (2)
	<p>(c) <math>F(x) = 0.75</math> ;                      or <math>F(2.5) = -\frac{1}{9} \times 2.5^2 + \frac{8}{9} \times 2.5 - \frac{7}{9}</math>   <math>-\frac{1}{9}x^2 + \frac{8}{9}x - \frac{7}{9} = 0.75</math>   <math>4x^2 - 32x + 55 = 0</math>   <math>-x^2 + 8x - 13.75 = 0</math>  <math>x = 2.5</math>                                      <math>= 0.75</math>                                      cso   and <math>F(x) = 0.25</math>  <math>-\frac{1}{9}x^2 + \frac{8}{9}x - \frac{7}{9} = 0.25</math>  <math>-x^2 + 8x - 7 = 2.25</math>  <math>-x^2 + 8x - 9.25 = 0</math>                                      quadratic 3 terms = 0  <math>x = \frac{-8 \pm \sqrt{8^2 - 4 \times -1 \times -9.25}}{2 \times -1}</math>  <math>x = 1.40</math></p>	M1; A1 M1 M1 dep M1 dep A1 (6)
	<p>(d) <math>Q_3 - Q_2 &gt; Q_2 - Q_1</math>  Or mode = 1 and mode &lt; median  Or mean = 2 and median &lt; mode  Sketch of pdf here or be referred to if in a different part of the question  Box plot with <math>Q_1, Q_2, Q_3</math> values marked on  Positive skew</p>	M1 A1 (2)

