Mark Scheme (Final)

January 2009

GCE

GCE Further Pure Mathematics FP1 (6667/01)

6667 Further Pure Mathematics FP1
January 2009 Advanced Subsidiary/Advanced Level in GCE Mathematics

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

January 2009
FP1(NEW)
Mark Scheme

Question Number	Scheme	Marks
1.	$x - 3$ is a factor	B1
	$f(x) = (x-3)(2x^2 - 2x + 1)$	M1 A1
	Attempt to solve quadratic i.e. $x = \dfrac{2 \pm \sqrt{4-8}}{4}$	M1
	$x = \dfrac{1 \pm i}{2}$	A1 (5 marks)

Notes:

First and last terms in second bracket required for first M1
Use of correct quadratic formula for their equation for second M1

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2.	(a)	$6\sum r^2 + 4\sum r - \sum 1 = 6\frac{n}{6}(n+1)(2n+1) + 4\frac{n}{2}(n+1), -n$	M1 A1, B1
		$= \frac{n}{6}(12n^2 + 18n + 6 + 12n + 12 - 6)$ or $n(n+1)(2n+1) + (2n+1)n$	M1
		$= \frac{n}{6}(12n^2 + 30n + 12) = n(2n^2 + 5n + 2) = n(n+2)(2n+1)$ *	A1 (5)
	(b)	$\sum_{r=1}^{20}(6r^2 + 4r - 1) - \sum_{r=1}^{10}(6r^2 + 4r - 1) = 20 \times 22 \times 41 - 10 \times 12 \times 21$	M1
		$= 15520$	A1 (2) (7 marks)

Notes:

(a) First M1 for first 2 terms, B1 for $-n$
Second M1 for attempt to expand and gather terms.
Final A1 for correct solution only

(b) Require (r from 1 to 20) subtract (r from 1 to 10) and attempt to substitute for M1

3	(a)	$xy = 25 = 5^2$ or $c = \pm 5$	B1 (1)
	(b)	A has co-ords (5, 5) and B has co-ords (25, 1) Mid point is at (15, 3)	B1 M1A1 (3) (4 marks)

Notes:

(a) $xy = 25$ only B1, $c^2 = 25$ only B1, $c = 5$ only B1

(b) Both coordinates required for B1
Add theirs and divide by 2 on both for M1

Question Number	Scheme	Marks
4.	When n = 1, LHS = $\frac{1}{1\times 2}=\frac{1}{2}$, RHS = $\frac{1}{1+1}=\frac{1}{2}$. So LHS = RHS and result true for n = 1	B1
	Assume true for $n = k$; $\sum_{r=1}^{k}\frac{1}{r(r+1)}=\frac{k}{k+1}$ and so $\sum_{r=1}^{k+1}\frac{1}{r(r+1)}=\frac{k}{k+1}+\frac{1}{(k+1)(k+2)}$	M1
	$\sum_{r=1}^{k+1}\frac{1}{r(r+1)}=\frac{k(k+2)+1}{(k+1)(k+2)}=\frac{k^2+2k+1}{(k+1)(k+2)}=\frac{(k+1)^2}{(k+1)(k+2)}=\frac{k+1}{k+2}$	M1 A1
	and so result is true for $n = k + 1$ (and by induction true for $n \in \mathbf{Z}^+$)	B1 (5 marks)

Notes:

Evaluate both sides for first B1
Final two terms on second line for first M1
Attempt to find common denominator for second M1.
Second M1 dependent upon first.
$\frac{k+1}{k+2}$ for A1
'Assume true for $n = k$ 'and 'so result true for $n = k + 1$' and correct solution for final B1

5.	(a)	attempt evaluation of f(1.1) and f(1.2) (– looking for sign change)	M1
		f(1.1) = 0.30875 , f(1.2) = -0.28199 Change of sign in f(x) $\Rightarrow$ root in the interval	A1
			(2)
	(b)	$f'(x) = \frac{3}{2}x^{-\frac{1}{2}} - 9x^{-1\frac{1}{2}}$	M1 A1 A1
			(3)
	(c)	f (1.1) = 0.30875.. f′(1.1) = -6.37086…	B1 B1
		$x_1 = 1.1 - \frac{0.30875...}{-6.37086..}$	M1
		= 1.15(to 3 sig.figs.)	A1
			(4)
			(9 marks)

Notes:

(a) awrt 0.3 and -0.3 and indication of sign change for first A1
(b) Multiply by power and subtract 1 from power for evidence of differentiation and award of first M1
(c) awrt 0.309 B1and awrt -6.37 B1 if answer incorrect
Evidence of Newton-Raphson for M1
Evidence of Newton-Raphson and awrt 1.15 award 4/4

6.	At n =1, $u_n = 5\times 6^0 + 1 = 6$ and so result true for n = 1	B1
	Assume true for n = k; $u_k = 5\times 6^{k-1} + 1$,and so $u_{k+1} = 6(5\times 6^{k-1} + 1) - 5$	M1, A1
	$\therefore u_{k+1} = 5\times 6^k + 6 - 5 \quad \therefore u_{k+1} = 5\times 6^k + 1$	A1
	and so result is true for n = k + 1 and by induction true for $n \geq 1$	B1 (5 marks)

Notes:

6 and so result true for n = 1 award B1

Sub u_k into u_{k+1} or M1 and A1 for correct expression on right hand of line 2

Second A1 for $\therefore u_{k+1} = 5\times 6^k + 1$

'Assume true for n = k' and 'so result is true for n = k + 1' and correct solution for final B1

Question Number	Scheme	Marks
7. (a)	The determinant is a - 2	M1
	$\mathbf{X}^{-1} = \frac{1}{a-2}\begin{pmatrix} -1 & -a \\ 1 & 2 \end{pmatrix}$	M1 A1 (3)
(b)	$\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	B1
	Attempt to solve $2 - \frac{1}{a-2} = 1$, or $a - \frac{a}{a-2} = 0$, or $-1 + \frac{1}{a-2} = 0$, or $-1 + \frac{2}{a-2} = 1$	M1
	To obtain a =3 only	A1 cso (3) (6 marks)
Alternatives for (b)	If they use $\mathbf{X}^2 + \mathbf{I} = \mathbf{X}$ they need to identify **I** for B1, then attempt to solve suitable equation for M1 and obtain a = 3 for A1 If they use $\mathbf{X}^2 + \mathbf{X}^{-1} = \mathbf{O}$, they can score the B1then marks for solving If they use $\mathbf{X}^3 + \mathbf{I} = \mathbf{O}$ they need to identify **I** for B1, then attempt to solve suitable equation for M1 and obtain a = 3 for A1	

Notes:

(a) Attempt *ad-bc* for first M1

$\frac{1}{\det}\begin{pmatrix} -1 & -a \\ 1 & 2 \end{pmatrix}$ for second M1

(b) Final A1 for correct solution only

8	(a)	$\frac{dy}{dx} = a^{\frac{1}{2}}x^{-\frac{1}{2}}$ or $2y\frac{\mathrm{d}y}{\mathrm{d}x} = 4a$ The gradient of the tangent is $\frac{1}{q}$ The equation of the tangent is $y - 2aq = \frac{1}{q}(x - aq^2)$ So $yq = x + aq^2$ *	M1 A1 M1 A1 (4)
	(b)	R has coordinates (0, aq) The line l has equation $y - aq = -qx$	B1 M1A1 (3)
	(c)	When $y = 0$ $x = a$ (so line l passes through (a, 0) the focus of the parabola.)	B1 (1)
	(d)	Line l meets the directrix when $x = -a$: Then $y = 2aq$. So coordinates are ($-a$, $2aq$)	M1:A1 (2) (10 marks)

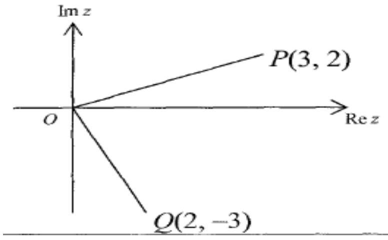
Notes:

(a) $\frac{dy}{dx} = \frac{2a}{2aq}$ OK for M1
Use of $y = mx + c$ to find c OK for second M1
Correct solution only for final A1

(b) -1/(their gradient in part a) in equation OK for M1

(c) They must attempt $y = 0$ or $x = a$ to show correct coordinates of R for B1

(d) Substitute $x = -a$ for M1.
Both coordinates correct for A1.

Question Number	Scheme	Marks
9. (a)	$z_2 = \frac{12-5\text{i}}{3+2\text{i}} \times \frac{3-2\text{i}}{3-2\text{i}} = \frac{36-24\text{i}-15\text{i}-10}{13}$	M1
	$= 2 - 3\text{i}$	A1 (2)
(b)		
	$Q(2, -3)$ P: B1, Q: B1ft	B1, B1ft (2)
(c)	grad. $OP \times$ grad. $OQ = \frac{2}{3} \times -\frac{3}{2}$	
	$= -1 \quad \Rightarrow \angle POQ = \frac{\pi}{2} \quad (\ast)$	
OR	$\angle POX = \tan^{-1}\frac{2}{3}$, $\angle QOX = \tan^{-1}\frac{3}{2}$	M1
	$\text{Tan}(\angle POQ) = \frac{\frac{2}{3}+\frac{3}{2}}{1-\frac{2}{3}\times\frac{3}{2}}$ M1	A1
	$\Rightarrow \angle POQ = \frac{\pi}{2} \quad (\ast)$ A1	(2)
(d)	$z = \frac{3+2}{2} + \frac{2+(-3)}{2}\text{i}$	M1
	$= \frac{5}{2} - \frac{1}{2}\text{i}$	A1 (2)
(e)	$r = \sqrt{\left(\frac{5}{2}\right)^2 + \left(-\frac{1}{2}\right)^2}$	M1
	$= \frac{\sqrt{26}}{2}$ or exact equivalent	A1 (2)
		(10 marks)

Notes:

(a) $\times \frac{3-2i}{3-2i}$ for M1

(b) Position of points not clear award B1B0

(c) Use of calculator / decimals award M1A0

(d) Final answer must be in complex form for A1

(e) Radius or diameter for M1

10 (a)	**A** represents an enlargement scale factor $3\sqrt{2}$ (centre *O*) **B** represents reflection in the line $y = x$ **C** represents a rotation of $\frac{\pi}{4}$, i.e.45° (anticlockwise) (about O)	M1 A1 B1 B1 (4)
(b)	$\begin{pmatrix} 3 & -3 \\ 3 & 3 \end{pmatrix}$	M1 A1 (2)
(c)	$\begin{pmatrix} 3 & -3 \\ 3 & 3 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -3 & 3 \\ 3 & 3 \end{pmatrix}$	B1 (1)
(d)	$\begin{pmatrix} -3 & 3 \\ 3 & 3 \end{pmatrix}\begin{pmatrix} 0 & -15 & 4 \\ 0 & 15 & 21 \end{pmatrix} = \begin{pmatrix} 0 & 90 & 51 \\ 0 & 0 & 75 \end{pmatrix}$ so (0, 0), (90 , 0) and (51 , 75)	M1A1A1A1 (4)
(e)	Area of $\Delta\, OR'S'$ is $\frac{1}{2}\times 90\times 75 = 3375$ Determinant of **E** is –18 or use area scale factor of enlargement So area of $\Delta\, ORS$ is $3375 \div 18 = 187.5$	B1 M1A1 (3) (14 marks)

Notes:

(a) Enlargement for M1
$3\sqrt{2}$ for A1

(b) Answer incorrect, require CD for M1

(c) Answer given so require DB as shown for B1

(d) Coordinates as shown or written as $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 90 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 51 \\ 75 \end{pmatrix}$ for each A1

(e) 3375 B1
Divide by theirs for M1