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Mark Scheme (Final)

January 2009

GCE

GCE Core Mathematics C4 (6666/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

January 2009
6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme		Marks
1. (a)	$C:\ \ y^2 - 3y = x^3 + 8$		
	$\left\{\cancel{\frac{\mathrm{d}y}{\mathrm{d}x}}\cancel{\times}\right\}\quad 2y\frac{\mathrm{d}y}{\mathrm{d}x} - 3\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2$	Differentiates implicitly to include either $\pm ky\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\pm 3\frac{\mathrm{d}y}{\mathrm{d}x}$. (Ignore $\left(\frac{\mathrm{d}y}{\mathrm{d}x} = \right)$.)	M1
		Correct equation.	A1
	$(2y-3)\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2$	A correct (condoning sign error) attempt to combine or factorise their '$2y\frac{\mathrm{d}y}{\mathrm{d}x} - 3\frac{\mathrm{d}y}{\mathrm{d}x}$'. Can be implied.	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x^2}{2y-3}$	$\underline{\frac{3x^2}{2y-3}}$	A1 oe
			[4]
(b)	$y = 3 \Rightarrow 9 - 3(3) = x^3 + 8$	Substitutes $y = 3$ into C.	M1
	$x^3 = -8 \Rightarrow \underline{x = -2}$	Only $\underline{x = -2}$	A1
	$(-2, 3) \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3(4)}{6-3} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 4$	$\frac{\mathrm{d}y}{\mathrm{d}x} = 4$ from correct working. Also can be ft using their 'x' value and $y = 3$ in the correct part (a) of $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x^2}{2y-3}$	A1 $\sqrt{}$
			[3]
			7 marks

1(b) final A1 $\sqrt{}$. Note if the candidate inserts their x value and $y = 3$ into $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x^2}{2y-3}$, then an answer of $\frac{\mathrm{d}y}{\mathrm{d}x} =$ their x^2, ***may*** indicate a correct follow through.

Question Number	Scheme		Marks
2. (a)	Area(R) = $\int_0^2 \frac{3}{\sqrt{(1+4x)}}\,dx = \int_0^2 3(1+4x)^{-\frac{1}{2}}\,dx$		
	$= \left[\frac{3(1+4x)^{\frac{1}{2}}}{\frac{1}{2}.4}\right]_0^2$	***Integrating*** $3(1+4x)^{-\frac{1}{2}}$ to give $\pm k(1+4x)^{\frac{1}{2}}$.	M1
		<u>Correct integration.</u> Ignore limits.	<u>A1</u>
	$= \left[\frac{3}{2}(1+4x)^{\frac{1}{2}}\right]_0^2$		
	$= \left(\frac{3}{2}\sqrt{9}\right) - \left(\frac{3}{2}(1)\right)$	Substitutes limits of 2 and 0 into a changed function and subtracts the correct way round.	M1
	$= \frac{9}{2} - \frac{3}{2} = \underline{3}\,(\text{units})^2$	<u>3</u>	<u>A1</u>
			[4]
	(Answer of 3 with no working scores M0A0M0A0.)		
(b)	Volume $= \underline{\pi \int_0^2 \left(\frac{3}{\sqrt{(1+4x)}}\right)^2 dx}$	Use of $\underline{V = \pi \int y^2\,dx}$. Can be implied. Ignore limits and dx.	B1
	$= (\pi)\int_0^2 \frac{9}{1+4x}\,dx$		
	$= (\pi)\left[\frac{9}{4}\ln\|1+4x\|\right]_0^2$	$\pm k\ln\|1+4x\|$	M1
		$\frac{9}{4}\ln\|1+4x\|$	A1
	$= (\pi)\left[\left(\frac{9}{4}\ln 9\right) - \left(\frac{9}{4}\ln 1\right)\right]$	Substitutes limits of 2 and 0 and subtracts the correct way round.	dM1
	So Volume $= \underline{\frac{9}{4}\pi\ln 9}$	$\underline{\frac{9}{4}\pi\ln 9}$ or $\underline{\frac{9}{2}\pi\ln 3}$ or $\underline{\frac{18}{4}\pi\ln 3}$	A1 oe isw
			[5]
			9 marks

Note the answer must be a one term exact value. Note, also you can ignore subsequent working here.

Note that $\ln 1$ can be implied as equal to 0.

Note that $= \frac{9}{4}\pi\ln 9 + c$ (oe.) would be awarded the final A0.

Question Number	Scheme		Marks
3. (a)	$27x^2+32x+16 \equiv A(3x+2)(1-x)+B(1-x)+C(3x+2)^2$	Forming this identity	M1
	$x=-\frac{2}{3},\quad 12-\frac{64}{3}+16=\left(\frac{5}{3}\right)B \Rightarrow \frac{20}{3}=\left(\frac{5}{3}\right)B \Rightarrow B=4$ $x=1,\quad 27+32+16=25C \Rightarrow 75=25C \Rightarrow C=3$	Substitutes either $x=-\frac{2}{3}$ or $x=1$ into their identity or equates 3 terms or substitutes in values to write down three simultaneous equations.	M1
		Both $B=4$ and $C=3$ **(Note the A1 is dependent on both method marks in this part.)**	A1
	Equate x^2: $27=-3A+9C \Rightarrow 27=-3A+27 \Rightarrow 0=-3A \Rightarrow A=0$ $x=0,\quad 16=2A+B+4C \Rightarrow 16=2A+4+12 \Rightarrow 0=2A \Rightarrow A=0$	Compares coefficients or substitutes in a third x-value or uses simultaneous equations to show $A=0$.	B1
			[4]
(b)	$\mathrm{f}(x)=\dfrac{4}{(3x+2)^2}+\dfrac{3}{(1-x)}$		
	$=4(3x+2)^{-2}+3(1-x)^{-1}$	Moving powers to top on any one of the two expressions	M1
	$=4\left[2\left(1+\frac{3}{2}x\right)^{-2}\right]+3(1-x)^{-1}$		
	$=1\left(1+\frac{3}{2}x\right)^{-2}+3(1-x)^{-1}$		
	$=1\left\{\underline{1+(-2)(\tfrac{3x}{2});+\frac{(-2)(-3)}{2!}(\tfrac{3x}{2})^2+\ldots}\right\}$	Either $1\pm(-2)(\frac{3x}{2})$ or $1\pm(-1)(-x)$ from either first or second expansions respectively	dM1;
	$+3\left\{\underline{1+(-1)(-x);+\frac{(-1)(-2)}{2!}(-x)^2+\ldots}\right\}$	Ignoring 1 and 3, any one correct $\{\underline{\ldots\ldots\ldots}\}$ expansion.	A1
		Both $\{\underline{\ldots\ldots\ldots}\}$ correct.	A1
	$=\left\{1-3x+\frac{27}{4}x^2+\ldots\right\}+3\left\{1+x+x^2+\ldots\right\}$		
	$=4+0x\,;+\frac{39}{4}x^2$	$4+(0x)\,;\ \frac{39}{4}x^2$	A1; A1
			[6]

Question Number	Scheme		Marks
3. (c)	$\text{Actual} = \text{f}(0.2) = \dfrac{1.08+6.4+16}{(6.76)(0.8)}$ $= \dfrac{23.48}{5.408} = 4.341715976... = \dfrac{2935}{676}$	Attempt to find the actual value of f(0.2) or seeing awrt 4.3 and believing it is candidate's actual f(0.2).	
	Or $\text{Actual} = \text{f}(0.2) = \dfrac{4}{(3(0.2)+2)^2} + \dfrac{3}{(1-0.2)}$ $= \dfrac{4}{6.76} + 3.75 = 4.341715976... = \dfrac{2935}{676}$	Candidates can also attempt to find the actual value by using $\dfrac{A}{(3x+2)} + \dfrac{B}{(3x+2)^2} + \dfrac{C}{(1-x)}$ with their A, B and C.	M1
	$\text{Estimate} = \text{f}(0.2) = 4 + \tfrac{39}{4}(0.2)^2$ $= 4 + 0.39 = 4.39$	Attempt to find an estimate for f(0.2) using their answer to (b)	M1 $\sqrt{}$
	$\%\text{age error} = \dfrac{\lvert 4.39 - 4.341715976... \rvert}{4.341715976...} \times 100$	$\left\lvert \dfrac{\text{their estimate - actual}}{\text{actual}} \right\rvert \times 100$	M1
	$= 1.112095408... = 1.1\%\,(2\text{sf})$	1.1%	A1 **cao** **[4]**
			14 marks

Question Number	Scheme		Marks
4. (a)	$\mathbf{d}_1 = -2\mathbf{i} + \mathbf{j} - 4\mathbf{k}$, $\mathbf{d}_2 = q\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$		
	As $\left\{\mathbf{d}_1 \bullet \mathbf{d}_2 = \begin{pmatrix}-2\\1\\-4\end{pmatrix} \bullet \begin{pmatrix}q\\2\\2\end{pmatrix}\right\} = \underline{(-2\times q) + (1\times 2) + (-4\times 2)}$	Apply dot product calculation between two direction vectors, ie. $\underline{(-2\times q) + (1\times 2) + (-4\times 2)}$	M1
	$\mathbf{d}_1 \bullet \mathbf{d}_2 = 0 \Rightarrow -2q + 2 - 8 = 0$ $-2q = 6 \Rightarrow \underline{q = -3}$ AG	Sets $\mathbf{d}_1 \bullet \mathbf{d}_2 = 0$ and solves to find $\underline{q = -3}$	A1 **cso**
			[2]
(b)	Lines meet where:		
	$\begin{pmatrix}11\\2\\17\end{pmatrix} + \lambda\begin{pmatrix}-2\\1\\-4\end{pmatrix} = \begin{pmatrix}-5\\11\\p\end{pmatrix} + \mu\begin{pmatrix}q\\2\\2\end{pmatrix}$		
	First two of $\mathbf{i}$: $11 - 2\lambda = -5 + q\mu$ (1) $\mathbf{j}$: $2 + \lambda = 11 + 2\mu$ (2) $\mathbf{k}$: $17 - 4\lambda = p + 2\mu$ (3)	Need to see equations (1) and (2). Condone one slip. (Note that $q = -3$.)	M1
	(1) + 2**(2)** gives: $15 = 17 + \mu \Rightarrow \mu = -2$	Attempts to solve (1) and (2) to find one of either λ or μ	dM1
	(2) gives: $2 + \lambda = 11 - 4 \Rightarrow \lambda = 5$	Any one of $\underline{\lambda = 5}$ or $\underline{\mu = -2}$	A1
		Both $\underline{\lambda = 5}$ and $\underline{\mu = -2}$	A1
	(3) $\Rightarrow 17 - 4(5) = p + 2(-2)$	Attempt to substitute their λ and μ into their $\mathbf{k}$ component to give an equation in p alone.	ddM1
	$\Rightarrow p = 17 - 20 + 4 \Rightarrow \underline{p = 1}$	$\underline{p = 1}$	A1 **cso**
			[6]
(c)	$\mathbf{r} = \begin{pmatrix}11\\2\\17\end{pmatrix} + 5\begin{pmatrix}-2\\1\\-4\end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix}-5\\11\\1\end{pmatrix} - 2\begin{pmatrix}-3\\2\\2\end{pmatrix}$	Substitutes their value of λ or μ into the correct line l_1 or l_2.	M1
	Intersect at $\mathbf{r} = \underline{\begin{pmatrix}1\\7\\-3\end{pmatrix}}$ or $\underline{(1, 7, -3)}$	$\underline{\begin{pmatrix}1\\7\\-3\end{pmatrix}}$ or $\underline{(1, 7, -3)}$	A1
			[2]

Question Number	Scheme		Marks
(d)	Let $\overrightarrow{OX} = \mathbf{i} + 7\mathbf{j} - 3\mathbf{k}$ be point of intersection		
	$\overrightarrow{AX} = \overrightarrow{OX} - \overrightarrow{OA} = \begin{pmatrix}1\\7\\-3\end{pmatrix} - \begin{pmatrix}9\\3\\13\end{pmatrix} = \begin{pmatrix}-8\\4\\-16\end{pmatrix}$	Finding vector $\overrightarrow{AX}$ by finding the difference between $\overrightarrow{OX}$ and $\overrightarrow{OA}$. Can be ft using candidate's $\overrightarrow{OX}$.	M1 $\sqrt{\ }$ ±
	$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OA} + 2\overrightarrow{AX}$		
	$\overrightarrow{OB} = \begin{pmatrix}9\\3\\13\end{pmatrix} + 2\begin{pmatrix}-8\\4\\-16\end{pmatrix}$	$\begin{pmatrix}9\\3\\13\end{pmatrix} + 2\begin{pmatrix}\text{their } \overrightarrow{AX}\end{pmatrix}$	dM1 $\sqrt{\ }$
	Hence, $\overrightarrow{OB} = \underline{\begin{pmatrix}-7\\11\\-19\end{pmatrix}}$ or $\overrightarrow{OB} = \underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$	$\underline{\begin{pmatrix}-7\\11\\-19\end{pmatrix}}$ or $\underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$ or $\underline{(-7, 11, -19)}$	A1
			[3]
			13 marks

Question Number	Scheme		Marks
5. (a)	Similar triangles $\Rightarrow \underline{\dfrac{r}{h} = \dfrac{16}{24}} \Rightarrow \underline{r = \dfrac{2h}{3}}$	Uses similar triangles, ratios or trigonometry to find either one of these two expressions oe.	M1
	$V = \dfrac{1}{3}\pi r^2 h = \dfrac{1}{3}\pi\left(\dfrac{2h}{3}\right)^2 h = \dfrac{4\pi h^3}{27}$ **AG**	Substitutes $r = \frac{2h}{3}$ into the formula for the volume of water V.	A1
			[2]
(b)	From the question, $\dfrac{dV}{dt} = 8$	$\dfrac{dV}{dt} = 8$	B1
	$\underline{\dfrac{dV}{dh}} = \underline{\dfrac{12\pi h^2}{27}} = \underline{\dfrac{4\pi h^2}{9}}$	$\underline{\dfrac{dV}{dh}} = \underline{\dfrac{12\pi h^2}{27}}$ or $\underline{\dfrac{4\pi h^2}{9}}$	B1
	$\dfrac{dh}{dt} = \dfrac{dV}{dt} \div \dfrac{dV}{dh} = \underline{8 \times \dfrac{9}{4\pi h^2}} = \underline{\dfrac{18}{\pi h^2}}$	Candidate's $\dfrac{dV}{dt} \div \dfrac{dV}{dh}$;	M1;
		$\underline{8 \div \left(\dfrac{12\pi h^2}{27}\right)}$ or $\underline{8 \times \dfrac{9}{4\pi h^2}}$ or $\underline{\dfrac{18}{\pi h^2}}$ oe	A1
	When $h = 12$, $\dfrac{dh}{dt} = \underline{\dfrac{18}{144\pi}} = \underline{\dfrac{1}{8\pi}}$	$\underline{\dfrac{18}{144\pi}}$ or $\underline{\dfrac{1}{8\pi}}$	A1 oe isw
			[5]
			7 marks

Note the answer must be a one term exact value.

Note, also you can ignore subsequent working after $\underline{\dfrac{18}{144\pi}}$.

Question Number	Scheme		Marks
6. (a)	$\int \tan^2 x \, dx$		
	$\left[NB: \underline{\sec^2 A = 1 + \tan^2 A} \text{ gives } \underline{\tan^2 A = \sec^2 A - 1} \right]$	The correct underlined identity.	M1 oe
	$= \int \sec^2 x - 1 \, dx$		
	$= \underline{\tan x} - x \, (+c)$	Correct integration with/without + c	A1
			[2]
(b)	$\int \frac{1}{x^3} \ln x \, dx$		
	$\left\{ \begin{array}{lll} u = \ln x & \Rightarrow & \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = x^{-3} & \Rightarrow & v = \frac{x^{-2}}{-2} = \frac{-1}{2x^2} \end{array} \right\}$		
	$= -\frac{1}{2x^2} \ln x - \int -\frac{1}{2x^2} . \frac{1}{x} \, dx$	Use of 'integration by parts' formula in the correct direction.	M1
		Correct expression.	A1
	$= -\frac{1}{2x^2} \ln x + \frac{1}{2} \int \frac{1}{x^3} \, dx$	An attempt to multiply through $\frac{k}{x^n}$, $n \in$ □ , $n \ldots 2$ by $\frac{1}{x}$ and an attempt to ...	
	$= \underline{-\frac{1}{2x^2} \ln x + \frac{1}{2}\left(-\frac{1}{2x^2}\right)} \, (+c)$	… "integrate"(process the result);	M1
		<u>correct solution</u> with/without + c	A1 oe
			[4]

Correct direction means that $u = \ln x$.

Question Number	Scheme		Marks
(c)	$\int \frac{e^{3x}}{1+e^x}\,dx$		
	$\left\{u = 1+e^x \Rightarrow \underline{\frac{du}{dx} = e^x}, \ \underline{\frac{dx}{du} = \frac{1}{e^x}}, \ \underline{\frac{dx}{du} = \frac{1}{u-1}}\right\}$	Differentiating to find any one of the <u>three underlined</u>	<u>B1</u>
	$= \int \frac{e^{2x}.e^x}{1+e^x}\,dx = \int \frac{(u-1)^2.e^x}{u}.\frac{1}{e^x}\,du$ or $= \int \frac{(u-1)^3}{u}.\frac{1}{(u-1)}\,du$	Attempt to substitute for $e^{2x} = f(u)$, their $\frac{dx}{du} = \frac{1}{e^x}$ and $u = 1+e^x$ or $e^{3x} = f(u)$, their $\frac{dx}{du} = \frac{1}{u-1}$ and $u = 1+e^x$.	M1*
	$= \int \frac{(u-1)^2}{u}\,du$	$\underline{\int \frac{(u-1)^2}{u}\,du}$	A1
	$= \int \frac{u^2-2u+1}{u}\,du$ $= \int u - 2 + \frac{1}{u}\,du$	An attempt to multiply out their numerator to give at least three terms and divide through each term by u	dM1*
	$= \frac{u^2}{2} - 2u + \ln u \ (+c)$	Correct integration with/without +c	A1
	$= \frac{(1+e^x)^2}{2} - 2(1+e^x) + \ln(1+e^x) + c$	Substitutes $u = 1+e^x$ back into their integrated expression with at least two terms.	dM1*
	$= \tfrac{1}{2} + e^x + \tfrac{1}{2}e^{2x} - 2 - 2e^x + \ln(1+e^x) + c$ $= \tfrac{1}{2} + e^x + \tfrac{1}{2}e^{2x} - 2 - 2e^x + \ln(1+e^x) + c$ $= \tfrac{1}{2}e^{2x} - e^x + \ln(1+e^x) - \tfrac{3}{2} + c$ $= \tfrac{1}{2}e^{2x} - e^x + \ln(1+e^x) + k$ **AG**	$\underline{\tfrac{1}{2}e^{2x} - e^x + \ln(1+e^x) + k}$ must use a $+c$ and "$-\tfrac{3}{2}$" combined.	A1 **cso** **[7]**
			13 marks

Question Number	Scheme		Marks
7. (a)	At A, $x=-1+8=7$ & $y=(-1)^2=1 \Rightarrow A(7,1)$	$A(7,1)$	B1 **[1]**
(b)	$x=t^3-8t$, $y=t^2$,		
	$\frac{dx}{dt}=3t^2-8$, $\frac{dy}{dt}=2t$		
	$\therefore \frac{dy}{dx}=\frac{2t}{3t^2-8}$	Their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ Correct $\frac{dy}{dx}$	M1 A1
	At A, $m(\mathbf{T})=\underline{\frac{2(-1)}{3(-1)^2-8}}=\underline{\frac{-2}{3-8}}=\underline{\frac{-2}{-5}}=\underline{\frac{2}{5}}$	Substitutes for t to give any of the four underlined oe:	<u>A1</u>
	$\mathbf{T}$: $y-(\text{their }1)=m_T\left(x-(\text{their }7)\right)$ or $1=\frac{2}{5}(7)+c \Rightarrow c=1-\frac{14}{5}=-\frac{9}{5}$ Hence $\mathbf{T}$: $y=\frac{2}{5}x-\frac{9}{5}$	Finding an equation of a tangent with their point and their tangent gradient or finds c and uses $y=(\text{their gradient})x+"c"$.	dM1
	gives $\mathbf{T}$: $\underline{2x-5y-9=0}$ **AG**	$\underline{2x-5y-9=0}$	A1 **cso** **[5]**
(c)	$2(t^3-8t)-5t^2-9=0$	Substitution of both $x=t^3-8t$ and $y=t^2$ into $\mathbf{T}$	M1
	$2t^3-5t^2-16t-9=0$		
	$(t+1)\{(2t^2-7t-9)=0\}$ $(t+1)\{(t+1)(2t-9)=0\}$	A realisation that $(t+1)$ is a factor.	dM1
	$\{t=-1 \text{ (at } A)\}$ $t=\frac{9}{2}$ at B	$t=\frac{9}{2}$	A1
	$x=\left(\frac{9}{2}\right)^2-8\left(\frac{9}{2}\right)=\frac{729}{8}-36=\frac{441}{8}=55.125$ or awrt 55.1	Candidate uses their value of t to find either the x or y coordinate	ddM1
	$y=\left(\frac{9}{2}\right)^2=\frac{81}{4}=20.25$ or awrt 20.3	One of either x or y correct. Both x and y correct.	A1 A1
	Hence $B\left(\frac{441}{8},\frac{81}{4}\right)$	awrt	**[6]**
			12 marks

- Note: dM1 denotes a method mark which is dependent upon the award of the previous method mark.
  ddM1 denotes a method mark which is dependent upon the award of the previous two method marks.
  Oe or equivalent.

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Appendix

Question 1

Question Number	Scheme		Marks
Aliter **1.** (a) **Way 2**	$C: \ y^2 - 3y = x^3 + 8$		
	$\left\{\frac{\cancel{\mathrm{d}x}}{\cancel{\mathrm{d}y}} \cancel{=}\right\} \quad 2y - 3 = 3x^2\frac{\mathrm{d}x}{\mathrm{d}y}$	Differentiates implicitly to include either $\pm kx^2\frac{\mathrm{d}x}{\mathrm{d}y}$. (Ignore $\left(\frac{\mathrm{d}x}{\mathrm{d}y} = \right)$.)	M1
		Correct equation.	A1
	$2y - 3 = 3x^2\frac{1}{\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)}$	Applies $\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{1}{\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)}$	dM1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x^2}{2y-3}$	$\underline{\frac{3x^2}{2y-3}}$	A1 oe
			[4]
Aliter **1.** (a) **Way 3**	$C: \ y^2 - 3y = x^3 + 8$		
	gives $x^3 = y^2 - 3y - 8$ $\Rightarrow \ x = \left(y^2 - 3y - 8\right)^{\frac{1}{3}}$		
	$\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{1}{3}\left(y^2 - 3y - 8\right)^{-\frac{2}{3}}(2y - 3)$	Differentiates in the form $\frac{1}{3}(\mathrm{f}(y))^{-\frac{2}{3}}(\mathrm{f}'(y))$.	M1
		Correct differentiation.	A1
	$\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{2y - 3}{3\left(y^2 - 3y - 8\right)^{\frac{2}{3}}}$		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3\left(y^2 - 3y - 8\right)^{\frac{2}{3}}}{2y - 3}$	Applies $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\left(\frac{\mathrm{d}x}{\mathrm{d}y}\right)}$	dM1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \underline{\frac{3\left(x^3\right)^{\frac{2}{3}}}{2y - 3}} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \underline{\frac{3x^2}{2y-3}}$	$\underline{\frac{3\left(x^3\right)^{\frac{2}{3}}}{2y - 3}}$ or $\underline{\frac{3x^2}{2y-3}}$	A1 oe
			[4]

Question 2

Question Number	Scheme		Marks
Aliter **2.** (a) **Way 2**	$\text{Area}(R) = \int_0^2 \frac{3}{\sqrt{(1+4x)}}\,\mathrm{d}x = \int_0^2 3(1+4x)^{-\frac{1}{2}}\,\mathrm{d}x$		
	{Using substitution $u = 1+4x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 4$ }		
	{change limits: When $x=0$, $u=1$ & when $x=2$, $u=9$ }		
	So, $\text{Area}(R) = \int_1^9 3u^{-\frac{1}{2}}\,\frac{1}{4}\mathrm{d}u$		
	$= \left[\frac{3}{4}\,\frac{u^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}\right]_0^2$	***Integrating*** $\pm\lambda u^{-\frac{1}{2}}$ to give $\pm k u^{\frac{1}{2}}$.	M1
		<u>Correct integration.</u> Ignore limits.	<u>A1</u>
	$= \left[\frac{3}{2}u^{\frac{1}{2}}\right]_1^9$		
	$= \left(\frac{3}{2}\sqrt{9}\right) - \left(\frac{3}{2}(1)\right)$	Substitutes limits of either $(u=9 \text{ and } u=1)$ or in x, $(x=2 \text{ and } x=0)$ into a changed function and subtracts the correct way round .	M1
	$= \frac{9}{2} - \frac{3}{2} = \underline{3}\,(\text{units})^2$	$\underline{3}$	<u>A1</u>
			[4]
Aliter **2.** (a) **Way 3**	$\text{Area}(R) = \int_0^2 \frac{3}{\sqrt{(1+4x)}}\,\mathrm{d}x = \int_0^2 3(1+4x)^{-\frac{1}{2}}\,\mathrm{d}x$		
	{Using substitution $u^2 = 1+4x \Rightarrow 2u\frac{\mathrm{d}u}{\mathrm{d}x} = 4 \Rightarrow \frac{1}{2}u\mathrm{d}u = \mathrm{d}x$ }		
	{change limits: When $x=0$, $u=1$ & when $x=2$, $u=3$ }		
	So, $\text{Area}(R) = \int_1^3 \frac{3}{u}\,\frac{1}{2}u\,\mathrm{d}u = \int_1^3 \frac{3}{2}\mathrm{d}u$		
	$= \left[\frac{3}{2}u\right]_1^3$	***Integrating*** $\pm\lambda$ to give $\pm k u$.	M1
		<u>Correct integration.</u> Ignore limits.	<u>A1</u>
	$= \left(\frac{3}{2}(3)\right) - \left(\frac{3}{2}(1)\right)$	Substitutes limits of either $(u=3 \text{ and } u=1)$ or in x, $(x=2 \text{ and } x=0)$ into a changed function and subtracts the correct way round .	M1
	$= \frac{9}{2} - \frac{3}{2} = \underline{3}\,(\text{units})^2$	$\underline{3}$	<u>A1</u>
			[4]

Question 3

Question Number	Scheme	Marks
Aliter **3.** (a) **Way 2**	$27x^2+32x+16 \equiv A(3x+2)(1-x)+B(1-x)+C(3x+2)^2$ Forming this identity	M1
	x^2 terms: $27=-3A+9C$ (1) x terms: $32=A-B+12C$ (2) constants: $16=2A+B+4C$ (3) equates 3 terms.	M1
	(2) + **(3)** gives $48=3A+16C$ **(4)**	
	(1) + **(4)** gives $75=25C \Rightarrow C=3$	
	(1) gives $27=-3A+27 \Rightarrow 0=-3A \Rightarrow A=0$	
	(2) gives $32=-B+36 \Rightarrow B=36-32=4$ Both $B=4$ and $C=3$ Decide to award B1 for $A=0$	A1 B1 **[4]**
3. (a)	If the candidate assumes $A=0$ and writes the identity $27x^2+32x+16 \equiv B(1-x)+C(3x+2)^2$ and goes on to find $B=4$ and $C=3$ then the candidate is awarded M0M1A0B0.	
3. (a)	If the candidate has the incorrect identity $27x^2+32x+16 \equiv A(3x+2)+B(1-x)+C(3x+2)^2$ and goes on to find $B=4$, $C=3$ and $A=0$ then the candidate is awarded M0M1A0B1.	
3. (a)	If the candidate has the incorrect identity $27x^2+32x+16 \equiv A(3x+2)^2(1-x)+B(1-x)+C(3x+2)^2$ and goes on to find $B=4$, $C=3$ and $A=0$ then the candidate is awarded M0M1A0B1.	

Question Number	Scheme		Marks
Aliter **3.** (b) ***Way 2***	$\mathrm{f}(x) = \dfrac{4}{(3x+2)^2} + \dfrac{3}{(1-x)}$		
	$= 4(3x+2)^{-2} + 3(1-x)^{-1}$	Moving powers to top on any one of the two expressions	M1
	$= 4(2+3x)^{-2} + 3(1-x)^{-1}$		
	$= 4\left\{\underline{(2)^{-2} + (-2)(2)^{-3}(3x); + \frac{(-2)(-3)}{2!}(2)^{-4}(3x)^2 +}\right\}$	Either $(2)^{-2} \pm (-2)(2)^{-3}(3x)$ or $1 \pm (-1)(-x)$ from either first or second expansions respectively	dM1;
	$+ 3\left\{\underline{1 + (-1)(-x); + \frac{(-1)(-2)}{2!}(-x)^2 + \dots}\right\}$	Ignoring 1 and 3, any one correct $\{\underline{\dots\dots\dots\dots}\}$ expansion.	A1
		Both $\{\underline{\dots\dots\dots\dots}\}$ correct.	A1
	$= 4\left\{\frac{1}{4} - \frac{3}{4}x + \frac{27}{16}x^2 + \dots\right\} + 3\left\{1 + x + x^2 + \dots\right\}$		
	$= 4 + 0x\ ; + \frac{39}{4}x^2$	$4 + (0x)\ ;\ \frac{39}{4}x^2$	A1; A1 **[6]**

Question Number	Scheme		Marks
Aliter **3.** (c) **Way 2**	Actual $= \mathrm{f}(0.2) = \dfrac{1.08+6.4+16}{(6.76)(0.8)}$ $= \dfrac{23.48}{5.408} = 4.341715976... = \dfrac{2935}{676}$	Attempt to find the actual value of f(0.2)	M1
	Estimate $= \mathrm{f}(0.2) = 4 + \frac{39}{4}(0.2)^2$ $= 4 + 0.39 = 4.39$	Attempt to find an estimate for f(0.2) using their answer to (b)	M1 $\sqrt{}$
	%age error $= \left\| 100 - \left(\dfrac{4.39}{4.341715976...} \times 100\right)\right\|$	$\left\|100 - \left(\left(\dfrac{\text{their estimate}}{\text{actual}}\right)\times 100\right)\right\|$	M1
	$= \|100 - 101.1120954\|$		
	$= \|-1.112095408...\| = 1.1\%(2\text{sf})$	1.1%	A1 **cao**
			[4]
3. (c)	Note that: %age error $= \dfrac{\|4.39 - 4.341715976...\|}{4.39} \times 100$ $= 1.0998638... = 1.1\%(2\text{sf})$	Should be awarded the final marks of M0A0	
3. (c)	Also note that: %age error $= \left\|100 - \left(\dfrac{4.341715976...}{4.39} \times 100\right)\right\|$ $= 1.0998638... = 1.1\%(2\text{sf})$	Should be awarded the final marks of M0A0	
	…so be wary of 1.0998638…		

Question 4

Question Number	Scheme		Marks
4. (a)	$-2q + 2 - 8$ is sufficient for M1.		
Aliter **4.** (b) **Way 2**	Only apply **Way 2** if candidate does not find both λ and μ. Lines meet where: $\begin{pmatrix}11\\2\\17\end{pmatrix} + \lambda\begin{pmatrix}-2\\1\\-4\end{pmatrix} = \begin{pmatrix}-5\\11\\p\end{pmatrix} + \mu\begin{pmatrix}q\\2\\2\end{pmatrix}$		
	First two of $\begin{array}{lll}\mathbf{i}: & 11 - 2\lambda = -5 + q\mu & (1)\\ \mathbf{j}: & 2 + \lambda = 11 + 2\mu & (2)\\ \mathbf{k}: & 17 - 4\lambda = p + 2\mu & (3)\end{array}$	Need to see equations (2) and (2). Condone one slip. (Note that $q = -3$.)	M1
	(2) gives $\lambda = 9 + 2\mu$		
	(1) gives $11 - 2(9 + 2\mu) = -5 - 3\mu$	Attempts to solve (1) and (2) to find one of either λ or μ	dM1
	$11 - 18 - 4\mu = -5 - 3\mu$		
	gives: $11 - 18 + 5 = \mu \Rightarrow \mu = -2$	Any one of $\underline{\lambda = 5}$ or $\underline{\mu = -2}$	A1
	(3) gives $17 - 4(9 + 2\mu) = p + 2\mu$	Candidate writes down a correct equation containing p and one of either λ or μ which has already been found.	A1
	(3) $\Rightarrow 17 - 4(9 + 2(-2)) = p + 2(-2)$	Attempt to substitute their value for $\lambda\ (= 9 + 2\mu)$ and μ into their **k** component to give an equation in p alone.	ddM1
	$\Rightarrow 17 - 20 = p - 4 \Rightarrow \underline{p = 1}$	$\underline{p = 1}$	A1 **cso** **[6]**
4. (c)	If no working is shown then any two out of the three coordinates can imply the first M1 mark.		M1
	Intersect at $\mathbf{r} = \underline{\begin{pmatrix}1\\7\\-3\end{pmatrix}}$ or $\underline{(1, 7, -3)}$	$\underline{\begin{pmatrix}1\\7\\-3\end{pmatrix}}$ or $\underline{(1, 7, -3)}$	A1 **[2]**

Question Number	Scheme		Marks
Aliter **4.** (d) **Way 2**	Let $\overrightarrow{OX} = \mathbf{i} + 7\mathbf{j} - 3\mathbf{k}$ be point of intersection		
	$\overrightarrow{AX} = \overrightarrow{OX} - \overrightarrow{OA} = \begin{pmatrix}1\\7\\-3\end{pmatrix} - \begin{pmatrix}9\\3\\13\end{pmatrix} = \begin{pmatrix}-8\\4\\-16\end{pmatrix}$	Finding the difference between their $\overrightarrow{OX}$ (can be implied) and $\overrightarrow{OA}$. $\overrightarrow{AX} = \pm\left(\begin{pmatrix}1\\7\\-3\end{pmatrix} - \begin{pmatrix}9\\3\\13\end{pmatrix}\right)$	M1$\sqrt{}$ ±
	$\overrightarrow{OB} = \overrightarrow{OX} + \overrightarrow{XB} = \overrightarrow{OX} + \overrightarrow{AX}$		
	$\overrightarrow{OB} = \begin{pmatrix}1\\7\\-3\end{pmatrix} + \begin{pmatrix}-8\\4\\-16\end{pmatrix}$	$\left(\text{their } \overrightarrow{OX}\right) + \left(\text{their } \overrightarrow{AX}\right)$	dM1$\sqrt{}$
	Hence, $\overrightarrow{OB} = \underline{\begin{pmatrix}-7\\11\\-19\end{pmatrix}}$ or $\overrightarrow{OB} = \underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$	$\underline{\begin{pmatrix}-7\\11\\-19\end{pmatrix}}$ or $\underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$ or $\underline{(-7, 11, -19)}$	A1
			[3]
Aliter **4.** (d) **Way 3**	At A, $\lambda = 1$. At X, $\lambda = 5$.		
	Hence at B, $\lambda = 5 + (5 - 1) = 9$	$\lambda_B = (\text{their } \lambda_X) + (\text{their } \lambda_X - \text{their } \lambda_A)$ $\lambda_B = 2(\text{their } \lambda_X) - (\text{their } \lambda_A)$	M1$\sqrt{}$
	$\overrightarrow{OB} = \begin{pmatrix}11\\2\\17\end{pmatrix} + 9\begin{pmatrix}-2\\1\\-4\end{pmatrix}$	Substitutes their value of λ into the line l_1.	dM1$\sqrt{}$
	Hence, $\overrightarrow{OB} = \underline{\begin{pmatrix}-7\\11\\-19\end{pmatrix}}$ or $\overrightarrow{OB} = \underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$	$\underline{\begin{pmatrix}-7\\11\\-19\end{pmatrix}}$ or $\underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$ or $\underline{(-7, 11, -19)}$	A1
			[3]

Question Number	Scheme		Marks
***Aliter* 4.** (d) **Way 4**	$\overrightarrow{OA} = 9\mathbf{i} + 3\mathbf{j} + 13\mathbf{k}$ and the point of intersection $\overrightarrow{OX} = \mathbf{i} + 7\mathbf{j} - 3\mathbf{k}$		
	$\begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} \to \begin{pmatrix} \text{Minus } 8 \\ \text{Plus } 4 \\ \text{Minus } 16 \end{pmatrix} \to \begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix}$	Finding the difference between their $\overrightarrow{OX}$ (can be implied) and $\overrightarrow{OA}$. $\left(\overrightarrow{AX} =\right) \pm \left(\begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix} - \begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix}\right)$	M1$\sqrt{}$ ±
	$\begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix} \to \begin{pmatrix} \text{Minus } 8 \\ \text{Plus } 4 \\ \text{Minus } 16 \end{pmatrix} \to \begin{pmatrix} -7 \\ 11 \\ -19 \end{pmatrix}$	$\left(\text{their } \overrightarrow{OX}\right) + \left(\text{their } \overrightarrow{AX}\right)$	dM1$\sqrt{}$
	Hence, $\overrightarrow{OB} = \underline{\begin{pmatrix} -7 \\ 11 \\ -19 \end{pmatrix}}$ or $\overrightarrow{OB} = \underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$	$\underline{\begin{pmatrix} -7 \\ 11 \\ -19 \end{pmatrix}}$ or $\underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$ or $\underline{(-7, 11, -19)}$	A1
			[3]
***Aliter* 4.** (d) **Way 5**	$\overrightarrow{OA} = 9\mathbf{i} + 3\mathbf{j} + 13\mathbf{k}$ and $\overrightarrow{OB} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ and the point of intersection $\overrightarrow{OX} = \mathbf{i} + 7\mathbf{j} - 3\mathbf{k}$ As X is the midpoint of AB, then		
	$(1, 7, -3) = \left(\frac{9+a}{2}, \frac{3+b}{2}, \frac{13+c}{2}\right)$	Writing down any two of these "equations" correctly.	M1$\sqrt{}$
	$a = 2(1) - 9 = -7$ $b = 2(7) - 3 = 11$ $c = 2(-3) - 13 = -19$	An attempt to find at least two of a, b or c.	dM1$\sqrt{}$
	Hence, $\overrightarrow{OB} = \underline{\begin{pmatrix} -7 \\ 11 \\ -19 \end{pmatrix}}$ or $\overrightarrow{OB} = \underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$	$\underline{\begin{pmatrix} -7 \\ 11 \\ -19 \end{pmatrix}}$ or $\underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$ or $\underline{(-7, 11, -19)}$ or $a = -7, b = 11, c = -19$	A1
			[3]

Question Number	Scheme		Marks
Aliter **4.** (d) **Way 6**	Let $\overrightarrow{OX} = \mathbf{i} + 7\mathbf{j} - 3\mathbf{k}$ be point of intersection		
	$\overrightarrow{AX} = \overrightarrow{OX} - \overrightarrow{OA} = \begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix} - \begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} = \begin{pmatrix} -8 \\ 4 \\ -16 \end{pmatrix}$ and $\lvert\overrightarrow{AX}\rvert = \sqrt{64 + 16 + 256} = \sqrt{336} = 4\sqrt{21}$	Finding the difference between their $\overrightarrow{OX}$ (can be implied) and $\overrightarrow{OA}$. $\overrightarrow{AX} = \pm\left(\begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix} - \begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix}\right)$ Note $\lvert\overrightarrow{AX}\rvert = \sqrt{336}$ would imply M1.	M1$\sqrt{\ }$ $\pm$
	$\overrightarrow{BX} = \overrightarrow{OX} - \overrightarrow{OB} = \begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix} - \begin{pmatrix} 11 - 2\lambda \\ 2 + \lambda \\ 17 - 4\lambda \end{pmatrix} = \begin{pmatrix} -10 + 2\lambda \\ 5 - \lambda \\ -20 + 4\lambda \end{pmatrix}$		
	Hence $\lvert\overrightarrow{BX}\rvert = \lvert\overrightarrow{AX}\rvert = \sqrt{336}$ gives $(-10 + 2\lambda)^2 + (5 - \lambda)^2 + (-20 + 4\lambda)^2 = 336$	Writes distance equation of $\lvert\overrightarrow{BX}\rvert^2 = 336$ where $\overrightarrow{BX} = \overrightarrow{OX} - \overrightarrow{OB}$ and $\overrightarrow{OB} = \begin{pmatrix} 11 - 2\lambda \\ 2 + \lambda \\ 17 - 4\lambda \end{pmatrix}$	dM1$\sqrt{\ }$
	$100 - 40\lambda + 4\lambda^2 + 25 - 10\lambda + \lambda^2 + 400 - 160\lambda + 16\lambda^2 = 336$ $21\lambda^2 - 210\lambda + 525 = 336$ $21\lambda^2 - 210\lambda + 189 = 0$ $\lambda^2 - 10\lambda + 9 = 0$ $(\lambda - 1)(\lambda - 9) = 0$		
	At A, $\lambda = 1$ and at B $\lambda = 9$, so, $\overrightarrow{OB} = \begin{pmatrix} 11 - 2(9) \\ 2 + 9 \\ 17 - 4(9) \end{pmatrix}$		
	Hence, $\overrightarrow{OB} = \underline{\begin{pmatrix} -7 \\ 11 \\ -19 \end{pmatrix}}$ or $\overrightarrow{OB} = \underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$	$\underline{\begin{pmatrix} -7 \\ 11 \\ -19 \end{pmatrix}}$ or $\underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$ or $\underline{(-7, 11, -19)}$	A1
			[3]

Question 5

Question Number	Scheme	Marks
5. (a)	Similar shapes $\Rightarrow$ either	
	$\underline{\dfrac{\frac{1}{3}\pi(16)^2 24}{V} = \left(\dfrac{24}{h}\right)^3}$ or $\underline{\dfrac{V}{\frac{1}{3}\pi(16)^2 24} = \left(\dfrac{h}{24}\right)^3}$	
	$\underline{\dfrac{\frac{1}{3}\pi r^2(24)}{V} = \left(\dfrac{24}{h}\right)^3}$ or $\underline{\dfrac{V}{\frac{1}{3}\pi r^2(24)} = \left(\dfrac{h}{24}\right)^3}$ Uses similar shapes to find either one of these two expressions oe.	M1
	$V = 2048\pi \times \left(\dfrac{h}{24}\right)^3 = \dfrac{4\pi h^3}{27}$ **AG** Substitutes their equation to give the correct formula for the volume of water V.	A1
		[2]
5. (a)	Candidates simply writing:	
	$V = \dfrac{4}{9} \times \dfrac{1}{3}\pi h^3$ or $V = \dfrac{1}{3}\pi\left(\dfrac{16}{24}\right)^2 h^3$ would be awarded M0A0.	
(b)	From question, $\dfrac{\mathrm{d}V}{\mathrm{d}t} = 8 \Rightarrow V = 8t\ (+c)$ $\dfrac{\mathrm{d}V}{\mathrm{d}t} = 8$ or $V = 8t$	B1
	$h = \left(\dfrac{27V}{4\pi}\right)^{\frac{1}{3}} \Rightarrow h = \underline{\left(\dfrac{27(8t)}{4\pi}\right)^{\frac{1}{3}}} = \underline{\left(\dfrac{54t}{\pi}\right)^{\frac{1}{3}}} = \underline{3\left(\dfrac{2t}{\pi}\right)^{\frac{1}{3}}}$ $\underline{\left(\dfrac{27(8t)}{4\pi}\right)^{\frac{1}{3}}}$ or $\underline{\left(\dfrac{54t}{\pi}\right)^{\frac{1}{3}}}$ or $\underline{3\left(\dfrac{2t}{\pi}\right)^{\frac{1}{3}}}$	B1
	$\dfrac{\mathrm{d}h}{\mathrm{d}t} = 3\left(\dfrac{2}{\pi}\right)^{\frac{1}{3}}\dfrac{1}{3}t^{-\frac{2}{3}}$ $\dfrac{\mathrm{d}h}{\mathrm{d}t} = \pm k t^{-\frac{2}{3}}$;	M1;
	$\dfrac{\mathrm{d}h}{\mathrm{d}t} = 3\left(\dfrac{2}{\pi}\right)^{\frac{1}{3}}\dfrac{1}{3}t^{-\frac{2}{3}}$	A1 oe
	When $h = 12$, $t = \left(\dfrac{12}{3}\right)^3 \times \dfrac{\pi}{2} = 32\pi$	
	So when $h = 12$, $\dfrac{\mathrm{d}h}{\mathrm{d}t} = \left(\dfrac{2}{\pi}\right)^{\frac{1}{3}}\left(\dfrac{1}{32\pi}\right)^{\frac{2}{3}} = \left(\dfrac{2}{1024\pi^3}\right)^{\frac{1}{3}} = \underline{\dfrac{1}{8\pi}}$ $\underline{\dfrac{1}{8\pi}}$	A1 oe
		[5]

Question 7

Question Number	Scheme		Marks
7. (a)	It is acceptable for a candidate to write $x = 7,\ y = 1,$ to gain B1.	$A(7,1)$	B1
			[1]
***Aliter* (c) Way 2**	$x = t^3 - 8t = t(t^2 - 8) = t(y - 8)$		
	So, $x^2 = t^2(y-8)^2 = y(y-8)^2$		
	$2x - 5y - 9 = 0 \ \Rightarrow\ 2x = 5y + 9 \ \Rightarrow\ 4x^2 = (5y+9)^2$		
	Hence, $4y(y-8)^2 = (5y+9)^2$	Forming an equation in terms of y only.	M1
	$4y(y^2 - 16y + 64) = 25y^2 + 90y + 81$		
	$4y^3 - 64y^2 + 256y = 25y^2 + 90y + 81$		
	$4y^3 - 89y^2 + 166y - 81 = 0$		
	$(y-1)(y-1)(4y-81) = 0$	A realisation that $(y-1)$ is a factor.	dM1
		Correct factorisation	A1
	$y = \frac{81}{4} = 20.25$ (or awrt 20.3)	***Correct y-coordinate (see below!)***	
	$x^2 = \frac{81}{4}(\frac{81}{4} - 8)^2$	Candidate uses their y-coordinate to find their x-coordinate.	ddM1
		Decide to award A1 here for correct y-coordinate.	A1
	$x = \frac{441}{8} = 55.125$ (or awrt 55.1)	Correct x-coordinate	A1
	Hence $B\left(\frac{441}{8}, \frac{81}{4}\right)$		**[6]**

Question Number	Scheme		Marks
Aliter 7. (c) **Way 3**	$t = \sqrt{y}$		
	So $x = \left(\sqrt{y}\right)^3 - 8\left(\sqrt{y}\right)$		
	$2x - 5y - 9 = 0$ yields		
	$2\left(\sqrt{y}\right)^3 - 16\left(\sqrt{y}\right) - 5y - 9 = 0$	Forming an equation in terms of y only.	M1
	$\Rightarrow 2\left(\sqrt{y}\right)^3 - 5y - 16\left(\sqrt{y}\right) - 9 = 0$		
	$\left(\sqrt{y}+1\right)\left\{\left(2y - 7\sqrt{y} - 9\right) = 0\right\}$	A realisation that $\left(\sqrt{y}+1\right)$ is a factor.	dM1
	$\left(\sqrt{y}+1\right)\left\{\left(\sqrt{y}+1\right)\left(2\sqrt{y} - 9\right) = 0\right\}$	Correct factorisation.	A1
	$y = \frac{81}{4} = 20.25$ (or awrt 20.3)	***Correct y-coordinate (see below!)***	
	$x = \left(\sqrt{\frac{81}{4}}\right)^3 - 8\left(\sqrt{\frac{81}{4}}\right)$	Candidate uses their y-coordinate to find their x-coordinate.	ddM1
		Decide to award A1 here for correct y-coordinate.	A1
	$x = \frac{441}{8} = 55.125$ (or awrt 55.1)	Correct x-coordinate	A1
	Hence $B\left(\frac{441}{8}, \frac{81}{4}\right)$		**[6]**