GCE

Edexcel GCE

Mathematics

Further Pure Mathematics 3 FP3 (6676)

June 2008

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Mark Scheme

Edexcel GCE

Mathematics

General Marking Guidance

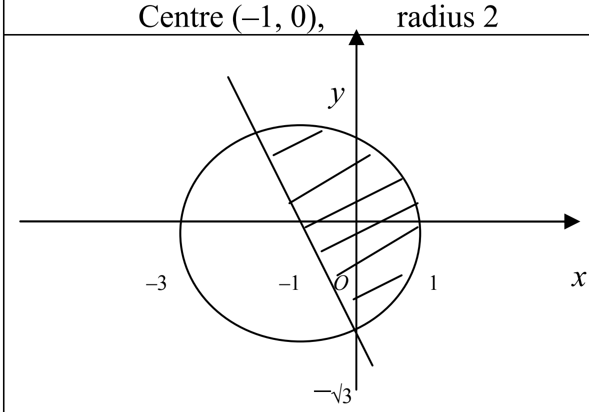
- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

6676 Further Pure FP3
Mark Scheme

Question Number	Scheme	Marks
1.	$\left(\frac{dy}{dx}\right)_0 = 0 + \cos 0.6 \ (= 0.825335\ldots)$ May be implicit	B1
	$y_1 \approx 0.05\left(\frac{dy}{dx}\right)_0 + y_0$ $(= 0.05 \times 0.825335\ldots + 0.6)$	M1
	$y_1 \approx 0.641266\ldots$ $= 0.6413$ (4 d.p.) Allow awrt	A1
	$\left(\frac{dy}{dx}\right)_1 = 0.05 + \cos 0.641266...$ $[\text{or } 0.05 + \cos(0.6 + 0.05\cos 0.6)]$ $= 0.851338\ldots$	A1ft
	$y_2 \approx 0.05\left(\frac{dy}{dx}\right)_1 + y_1$ $(= 0.05 \times 0.851338\ldots + 0.641266\ldots)$ Requires use of the differential equation to find $\left(\frac{dy}{dx}\right)_1$	M1
	$y_2 \approx 0.683833\ldots$ $= 0.6838$ (4 d.p.)	A1 **(6)**
	Degree mode in calculator: Gives answers: 0.6500 (0.64999…) 0.7025 (0.70248…) This can score B1 M1 A0 A1ft M1 A0	

Question Number	Scheme	Marks
2. (a)	$\begin{pmatrix}1 & p & 2\\0 & 3 & q\\2 & p & 1\end{pmatrix}\begin{pmatrix}1\\2\\1\end{pmatrix}=\lambda\begin{pmatrix}1\\2\\1\end{pmatrix}$ $\begin{pmatrix}1+2p+2\\6+q\\2+2p+1\end{pmatrix}=\begin{pmatrix}\lambda\\2\lambda\\\lambda\end{pmatrix}$ is M1 A1 (2 eqns implied) $\begin{pmatrix}3+2p\\6+q\\3+2p\end{pmatrix}\Rightarrow 6+q=2(3+2p)$ is M1 A1 (2 eqns, use of parameter implied) $1+2p+2=\lambda$ $\quad 6+q=2\lambda$ $\quad$ M: Two equations, one in p, one in q $\therefore\ 6+q=6+4p \Rightarrow q=4p$ (*)	 M1 A1 A1 (3)
(b)	$\begin{vmatrix}-4 & p & 2\\0 & -2 & 4p\\2 & p & -4\end{vmatrix}=0$ or $\begin{vmatrix}1-\lambda & p & 2\\0 & 3-\lambda & 4p\\2 & p & 1-\lambda\end{vmatrix}=0$ (or with q instead of $4p$) $[-4(8-4p^2)-p(0-8p)+2(0+4)=0]$ $\quad p^2=1$ or $pq=4$ $p<0\quad p=-1\quad q=-4$ $\quad$ M: Use $q=4p$ to find value of p and of q A1: Positive values must be rejected	M1 A1 dM1 A1 (4)
(c)	$-4x-y+2z=0,\ -2y-4z=0,\ 2x-y-4z=0$ Any 2 eqns, with value of p $2x=-y=2z$ $\quad$ (or 2 separate equations) E.vector is $k\begin{pmatrix}1\\-2\\1\end{pmatrix}$ $\quad$ (Any non-zero value of k)	M1 M1 A1 (3) **(10)**
	(a) Assuming a value for λ, e.g. $\lambda=1$, gives M1 A0 A0. (a) Assuming result and working 'backwards': $\begin{pmatrix}1 & p & 2\\0 & 3 & 4p\\2 & p & 1\end{pmatrix}\begin{pmatrix}1\\2\\1\end{pmatrix}=\begin{pmatrix}3+2p\\6+4p\\3+2p\end{pmatrix}=(3+2p)\begin{pmatrix}1\\2\\1\end{pmatrix}$, gives M1 A0 A0 (b) <u>Alternative</u>: $\begin{pmatrix}1 & p & 2\\0 & 3 & 4p\\2 & p & 1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=5\begin{pmatrix}x\\y\\z\end{pmatrix}$ or $\begin{pmatrix}-4 & p & 2\\0 & -2 & 4p\\2 & p & -4\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$ (or q instead of $4p$) $x+py+2z=5x,\ 3y+4pz=5y,\ 2x+py+z=5z$ $py+2z=4x$ (i), $2pz=y$ (ii), $2x+py=4z$ (iii) From (i) and (iii) $\quad py=2z$ From (ii) $\quad p^2=1$ $\quad$ (or equiv. in terms of p and/or q) $p<0,\ p=-1,\ q=-4$ $\quad$ A1: Positive values must be rejected (b) Using the eigenvector $\begin{pmatrix}1\\2\\1\end{pmatrix}$ scores <u>no marks</u> in this part.	 M1 A1 dM1 A1

Question Number	Scheme	Marks
3. (a)	$\left(x^2+1\right)\frac{d^3y}{dx^3}+2x\frac{d^2y}{dx^2}=4y\frac{dy}{dx}+(1-2x)\frac{d^2y}{dx^2}-2\frac{dy}{dx}$	M1 A1
	$\left(x^2+1\right)\frac{d^3y}{dx^3}=\left(1-4x\right)\frac{d^2y}{dx^2}+\left(4y-2\right)\frac{dy}{dx}$ (*)	A1 (3)
(b)	$\left(\frac{d^2y}{dx^2}\right)_0=3$	B1
	$\left(\frac{d^3y}{dx^3}\right)_0=5$ Follow through: $\frac{d^3y}{dx^3}=\frac{d^2y}{dx^2}+2$	B1ft
	$y=1+x+\frac{3}{2}x^2+\frac{5}{6}x^3....$	M1 A1 (4)
(c)	$x=-0.5,\quad y\approx 1-0.5+0.375-0.104166\ldots$ $=0.77$ (2 d.p.) [awrt 0.77]	B1 (1) **(8)**
	(a) M: Use of product rule (at least once) and implicit differentiation (at least once). (b) M: Use of series expansion with values for the derivatives (can be allowed without the first term 1, and can also be allowed if final term uses 3 rather than 3!)	

4.	(a)	$\lvert (x-3)+\mathrm{i}y \rvert = 2\lvert x+\mathrm{i}y \rvert \Rightarrow (x-3)^2+y^2 = 4x^2+4y^2$ $\therefore\ x^2+y^2+2x-3=0$ $(x+1)^2+y^2=4$ Centre (–1, 0), radius 2	M1 A1 M1 A1, A1 (5)
	(b)	 Circle, centre on x-axis B1 C (–1, 0), r = 2 dB1ft Follow through centre and radius, but dependent on first B1. There must be indication of their ‘–3’, ‘–1’ or ‘1’ on the x-axis and no contradictory evidence for their radius. Straight line B1 Straight line through (–1, 0), or perp. bisector of (–3, 0) and (0, $\sqrt{3}$). B1 Straight line through point of int. of circle & –ve y-axis, or through $(0, -\sqrt{3})$ B1	B1 dB1 B1 B1 B1 (5)
	(c)	Shading (only) inside circle Inside correct circle and all of the correct side of the correct line… this mark is dependent on <u>all</u> the previous B marks in parts (b) and (c).	B1 dB1 (2) **(12)**
		(a) 1st M: Use $z = x + \mathrm{i}y$, and attempt square of modulus of each side. Not squaring the 2 on the RHS would be M1 A0. 2nd M: Attempting to express in the form $(x-a)^2+(y-b)^2=k$, or attempting centre and radius from the form $x^2+y^2+2gx+2fy+c=0$	

Question Number	Scheme	Marks
5. (a)	$\begin{pmatrix} k & -2 \\ 1-k & k \end{pmatrix}\begin{pmatrix} t \\ 2t \end{pmatrix} = \begin{pmatrix} t(k-4) \\ t(1+k) \end{pmatrix}$	M1
	$t(1+k) = 2t(k-4)$	dM1
	$k = 9$	A1 (3)
(b)	det **A** $= k^2 + 2(1-k)$ (Must be seen in part (b))	M1
	$= (k-1)^2 + 1$, which is always positive	M1
	A is non-singular	A1cso (3)
(c)	$\mathbf{A}^{-1} = \dfrac{1}{k^2 - 2k + 2}\begin{pmatrix} k & 2 \\ k-1 & k \end{pmatrix}$	M1 A1 (2)
(d)	$k = 3, \quad \mathbf{A}^{-1} = \dfrac{1}{5}\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$	B1
	$\mathbf{Ap} = \mathbf{q} \Rightarrow \mathbf{p} = \mathbf{A}^{-1}\mathbf{q} \qquad \mathbf{p} = \dfrac{1}{5}\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 4 \\ -3 \end{pmatrix} = \dfrac{1}{5}\begin{pmatrix} 6 \\ -1 \end{pmatrix}$	M1 A1 (3)
	Alt. $\begin{pmatrix} 3 & -2 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \end{pmatrix} \Rightarrow 3x - 2y = 4, -2x + 3y = -3$ B1	
	M1 A1 for solving two sim. eqns. in x and y to give $x = 1.2$, $y = -0.2$ (o.e.)	
		(11)
	(b) 2[nd] M: Alternative is to use quadratic formula on the quadratic equation, or to use the discriminant, with a comment about ‘no real roots’, or ‘can’t equal zero’, or a comment about the condition for singularity. $\left(x = \dfrac{2 \pm \sqrt{4-8}}{2} \right)$ A1 Conclusion. (c) M: Need $\dfrac{1}{\text{their det A}}$, k's unchanged and attempt to change sign for either -2 (leaving as top right) or $1-k$ (leaving as bottom left). (d) M: Requires an attempt to multiply the matrices.	

| Question | Scheme | Marks |

Number		
6. (a)	$(\cos\theta + i\sin\theta)^1 = \cos\theta + i\sin\theta \quad \therefore$ true for $n = 1$	B1
	Assume true for $n = k$, $(\cos\theta + i\sin\theta)^k = \cos k\theta + i\sin k\theta$	
	$(\cos\theta + i\sin\theta)^{k+1} = (\cos k\theta + i\sin k\theta)(\cos\theta + i\sin\theta)$	M1
	$= \cos k\theta\cos\theta - \sin k\theta\sin\theta + i(\sin k\theta\cos\theta + \cos k\theta\sin\theta)$	M1
	(Can be achieved either from the line above or the line below)	
	$= \cos(k+1)\theta + i\sin(k+1)\theta$	A1
	Requires full justification of $(\cos\theta + i\sin\theta)^{k+1} = \cos(k+1)\theta + i\sin(k+1)\theta$	
	($\therefore$ true for $n = k + 1$ if true for $n = k$) $\quad \therefore$ true for $n \in \mathbb{Z}^+$ by induction	A1cso (5)
(b)	$\cos 5\theta = \text{Re}\left[(\cos\theta + i\sin\theta)^5\right]$	
	$= \cos^5\theta + 10\cos^3\theta\, i^2\sin^2\theta + 5\cos\theta\, i^4\sin^4\theta$	M1 A1
	$= \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$	M1
	$= \cos^5\theta - 10\cos^3\theta(1-\cos^2\theta) + 5\cos\theta(1-\cos^2\theta)^2$	M1
	$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$ (*)	A1cso (5)
(c)	$\dfrac{\cos 5\theta}{\cos\theta} = 0 \quad \Rightarrow \quad \cos 5\theta = 0$	M1
	$5\theta = \dfrac{\pi}{2} \qquad \theta = \dfrac{\pi}{10}$	A1
	$x = 2\cos\theta, \qquad x = 2\cos\dfrac{\pi}{10}$ is a root (*)	A1 (3)
		(13)
	(a) <u>Alternative</u>: For the 2nd M mark: $(e^{ik\theta})(e^{i\theta}) = e^{i\theta(k+1)}$	
	(b) <u>Alternative</u>:	
	$\left(z + \frac{1}{z}\right)^5 = z^5 + 5z^4\left(\frac{1}{z}\right) + 10z^3\left(\frac{1}{z}\right)^2 + 10z^2\left(\frac{1}{z}\right)^3 + 5z\left(\frac{1}{z}\right)^4 + \left(\frac{1}{z}\right)^5$ — M1	
	$= 2\cos 5\theta + 10\cos 3\theta + 20\cos\theta$ — A1	
	$(2\cos\theta)^5 = ...$ and attempt to put $\cos 3\theta$ in powers of $\cos\theta$ — M1	
	Correct method (or formula) for putting $\cos 3\theta$ in powers of $\cos\theta$ — M1	
	$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$ — A1cso	
	(c) <u>Alternatives</u>:	
	(i) Substitute given root into $x^4 - 5x^2 + 5$:	
	$\left(2\cos\frac{\pi}{10}\right)^4 - 5\left(2\cos\frac{\pi}{10}\right)^2 + 5 = 2^4\left(\cos\frac{\pi}{10}\right)^4 - 5\times 2^2\left(\cos\frac{\pi}{10}\right)^2 + 5$ — M1	
	'Multiply by $\cos\theta$' and use result from part (b): $... = \cos\dfrac{5\pi}{10}$ — A1	
	$= 0$ and conclusion — A1	
	(ii) Use $5\theta = \dfrac{\pi}{2}$ in result from part (b) — M1	
	$16\left(\cos\frac{\pi}{10}\right)^5 - 20\left(\cos\frac{\pi}{10}\right)^3 + 5\left(\cos\frac{\pi}{10}\right)$ and divide by $\cos\theta$ — A1	
	$= 0$ and conclusion — A1	

Question Number	Scheme	Marks
7. (a)	$\overrightarrow{PQ} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$, $\overrightarrow{PR} = 2\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$	B1
	$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 2 \\ 2 & -3 & 3 \end{vmatrix} = 3\mathbf{i} + \mathbf{j} - \mathbf{k}$	M1 A1 (3)
(b)	$\mathbf{r} \cdot (3\mathbf{i} + \mathbf{j} - \mathbf{k}) = (\mathbf{i} - \mathbf{k}) \cdot (3\mathbf{i} + \mathbf{j} - \mathbf{k})$ [may use $\overrightarrow{OQ}$ or $\overrightarrow{OR}$]	M1
	$\mathbf{r} \cdot (3\mathbf{i} + \mathbf{j} - \mathbf{k}) = 4$ o.e. ft from (a)	A1ft (2)
(c)	$3x + y - z = 4$ (i), $x - 2y - 5z = 6$ (ii)	
	(i) × 2 + (ii) $7x - 7z = 14$, $x = z + 2$ (M: Eliminate one variable)	M1
	In (ii) $z + 2 - 2y - 5z = 6$, $y + 2 = -2z$ (M: Substitute back)	M1
	$\therefore x = z + 2$ and $y + 2 = -2z$ o.e. ($y = 2 - 2x$) (Two correct '3-term' equations)	A1
	$\dfrac{x-2}{(1)} = \dfrac{y+2}{-2} = \dfrac{z}{(1)}$ o.e. (M: Form cartesian equations)	M1 A1 (5)
(d)	Writing down direction vector of $\overrightarrow{PS}$ from part (c).	M1
	$\overrightarrow{QR} = \mathbf{i} - 2\mathbf{j} + \mathbf{k} = \overrightarrow{PS}$ $\therefore PS$ // QR (or cross-product = 0)	A1 (2)
(e)	$\overrightarrow{PT} = 4\mathbf{i} + 2\mathbf{j}$ (or $\overrightarrow{QT} = 3\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ or $\overrightarrow{RT} = 2\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$)	
	Volume = $\dfrac{1}{3}\left\lvert \overrightarrow{PQ} \times \overrightarrow{PR} . \overrightarrow{PT} \right\rvert = \dfrac{1}{3} \lvert (3\mathbf{i} + \mathbf{j} - \mathbf{k}) \cdot (4\mathbf{i} + 2\mathbf{j}) \rvert$ ft from (a) (Instead of $\overrightarrow{PQ} \times \overrightarrow{PR}$, it could be $\overrightarrow{PQ} \times \overrightarrow{QR}$ or $\overrightarrow{PR} \times \overrightarrow{QR}$)	M1 A1ft
	$= \dfrac{1}{3}(12 + 2)$	
	$= 4\frac{2}{3}$ o.e.	A1 (3)
		(15)
	(a) If both vectors are 'reversed', B0 M1 A1 is possible (c) <u>Alternative</u>: Direction of line: $\begin{pmatrix} 1 \\ -2 \\ -5 \end{pmatrix} \times \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} = 7\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ M2 A1 Through P $(1, 0, -1)$: $\dfrac{x-1}{1} = \dfrac{y}{-2} = \dfrac{z+1}{1}$ M1 A1 (e) <u>Alternative</u>: $\dfrac{1}{3}\begin{vmatrix} 4 & 2 & 0 \\ 1 & -1 & 2 \\ 2 & -3 & 3 \end{vmatrix}$ gives M1 A1 directly. Here ft from 1st line of part (a). <u>Special case</u>: $\dfrac{1}{6}$ or $\dfrac{1}{2}$ instead of $\dfrac{1}{3}$, but method otherwise correct: M1 A0 A0	