GCE

Edexcel GCE

Mathematics

Further Pure Mathematics (FP1/ 6674)

June 2008

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Mark Scheme (Final)

Edexcel GCE

Mathematics

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

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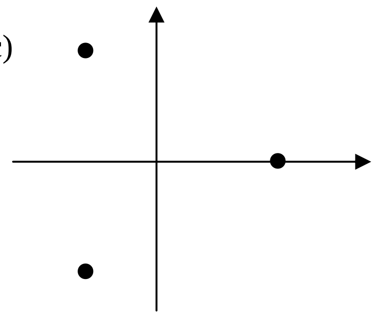
Mark Scheme

Question number	Scheme	Marks	
1.	(a) 4	B1	(1)
	(b) $(x-4)(x^2+4x+16)$	M1 A1	
	$x = \dfrac{-4 \pm \sqrt{16-64}}{2}$, $\quad x = -2 \pm 2\sqrt{3}\,\mathrm{i}$ (or equiv. surd for $2\sqrt{3}$)	M1, A1	(4)
	(c)  Root on + ve real axis, one other in correct quad.	B1	
	Third root in conjugate complex position	B1ft	(2)
			7
	M1 in part (b) needs(x-"their 4") times quadratic($x^2+ax+..$)or times(x^2+16)		
	M1 needs solution of three term quadratic		
	So (x^2+16) special case, results in B1M1A0M0A0B0B1 possibly		
	Alternative scheme for (b)		
	$(a+ib)^3 = 64$, so $a^3+3a^2ib+3a(ib)^2+(ib)^3 = 64$ and equate real, imaginary parts	M1	
	so $a^3-3ab^2 = 64$ and $3a^2b - b^3 = 0$	A1	
	Solve to obtain $a=-2,\ b=\sqrt{12}$	M1A1	
	Alternative ii		
	$(x-4)(x-a-ib)(x-a+ib)=0$ expand and compare coefficients	M1	
	two of the equations $-2a-4=0,\ 8a+a^2+b^2=0,\ 4(a^2+b^2)=64$	A1	
	Solve to obtain $a=-2,\ b=\sqrt{12}$	M1A1	
	(c)Allow vectors, line segments or points in Argand diagram.		
	Extra points plotted in part (c) – lose last B mark		
	Part (c) answers are independent of part (b)		

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2.	(a) $f(1.6) = ...$ $\quad f(1.7) = ...$ (Evaluate both)	M1	
	$0.08\ldots$ (or 0.09), $-0.3\ldots$ One +ve, one –ve or Sign change, $\therefore$ root	A1	(2)
	(b) $f'(x) = -4\sin x - e^{-x}$	B1	
	$1.6 - \dfrac{f(1.6)}{f'(1.6)}$	M1	
	$= 1.6 - \dfrac{4\cos 1.6 + e^{-1.6}}{(-4\sin 1.6 - e^{-1.6})}$ $\quad \left(= 1.6 - \dfrac{0.085...}{-4.2...} \right)$	A1	
	$= 1.62$	A1	(4)
			6
	(a) Any errors seen in evaluation of f(1.6) or f(1.7) lose A mark so –0.32 is A0 Values are 0.0851 and –0.3327 Need concluding statement also. (b) B1 may be awarded if seen in N-R as $-4\sin 1.6 - e^{-1.6}$ or as –4.2 M1 for statement of Newton Raphson (sign error in rule results in M 0) First A1 may be implied by correct work previously followed by correct answer Do not accept 1.620 for final A1. It must be given and correct to 3sf. 1.62 may follow incorrect work and is A0 No working at all in part (b) is zero marks.		

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3.	(a) $z = \frac{(a+2\text{i})(a+\text{i})}{(a-\text{i})(a+\text{i})} = \frac{a^2+3a\text{i}-2}{a^2+1}$	M1 A1	
	$\frac{a^2-2}{a^2+1} = \frac{1}{2}$, $2a^2-4 = a^2+1$ $a=\sqrt{5}$ (presence of $-\sqrt{5}$ also is A0)	M1, A1	(4)
	(b) Evaluating their " $\frac{3a}{a^2+1}$ ", or " $3a$ " $\left(\frac{\sqrt{5}}{2}\text{ or } 3\sqrt{5}\right)$ (ft errors in part a)	B1ft	
	$\tan\theta = \frac{3a}{a^2-2}$ $\left(=\frac{3\sqrt{5}}{3}\right)$, arg $z = 1.15$ (accept answers which round to 1.15)	M1, A1	(3)
			7
	(b) B mark is treated here as a method mark The M1 is for tan (argz) = Imaginary part / real part answer in degrees is A0 <u>Alternative method</u>:		
	(a) $\left(\frac{1}{2}+\text{i}y\right)(a-\text{i}) = a+2\text{i} \Rightarrow \frac{1}{2}a+y=a$ and $ay-\frac{1}{2}=2$	M1 A1	
	$y=\frac{1}{2}a$ and $ay=\frac{5}{2} \Rightarrow \frac{1}{2}a^2=\frac{5}{2} \Rightarrow a=\sqrt{5}$	M1 A1	(4)
	(b) $y=\frac{\sqrt{5}}{2}$ (May be seen in part (a))	B1ft	
	$\tan\theta=\sqrt{5}$ arg $z = 1.15$	M1 A1	(3)
	<u>Further Alternative method in (b)</u>		
	Use $\arg(a+2\text{i})-\arg(a-\text{i})$	B1	
	$=0.7297-(-0.4205)=1.15$	M1A1	(3)

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Question number	Scheme	Marks	
4.	(a) $m^2 + 4m + 3 = 0$ $\quad m = -1,\ m = -3$	M1 A1	
	C.F. $(x =)Ae^{-t} + Be^{-3t}$ $\quad$ must be function of t, not x	A1	
	P.I. $x = pt + q$ **(or** $x = at^2 + bt + c$)	B1	
	$4p + 3(pt + q) = kt + 5$ $\quad 3p = k$ $\quad$ (Form at least one eqn. in p and/or q)	M1	
	$4p + 3q = 5$		
	$p = \frac{k}{3}$, $\quad q = \frac{5}{3} - \frac{4k}{9}\left(= \frac{15 - 4k}{9}\right)$	A1	
	General solution: $x = Ae^{-t} + Be^{-3t} + \frac{kt}{3} + \frac{15 - 4k}{9}$ must include x = and be function of t	A1 ft	(7)
	(b) When $k = 6$, $\quad x = 2t - 1$	M1 A1cao	(2)
			9
	(a) M1 for auxiliary equation substantially correct B1 not awarded for $x = kt$+constant (b) M mark for using $k = 6$ to derive a linear expression in t. (cf must have involved negative exponentials only) so e.g. $y = 2t - 1$ is M1 A0		

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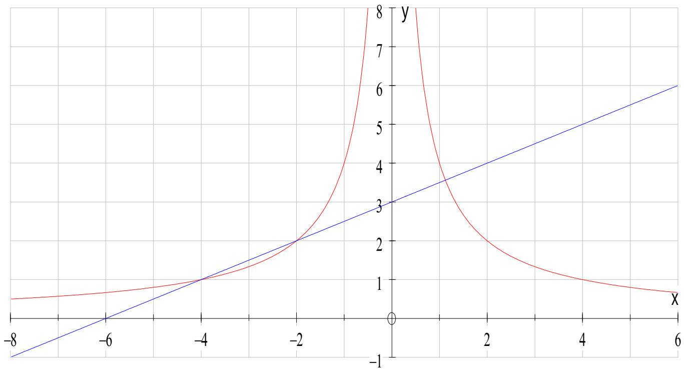
5.	(a) $\frac{4}{x} = \frac{x}{2} + 3$ $\quad x^2 + 6x - 8 = 0 \quad x = ..., \left(\frac{-6 \pm \sqrt{68}}{2}\right)$ $\quad -3 \pm \sqrt{17}$ - root not needed	M1, A1
	$-\frac{4}{x} = \frac{x}{2} + 3,$ $\quad x^2 + 6x + 8 = 0$ $\quad x = -4$ and -2	M1, A1
	Three correct solutions (and no extras): $-4,\ -2,\ -3 + \sqrt{17}$	A1 (5)
	(b)  Line through point on –ve x axis and + y axis	B1
	Curve	B1
	3 Intersections in correct quadrants	B1 (3)
	(c) $-4 < x < -2,$ $\quad x > -3 + \sqrt{17}$ $\quad$ o.e.	B1, B1 (2)
		10
	(a) <u>Alternative using squaring method</u> Square both sides and attempt to find roots	M1
	$x^4 + 12x^3 + 36x^2 - 64 = 0$ gives x = -2 and x = -4	A1
	Obtain quadratic factor, divide find solutions of quadratic and obtain $\left(-3 \pm \sqrt{17}\right)$	M1 A1
	Last mark as before	
	(c) Use of $\leq$ instead of < lose last B1 $\quad$ Extra inequalities lose last B1	

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6.	(a) $\frac{2}{(r+1)(r+3)} = \frac{1}{r+1} - \frac{1}{r+3}$ M: $\frac{2}{(r+1)(r+3)} = \frac{A}{r+1} + \frac{B}{r+3}$	M1 A1 (2)
	(b) $r = 1$: $\left(\frac{2}{2\times 4}\right) = \frac{1}{2} - \frac{1}{4}$	M1
	$r = 2$: $\left(\frac{2}{3\times 5}\right) = \frac{1}{3} - \frac{1}{5}$	
	… $r = n-1$: $\left(\frac{2}{n(n+2)}\right) = \frac{1}{n} - \frac{1}{n+2}$	
	$r = n$: $\left(\frac{2}{(n+1)(n+3)}\right) = \frac{1}{n+1} - \frac{1}{n+3}$	A1 ft
	Summing: $\sum = \frac{1}{2} + \frac{1}{3} - \frac{1}{n+2} - \frac{1}{n+3}$	M1 A1
	$= \frac{5(n+2)(n+3) - 6(n+3) - 6(n+2)}{6(n+2)(n+3)} = \frac{n(5n+13)}{6(n+2)(n+3)}$	**d** M1 A1cso (6)
	(c) $\sum_{21}^{30} = \sum_{1}^{30} - \sum_{1}^{20} = \frac{30\times 163}{6\times 32\times 33} - \frac{20\times 113}{6\times 22\times 23}$, $= 0.02738$	M1A1ft,A1cso (3)
		(11)
	(b) The first M1 requires list of first two and last two terms The A1 must be correct but ft on their *A* and *B* The second M1 requires terms to be eliminated and A1 is cao (c) The M mark is also allowed for $\sum_{1}^{30} - \sum_{1}^{21}$ applied with numbers included Using $u_{30} - u_{20}$ scores M0 A0 A0 The first A1 is ft their *A* and *B* or could include *A* and *B*, but final A1 is cao but accept 0.027379775599 to 5 or more decimal places..	

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7.	(a) $\frac{dy}{dx} = v + x\frac{dv}{dx}$	B1
	$\left(v + x\frac{dv}{dx}\right) = \frac{x}{vx} + \frac{3vx}{x} \quad \Rightarrow \quad x\frac{dv}{dx} = 2v + \frac{1}{v}$ (*)	M1 A1 (3)
	(b) $\int \frac{v}{1+2v^2} dv = \int \frac{1}{x} dx$	M1
	$\frac{1}{4}\ln(1+2v^2), \quad = \ln x \ (+C)$	dM1 A1, B1
	$Ax^4 = 1 + 2v^2$	d M1
	$Ax^4 = 1 + 2\left(\frac{y}{x}\right)^2$ so $y = \sqrt{\frac{Ax^6 - x^2}{2}}$ or $y = x\sqrt{\frac{Ax^4 - 1}{2}}$ or $y = x\sqrt{\left(\frac{1}{2}e^{4\ln x + 4c} - \frac{1}{2}\right)}$	M1 A1 (7)
	(c) $x = 1$ at $y = 3$: $\quad 3 = \sqrt{\frac{A-1}{2}} \quad A = \ldots$	M1
	$y = \sqrt{\frac{19x^6 - x^2}{2}}$ or $y = x\sqrt{\frac{19x^4 - 1}{2}}$	A1 (2) **12**
	(a) B1 for statement printed or for $\frac{dy}{dx} = (x + v\frac{dx}{dv})\frac{dv}{dx}$ First M1 is for RHS of equation only but for A1 need whole answer correct . (b) First M1 accept $\int \frac{1}{2v + \frac{1}{v}} dv = \int \frac{1}{x} dx$ Second M1 requires an integration of correct form ¼ may be missing A1 for LHS correct with ¼ and B1 is independent and is for ln$x$ Third M1 is **dependent** and needs correct application of log laws Fourth M1 is independent and merely requires return to y/x for v **<u>N.B. There is an IF method possible after suitable rearrangement – see note</u>.**	

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8.	(a) $r\cos\theta = 4(\cos\theta - \cos^2\theta)$ or $r\cos\theta = 4\cos\theta - 2\cos 2\theta - 2$	B1
	$\frac{d(r\cos\theta)}{d\theta} = 4(-\sin\theta + 2\cos\theta\sin\theta)$ or $\frac{d(r\cos\theta)}{d\theta} = 4(-\sin\theta + \sin 2\theta)$	M1 A1
	$4(-\sin\theta + 2\cos\theta\sin\theta) = 0 \Rightarrow \cos\theta = \frac{1}{2}$ which is satisfied by $\theta = \frac{\pi}{3}$ and $r = 2$(*)	d M1 A1 (5)
	(b) $\frac{1}{2}\int r^2\, d\theta = (8)\int\left(1 - 2\cos\theta + \cos^2\theta\right)d\theta$	M1
	$= (8)\left[\theta - 2\sin\theta + \frac{\sin 2\theta}{4} + \frac{\theta}{2}\right]$	M1 A1
	$8\left[\frac{3\theta}{2} - 2\sin\theta + \frac{\sin 2\theta}{4}\right]_{\pi/3}^{\pi/2} = 8\left(\left(\frac{3\pi}{4} - 2\right) - \left(\frac{\pi}{2} - \sqrt{3} + \frac{\sqrt{3}}{8}\right)\right) = 2\pi - 16 + 7\sqrt{3}$	M1
	Triangle: $\frac{1}{2}(r\cos\theta)(r\sin\theta) = \frac{1}{2}\times 1\times\sqrt{3} = \frac{\sqrt{3}}{2}$	M1 A1
	Total area: $\left(2\pi - 16 + 7\sqrt{3}\right) + \frac{\sqrt{3}}{2} = (2\pi - 16) + \frac{15\sqrt{3}}{2}$	(A1) A1 (8)
		13
	(a) <u>Alternative for first 3 marks</u>: $\frac{dr}{d\theta} = 4\sin\theta$ B1 $\frac{dx}{d\theta} = -r\sin\theta + \cos\theta\frac{dr}{d\theta} = -4\sin\theta + 8\sin\theta\cos\theta$ M1 A1 Substituting r = 2 and $\theta = \frac{\pi}{3}$ into **original equation** scores 0 marks. (b) M1 needs attempt to expand $(1-\cos\theta)^2$ giving three terms (allow slips) Second M1 needs integration of $\cos^2\theta$ using $\cos 2\theta \pm 1$ Third M1 needs correct limits- may evaluate two areas and subtract M1 needs attempt at area of triangle and A1 for cao Next A1 is for value of area within curve, then final A1 is cao, must be exact but allow 4 terms and isw for incorrect collection of terms Special case for use of rsinθ gives B0M1A0M0A0	

Note on Integrating Factor Method for qu 7

This is unusual, but just in case….

Writes $\frac{dx}{dv} = \frac{vx}{1+2v^2}$

$\therefore \frac{dx}{dv} - \frac{vx}{1+2v^2} = 0$

$IF = e^{\int -\frac{vdv}{1+2v^2}}$ M1

$= \mathrm{e}^{-\frac{1}{4}\ln(1+2v^2)}$ M1A1

$x(1+2v^2)^{-\frac{1}{4}} = k$ $B1$

$Ax^4 = 1+2v^2$ dM1