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GCE

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Mathematics

Statistics 2 S2 (6684)

June 2008

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Mark Scheme (Final)

Edexcel GCE Mathematics

6684/01 Statistics S2
June 2008 Advanced Subsidiary/Advanced Level in GCE Mathematics

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

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June 2008
6684 Statistics S2
Mark Scheme

Question Number	Scheme	Marks
1(a)	$E(X) = 5$	B1
	$\text{Var}(X) = \frac{1}{12}(10-0)^2$ or attempt to use $\int \frac{x^2}{10}dx - \mu^2$	M1
	$= \frac{100}{12} = \frac{25}{3} = 8\frac{1}{3} = 8.\dot{3}$ awrt 8.33	A1
		(3)
(b)	$P(X \le 2) = (2-0)\times\frac{1}{10} = \frac{1}{5}$ or $\frac{2}{10}$ or 0.2	M1 A1
		(2)
(c)	$\left(\frac{1}{5}\right)^5 = 0.00032$ or $\frac{1}{3125}$ or 3.2×10^{-4} o.e.	M1 A1
		(2)
(d)	$P(X \ge 8)$ or $P(X > 8)$	M1
	$P(X \ge 8 \mid X \ge 5) = \frac{P(X \ge 8)}{P(X \ge 5)}$	M1
	$= \frac{2/10}{5/10}$	
	$= \frac{2}{5}$	A1
		(3)
	alternative	
	remaining time ~ U[0,5] or U[5,10] $P(X \ge 3 \text{ or } 8) = \frac{2}{5}$	M1 M1 A1
		(Total 10)

Notes

(a) B1 cao

M1 using the correct formula $\frac{(a-b)^2}{12}$ and subst in 10 or 0

or for an attempt at the integration they must increase the power of x by 1 and subtract their $E(X)$ squared.

A1 cao

(b) M1 for $P(X \le 2)$ or $P(X < 2)$

A1 cao

(c) M1 (their b) ^{5}. If the answer is incorrect we must see this. No need to check with your calculator

A1 cao

	(d) writing $P(X \geq 8)$ (may use > sign). If they do not write $P(X \geq 8)$ then it must be clear from their working that they are finding it. 0.2 on its own with no working gets M0 M1 For attempting to use a correct conditional probability. NB this is an A mark on EPEN A1 2/5 Full marks for 2/5 on its own with no incorrect working Alternative M1 for $P(X \geq 3)$ or $P(X \geq 8)$ may use > sign M1 using either U[0,5] or U[5,10] A1 2/5	

Question Number	Scheme	Marks
2	$X \sim B(100,0.58)$ $Y \sim N(58, 24.36)$	B1 B1 B1
	$[P(X > 50) = P(X \geq 51)]$	
	using 50.5 or 51.5 or 49.5 or 48.5	M1
	$= P\left(z \geq \pm\left(\frac{50.5-58}{\sqrt{24.36}}\right)\right)$ standardising 50.5, 51, 51.5 ,48.5,49, 49.5 and their μ and σ for M1	M1
	$= P(z \geq -1.52\ldots)$	A1
	$= 0.9357$	A1
	alternative $X \sim B(100,0.58)$ $Y \sim N(42, 24.36)$	(7) B1 B1 B1
	$[P(X < 50) = P(X \leq 49)]$	
	using 50.5 or 51.5 or 49.5 or 48.5	M1
	$= P\left(z \leq \pm\left(\frac{49.5-42}{\sqrt{24.36}}\right)\right)$ standardising 50.5, 51, 51.5 ,48.5,49, 49.5 and their μ and σ for M1	M1 A1
	$= P(z \leq 1.52\ldots)$	
	$= 0.9357$	A1 (Total 7)
	Notes The first 3 marks may be given if the following figures are seen in the standardisation formula :- 58 or 42, 24.36 or √24.36 or √24.4 or awrt 4.94. Otherwise B1 normal B1 58 or 42 B1 24.36 M1 using 50.5 or 51.5 or 49.5 or 48.5. ignore the direction of the inequality. M1 standardising 50.5, 51, 51.5 ,48.5, 49, 49.5 and their μ and σ . They may use √ 24 or √24.36 or √24.4 or awrt 4.94 for σ or the √of their variance. A1 ± 1.52. may be awarded for $\pm\left(\frac{50.5-58}{\sqrt{24.36}}\right)$ or $\pm\left(\frac{49.5-42}{\sqrt{24.36}}\right)$ o.e. A1 awrt 0.936	

Question Number	Scheme	Marks
3(a)	$X \sim$ Po (9) may be implied by calculations in part a or b	M1
	P($X \leq 3$) = 0.0212 P($X \geq 16$) = 0.0220	
	CR $X \leq 3$; $\cup$ $X \geq 16$	A1; A1 (3)
(b)	P(rejecting Ho) = 0.0212 + 0.0220	M1
	= 0.0432 or 0.0433	A1 cao (2)
		Total 5

Notes

(a) M1 for using Po (9) – other values you might see which imply Po (9) are 0.0550, 0.0415, 0.9780, 0.9585, 0.9889,0.0111,0.0062 or may be assumed by at least one correct region.

A1 for $X \leq 3$ or $X < 4$ condone c1 or CR instead of X

A1 for $X \geq 16$ or $X > 15$

They must identify the critical regions at the end and not just have them as part of their working. Do not accept P($X \leq 3$) etc gets A0

(b) if they use 0.0212 and 0.0220 they can gain these marks regardless of the critical regions in part a. If they have not got the correct numbers they must be adding the values for their critical regions.(both smaller than 0.05) You may need to look these up. The most common table values for lambda = 9 are in this table

x	2	3	4	5	14	15	16	17	18
	0.0062	0.0212	0.0550	0.1157	0.9585	0.9780	0.9889	0.9947	0.9976

A1 awrt 0.0432 or 0.0433

Special case

If you see 0.0432 / 0.0433 and then they go and do something else with it eg 1 – 0.0432 award M1 A0

Question Number	Scheme	Marks
4(a)	$X \sim B(11000, 0.0005)$	M1 A1 (2)
(b)	$E(X) = 11000 \times 0.0005 = 5.5$	B1
	$\text{Var}(X) = 11000 \times 0.0005 \times (1 - 0.0005)$ $= 5.49725$	B1 (2)
(c)	$X \sim Po(5.5)$	M1 A1
	$P(X \leq 2) = 0.0884$	dM1 A1 (4)
		Total 8
	Notes (a) M1 for Binomial, A1 fully correct These cannot be awarded unless seen in part a (b)B1 cao B1 also allow 5.50, 5.497, 5.4973, do **not** allow 5.5 (c) M1 for Poisson A1 for **using** Po (5.5) M1 this is dependent on the previous M mark. It is for attempting to find $P(X \leq 2)$ A1 awrt 0.0884 Correct answer with no working gets full marks Special case If they use normal approximation they could get M0 A0 M1 A0 if they use 2.5 in their standardisation. NB exact binomial is 0.0883	

Question Number	Scheme	Marks
5(a)	$X \sim B(15, 0.5)$	B1 B1 (2)
(b)	P ($X = 8$) = P ($X \leq 8$) – P($X \leq 7$) or $\left(\frac{15!}{8!7!}(p)^8(1-p)^7\right)$	M1
	= 0.6964 – 0.5	
	= 0.1964 awrt 0.196	A1 (2)
(c)	P ($X \geq 4$) = 1 – P($X \leq 3$)	M1
	= 1 – 0.0176	
	= 0.9824	A1 (2)
(d)	$H_o : p = 0.5$ $H_1 : p > 0.5$	B1 B1
	$X \sim B(15, 0.5)$	
	P($X \geq 13$) = 1 − P($X \leq 12$) = 1 − 0.9963 = 0.0037 | [P($X \geq 12$) = 1− 0.9824 = 0.0176] P($X \geq 13$) = 1− 0.9963 = 0.0037 CR $X \geq 13$ att P($X \geq 13$) awrt 0.0037/ CR $X \geq 13$	M1 A1
	0.0037 < 0.01 | 13 ≥ 13	
	Reject H_0 or it is significant or a correct statement in context from their values	M1
	There is sufficient evidence at the 1% significance level that the coin is biased in favour of heads Or There is evidence that Sues belief is correct	A1 (6)
	Notes (a) B1 for Binomial B1 for 15 and 0.5 must be in part a This need not be in the form written (b) M1 attempt to find P ($X = 8$) any method. Any value of p A1 awrt 0.196 Answer only full marks (c) M1 for 1 - P ($X \leq 3$). A1 awrt 0.982	

(d) B1 for correct H_0. must use p or π
B1 for correct H_1 must be one tail must use p or π
M1 attempt to find P($X \geq 13$) correctly. E.g. $1 - P(X \leq 12)$
A1 correct probability or CR

To get the next 2 marks the null hypothesis must state or imply that (p) = 0.5

M1 for correct statement based on their probability or critical region or a correct contextualised statement that implies that. not just 13 is in the critical region.

A1 This depends on their M1 being awarded for rejecting H_0. Conclusion in context. Must use the words biased in favour of heads or biased against tails or sues belief is correct .
NB this is a B mark on EPEN.

They may also attempt to find P($X < 13$) = 0.9963 and compare with 0.99

Question Number	Scheme	Marks
6(a)	Calls occur singly Calls occur at a constant rate Calls occur independently or randomly.	any two of the 3 only need calls once B1 B1 (2)
(b) (i)	X~ Po(4.5) used or seen in (i) or (ii) P ($X = 5$) = P ($X \leq 5$) – P($X \leq 4$) = 0.7029 – 0.5321 = 0.1708	M1 M1 A1 (3)
(ii)	P ($X > 8$) = 1 – P ($X \leq 8$) = 1 – 0.9597 = 0.0403	M1 A1 (2)
(c)	H_o : $\lambda = 9$ ($\lambda = 18$) may use λor μ H_1 : $\lambda > 9$ ($\lambda > 18$)	B1
	$X \sim$ Po (9) may be implied	B1
	P($X \geq 14$) = 1 − P($X \leq 13$) = 1 − 0.9261 = 0.0739 0.0739 > 0.05 [P($X \geq 14$) = 1− 0.9261 = 0.0739] att P($X \geq 14$) P($X \geq 15$) P($X \geq 15$) = 1− 0.9780 = 0.0220 CR $X \geq 15$ awrt 0.0739 $14 \leq 15$	M1 A1
	Accept H_0. or it is not significant or a correct statement in context from their values There is insufficient evidence to say that the number of calls per hour handled by the agent has increased.	M1 A1 (6)

Notes

(a) B1 B1 They must use calls at least once. Independently and randomly are the same reason.
Award the first B1 if they only gain 1 mark.
Special case if they don't put in the word calls but write two correct statements award B0B1

(b) correct answers only score full marks
(i) M1 Po (4.5) may be implied by them using it in their calculations in (i) or (ii)
M1 for P($X \leq 5$) – P($X \leq 4$) **or** $\frac{e^{-\lambda}\lambda^5}{5!}$
A1 only awrt 0.171

(ii) M1 for $1 - P(X \leq 8)$
A1 only awrt 0.0403

(c) B1 both . Must be one tail test. They may use λ or μ and either 9 or 18 and match H_0 and H_1

M1 Po (9) may be implied by them using it in their calculations.
M1 attempt to find $P(X \geq 14)$ eg $1 - P(X \leq 13)$ or $1 - P(X < 14)$
A1 correct probability or CR

To get the next2 marks the null hypothesis must state or imply that $(\lambda) = 9$ or 18

M1 for a correct statement based on their probability or critical region
or a correct contextualised statement that implies that.

A1. This depends on their M1 being awarded for accepting H_0. Conclusion in context. Must have calls per hour has not increased. Or the rate of calls has not increased.
Any statement that has the word **calls** in and implies the **rate not increasing**
e.g. no evidence that the rate of calls handled has increased
Saying the number of calls has not increased gains A0 as it does not imply rate
NB this is an A mark on EPEN

They may also attempt to find $P(X < 14) = 0.9261$ and compare with 0.95

Question Number	Scheme	Marks
7(a)	$\int_0^1 \frac{1}{2}x\ \mathrm{d}x = \left[\frac{1}{4}x^2\right]_0^1 = \frac{1}{4}$ oe — attempt to integrate both parts	M1
	$\int_1^2 kx^3\ \mathrm{d}x \left[\frac{1}{4}kx^4\right]_1^2 = 4k - \frac{1}{4}k$ oe — both answer correct	A1
	$\frac{1}{4} + 4k - \frac{1}{4}k = 1$ — adding two answers and putting = 1	dM1dep on previous M
	$\frac{15k}{4} = \frac{3}{4}$	
	$k = \frac{1}{5}$ *	A1 (4)
(b)	$\int_0^1 \frac{1}{2}x^2\ \mathrm{d}x = \left[\frac{1}{6}x^3\right]_0^1 = \frac{1}{6}$ — attempt to integrate $xf(x)$ for one part	M1
	1/6	A1
	$\int_1^2 \frac{1}{5}x^4\ \mathrm{d}x = \left[\frac{1}{25}x^5\right]_1^2 = \frac{32}{25} - \frac{1}{25}$	
	$= \frac{31}{25}$ or 1.24	A1
	$\mathrm{E}(X) = \frac{1}{6} + \frac{31}{25}$	
	$= \frac{211}{150} = 1\frac{61}{150} = 1.40\dot{6}$	A1 (4)
(c)		
	$\mathrm{F}(x) = \int_0^x \frac{1}{2}t\ \mathrm{dt}$ (for $0 \le x \le 1$) — ignore limits for M	M1
	$= \frac{1}{4}x^2$ — must use limit of 0	A1
	$\mathrm{F}(x) = \int_1^x \frac{1}{5}t^3\ \mathrm{dt}; + \int_0^1 \frac{1}{2}t\ \mathrm{dt}$ (for $1 < x \le 2$) — need limit of 1 and variable upper limit; need limit 0 and 1	M1; M1
	$= \frac{1}{20}x^4 + \frac{1}{5}$	A1

	$F(x)\begin{cases} 0 & x<0 \\ \frac{1}{4}x^2 & 0\le x\le 1 \\ \frac{1}{20}x^4+\frac{1}{5} & 1<x\le 2 \\ 1 & x>2 \end{cases}$ middle pair ends	B1 ft B1 (7)
(d)	F(m) = 0.5 either eq $\frac{1}{20}m^4+\frac{1}{5}=0.5$ eq for their $1\le x\le 2$ $m = \sqrt[4]{6}$ or 1.57 or awrt 1.57	M1 A1ft A1 (3)
(e)	negative skew This depends on the previous B1 being awarded. One of the following statements which must be compatible with negative skew and their figures. If they use mode then they must have found a value for it Mean < Median Mean < mode Mean < median (< mode) Median < mode Sketch of the pdf.	B1 dB1 (2)
	Notes (a) M1 for adding two integrals together =1, ignore limits A1 for correct integration, ignore limits M1 using correct limits A1 cso (b) M1 attempting to use integral of x f(x) A1 correct two integrals added with limits A1 correct integration ignore limits A1 awrt 1.41 (c) M1 Att to integrate $\frac{1}{2}t$ (they need to increase the power by 1). Ignore limits for method mark A1 $\frac{1}{4}x^2$ allow use of t. must have used/implied use of limit of 0. This must be on its own without anything else added M1 att to integrate $\int_1^x \frac{1}{5}t^3\,dt$ and correct limits.	

M1 $\int_0^1 \frac{1}{2}t\ \mathrm{dt} +$ Att to integrate using limits 0 and 1. no need to see them put 0 in .

they must add this to their $\int_1^x \frac{1}{5}t^3\ \mathrm{dt}$. may be given if they add 1/4

(Alternative method for these last two M marks

M1 for att to $\int \frac{1}{5}\boldsymbol{t^3}\ \mathrm{dt}$ and putting + C

M1 use of F(2) = 1 to find C)

A1 $\frac{1}{20}x^4 + \frac{1}{5}$ must be correct

B1 middle pair followed through from their answers. condone them using < or $\leq$ incorrectly they do not need to match up

B1 end pairs. condone them using < or $\leq$. They do not need to match up

NB if they show no working and just write down the distribution. If it is correct they get full marks. If it is incorrect then they cannot get marks for any incorrect part. So if $0<x<1$ is correct they can get M1 A1 otherwise M0 A0. if $3<x<4$ is correct they can get M1 A1A1 otherwise M0 A0A0. you cannot award B1ft if they show no working unless the middle parts are correct.

(d) M1 either of their $\frac{1}{4}x^2$ or $\frac{1}{20}x^4 + \frac{1}{5} = 0.5$

A1 for their F(X) $1<x<2 = 0.5$

A1 cao

If they add both their parts together and put = 0.5 they get M0
If they work out both parts separately and do not make the answer clear they can get M1 A1 A0

(e) B1 negative skew only

B1 Dependent on getting the previous B1. their reason must follow through from their figures.