Edexcel GCE

Mechanics M5

Advanced/Advanced Subsidiary

Friday 20 June 2008 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination
Mathematical Formulae (Green)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions. Write your answers in the spaces provided in this question paper.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 7 questions in this question paper. The total mark for this paper is 75.
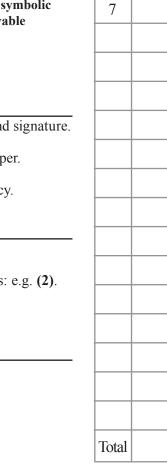There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

Turn over

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1. [*In this question* **i** *and* **j** *are horizontal unit vectors.*]

A small bead of mass 0.5 kg is threaded on a smooth horizontal wire. The bead is initially at rest at the point with position vector $(\mathbf{i} - 6\mathbf{j})$ m. A constant horizontal force **P** N then acts on the bead causing it to move along the wire. The bead passes through the point with position vector $(7\mathbf{i} - 14\mathbf{j})$ m with speed $2\sqrt{7}$ m s^{-1}.

Given that **P** is parallel to $(6\mathbf{i} + \mathbf{j})$, find **P**.

(6)

Question 1 continued

2. The velocity **v** m s^{-1} of a particle P at time t seconds satisfies the vector differential equation

$$\frac{d\mathbf{v}}{dt} + 4\mathbf{v} = 0 \ .$$

The position vector of P at time t seconds is **r** metres.

Given that at $t = 0$, $\mathbf{r} = (\mathbf{i} - \mathbf{j})$ and $\mathbf{v} = (-8\mathbf{i} + 4\mathbf{j})$, find **r** at time t seconds.

(7)

Question 2 continued

3. A system of forces consists of two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ acting on a rigid body.

$\mathbf{F}_1 = (-2\mathbf{i} + \mathbf{j} - \mathbf{k})$ N and acts at the point with position vector $\mathbf{r}_1 = (\mathbf{i} - \mathbf{j} + \mathbf{k})$ m.

$\mathbf{F}_2 = (3\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ N and acts at the point with position vector $\mathbf{r}_2 = (4\mathbf{i} - \mathbf{j} - 2\mathbf{k})$ m.

Given that the system is equivalent to a single force $\mathbf{R}$ N, acting at the point with position vector $(5\mathbf{i} + \mathbf{j} - \mathbf{k})$ m, together with a couple $\mathbf{G}$ N m, find

(a) $\mathbf{R}$,

(2)

(b) the magnitude of $\mathbf{G}$.

(9)

Question 3 continued

Q3

(Total 11 marks)

4. At time $t = 0$ a rocket is launched from rest vertically upwards. The rocket propels itself upwards by expelling burnt fuel vertically downwards with constant speed U m s^{-1} relative to the rocket. The initial mass of the rocket is M_0 kg. At time t seconds, where $t < 2$, its mass is $M_0(1 - \frac{1}{2}t)$ kg, and it is moving upwards with speed v m s^{-1}.

(a) Show that

$$\frac{dv}{dt} = \frac{U}{(2-t)} - 9.8$$

(7)

(b) Hence show that $U > 19.6$

(2)

(c) Find, in terms of U, the speed of the rocket one second after its launch.

(5)

Question 4 continued

H 2 9 9 4 7 R A 0 9 2 4

Question 4 continued

Question 4 continued

(Total 14 marks)

Q4

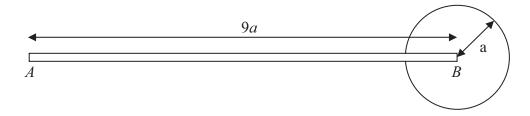**Turn over**

5.

$9a$

a

A

B

Figure 1

A pendulum P is modelled as a uniform rod AB, of length $9a$ and mass m, rigidly fixed to a uniform circular disc of radius a and mass $2m$. The end B of the rod is attached to the centre of the disc, and the rod lies in the plane of the disc, as shown in Figure 1. The pendulum is free to rotate in a vertical plane about a fixed smooth horizontal axis L which passes through the end A and is perpendicular to the plane of the disc.

(a) Show that the moment of inertia of P about L is $190ma^2$.

(4)

The pendulum makes small oscillations about L.

(b) By writing down an equation of motion for P, find the approximate period of these small oscillations.

(7)

Question 5 continued

H 2 9 9 4 7 R A 0 1 3 2 4

Question 5 continued

Question 5 continued

(Total 11 marks)

H 2 9 9 4 7 R A 0 1 5 2 4

6. A uniform solid right circular cylinder has mass M, height h and radius a. Find, using integration, its moment of inertia about a diameter of one of its circular ends.

[*You may assume without proof that the moment of inertia of a uniform circular disc, of mass m and radius a, about a diameter is* $\frac{1}{4}ma^2$.]

(10)

Question 6 continued

H 2 9 9 4 7 R A 0 1 7 2 4

Question 6 continued

H 2 9 9 4 7 R A 0 1 8 2 4

Question 6 continued

Q6

H 2 9 9 4 7 R A 0 1 9 2 4

7. A uniform square lamina *ABCD*, of mass $2m$ and side $3a\sqrt{2}$, is free to rotate in a vertical plane about a fixed smooth horizontal axis *L* which passes through *A* and is perpendicular to the plane of the lamina. The moment of inertia of the lamina about *L* is $24ma^2$.

The lamina is at rest with *C* vertically above *A*. At time $t = 0$ the lamina is slightly displaced. At time *t* the lamina has rotated through an angle θ.

(a) Show that

$$2a\left(\frac{d\theta}{dt}\right)^2 = g(1 - \cos\theta).$$

(4)

(b) Show that, at time *t*, the magnitude of the component of the force acting on the lamina at *A*, in a direction perpendicular to *AC*, is $\frac{1}{2}mg\sin\theta$.

(7)

When the lamina reaches the position with *C* vertically below *A*, it receives an impulse which acts at *C*, in the plane of the lamina and in a direction which is perpendicular to the line *AC*. As a result of this impulse the lamina is brought immediately to rest.

(c) Find the magnitude of the impulse.

(5)

Question 7 continued

H 2 9 9 4 7 R A 0 2 1 2 4

Question 7 continued

Question 7 continued

(Total 16 marks)

TOTAL FOR PAPER: 75 MARKS

END

H 2 9 9 4 7 R A 0 2 3 2 4

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