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Mark Scheme (Final)

Summer 2008

GCE

GCE Mathematics (6665/01)

A PEARSON COMPANY

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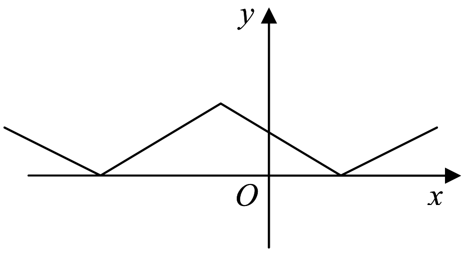
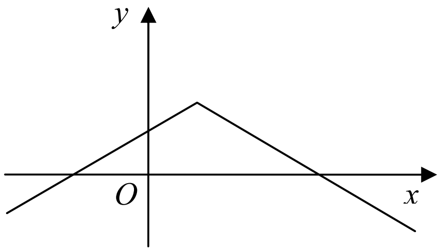
General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate’s response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

June 2008
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme	Marks
1.	(a) $e^{2x+1} = 2$	
	$2x + 1 = \ln 2$	M1
	$x = \frac{1}{2}(\ln 2 - 1)$	A1 (2)
	(b) $\frac{dy}{dx} = 8e^{2x+1}$	B1
	$x = \frac{1}{2}(\ln 2 - 1) \quad \Rightarrow \quad \frac{dy}{dx} = 16$	B1
	$y - 8 = 16\left(x - \frac{1}{2}(\ln 2 - 1)\right)$	M1
	$y = 16x + 16 - 8\ln 2$	A1 (4)
		[6]

Question Number	Scheme		Marks	
2.	(a) $R^2 = 5^2 + 12^2$		M1	
	$R = 13$		A1	
	$\tan\alpha = \dfrac{12}{5}$		M1	
	$\alpha \approx 1.176$	cao	A1	(4)
	(b) $\cos(x-\alpha) = \dfrac{6}{13}$		M1	
	$x-\alpha = \arccos\dfrac{6}{13} = 1.091\ \ldots$		A1	
	$x = 1.091\ \ldots + 1.176\ \ldots \approx 2.267\ldots$	awrt 2.3	A1	
	$x-\alpha = -1.091\ \ldots$	accept … = 5.19 … for M	M1	
	$x = -1.091\ \ldots + 1.176\ \ldots \approx 0.0849\ldots$	awrt 0.084 or 0.085	A1	(5)
	(c)(i) $R_{\max} = 13$	ft their R	B1 ft	
	(ii) At the maximum, $\cos(x-\alpha) = 1$ *or* $x-\alpha = 0$		M1	
	$x = \alpha = 1.176\ \ldots$	awrt 1.2, ft their α	A1ft	(3)
				[12]

Question Number	Scheme		Marks	
3.	(a) 	 shape	B1	
		Vertices correctly placed	B1	(2)
	(b) 	 shape	B1	
		Vertex and intersections with axes correctly placed	B1	(2)
	(c) $P:(-1,2)$		B1	
	$Q:(0,1)$		B1	
	$R:(1,0)$		B1	(3)
	(d) $x>-1$; $\quad 2-x-1=\frac{1}{2}x$		M1 A1	
	Leading to $x=\frac{2}{3}$		A1	
	$x<-1$; $\quad 2+x+1=\frac{1}{2}x$		M1	
	Leading to $x=-6$		A1	(5)
				[12]

Question Number	Scheme		Marks	
4.	(a) $x^2-2x-3=(x-3)(x+1)$		B1	
	$f(x)=\dfrac{2(x-1)-(x+1)}{(x-3)(x+1)}$ $\left(or\ \dfrac{2(x-1)}{(x-3)(x+1)}-\dfrac{x+1}{(x-3)(x+1)} \right)$		M1 A1	
	$=\dfrac{x-3}{(x-3)(x+1)}=\dfrac{1}{x+1}$ $\ast$	cso	A1	(4)
	(b) $\left(0,\dfrac{1}{4}\right)$ Accept $0<y<\dfrac{1}{4},\ 0<f(x)<\dfrac{1}{4}$ etc.		B1 B1	(2)
	(c) Let $y=f(x)$ $y=\dfrac{1}{x+1}$			
	$x=\dfrac{1}{y+1}$			
	$yx+x=1$			
	$y=\dfrac{1-x}{x}$	or $\dfrac{1}{x}-1$	M1 A1	
	$f^{-1}(x)=\dfrac{1-x}{x}$			
	Domain of f^{-1} is $\left(0,\dfrac{1}{4}\right)$	ft their part (b)	B1 ft	(3)
	(d) $fg(x)=\dfrac{1}{2x^2-3+1}$			
	$\dfrac{1}{2x^2-2}=\dfrac{1}{8}$		M1	
	$x^2=5$		A1	
	$x=\pm\sqrt{5}$	both	A1	(3)
				[12]

Question Number	Scheme	Marks
5.	(a) $\sin^2\theta+\cos^2\theta=1$	
	$\div\sin^2\theta$ $\quad \dfrac{\sin^2\theta}{\sin^2\theta}+\dfrac{\cos^2\theta}{\sin^2\theta}=\dfrac{1}{\sin^2\theta}$	M1
	$1+\cot^2\theta=\operatorname{cosec}^2\theta$ ∗ cso	A1 (2)
	Alternative for (a)	
	$1+\cot^2\theta=1+\dfrac{\cos^2\theta}{\sin^2\theta}=\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta}=\dfrac{1}{\sin^2\theta}$	M1
	$=\operatorname{cosec}^2\theta$ ∗ cso	A1
	(b) $2\left(\operatorname{cosec}^2\theta-1\right)-9\operatorname{cosec}\theta=3$	M1
	$2\operatorname{cosec}^2\theta-9\operatorname{cosec}\theta-5=0$ *or* $5\sin^2\theta+9\sin\theta-2=0$	M1
	$(2\operatorname{cosec}\theta+1)(\operatorname{cosec}\theta-5)=0$ *or* $(5\sin\theta-1)(\sin\theta+2)=0$	M1
	$\operatorname{cosec}\theta=5$ *or* $\sin\theta=\dfrac{1}{5}$	A1
	$\theta=11.5°, 168.5°$	A1 A1 (6)
		[8]

Question Number	Scheme	Marks
6.	(a)(i) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{3x}\left(\sin x+2\cos x\right)\right)=3\mathrm{e}^{3x}\left(\sin x+2\cos x\right)+\mathrm{e}^{3x}\left(\cos x-2\sin x\right)$	M1 A1 A1 (3)
	$\left(=\mathrm{e}^{3x}\left(\sin x+7\cos x\right)\right)$	
	(ii) $\frac{\mathrm{d}}{\mathrm{d}x}\left(x^3\ln\left(5x+2\right)\right)=3x^2\ln\left(5x+2\right)+\frac{5x^3}{5x+2}$	M1 A1 A1 (3)
	(b) $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\left(x+1\right)^2\left(6x+6\right)-2\left(x+1\right)\left(3x^2+6x-7\right)}{\left(x+1\right)^4}$	M1 $\frac{\text{A1}}{\text{A1}}$
	$=\frac{\left(x+1\right)\left(6x^2+12x+6-6x^2-12x+14\right)}{\left(x+1\right)^4}$	M1
	$=\frac{20}{\left(x+1\right)^3}$ * cso	A1 (5)
	(c) $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\frac{60}{\left(x+1\right)^4}=-\frac{15}{4}$	M1
	$\left(x+1\right)^4=16$	M1
	$x=1,-3$ both	A1 (3) **[14]**
	Note: The simplification in part (b) can be carried out as follows	
	$\frac{\left(x+1\right)^2\left(6x+6\right)-2\left(x+1\right)\left(3x^2+6x-7\right)}{\left(x+1\right)^4}$	
	$=\frac{\left(6x^3+18x^2+18x+6\right)-\left(6x^3+18x^2-2x-14\right)}{\left(x+1\right)^4}$	
	$=\frac{20x+20}{\left(x+1\right)^4}=\frac{20\left(x+1\right)}{\left(x+1\right)^4}=\frac{20}{\left(x+1\right)^3}$	M1 A1

Question Number	Scheme	Marks
7.	(a) $\mathrm{f}(1.4) = -0.568\ \ldots < 0$	
	$\mathrm{f}(1.45) = 0.245\ \ldots > 0$	M1
	Change of sign (and continuity) $\Rightarrow\ \alpha \in (1.4, 1.45)$	A1 (2)
	(b) $3x^3 = 2x + 6$	
	$x^3 = \frac{2x}{3} + 2$	
	$x^2 = \frac{2}{3} + \frac{2}{x}$	M1 A1
	$x = \sqrt{\left(\frac{2}{x} + \frac{2}{3}\right)}$ * cso	A1 (3)
	(c) $x_1 = 1.4371$	B1
	$x_2 = 1.4347$	B1
	$x_3 = 1.4355$	B1 (3)
	(d) Choosing the interval $(1.4345, 1.4355)$ or appropriate tighter interval.	M1
	$\mathrm{f}(1.4345) = -0.01$ ...	
	$\mathrm{f}(1.4355) = 0.003\ \ldots$	M1
	Change of sign (and continuity) $\Rightarrow\ \alpha \in (1.4345, 1.4355)$	
	$\Rightarrow \alpha = 1.435$, correct to 3 decimal places * cso	A1 (3) **[11]**
	Note: $\alpha = 1.435\,304\,553\ \ldots$	