

Mark Scheme (Final)

January 2008

GCE

GCE Mathematics (6683/01)

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

January 2008
6683 Statistics Mathematics
Mark Scheme

Question Number	Scheme	Marks
1. (a)	$\sum x = 773, \sum y = 724$ $r = \frac{10 \times 56076 - 773 \times 724}{\sqrt{(10 \times 60475 - 773^2)(10 \times 53122 - 724^2)}} \quad \text{o.e.}$ $r = 0.155357\dots$	B1, B1 M1 A1ft A1 (5)
(b)	Both weak correlation Neither score is a good indication of future performance Interview test is slightly better since correlation is positive	B1g B1h (2) Total 7 marks
NB (a)	$S_{xx} = 60475 - \frac{(773)^2}{10} = 722.1, \quad S_{yy} = 53122 - \frac{(724)^2}{10} = 704.4, \quad S_{xy} = 56076 - \frac{773 \times 724}{10} = 110.8$ <p>1st B1 for $\sum x$ and 2nd B1 for $\sum y$, should be seen or implied.</p> <p>M1 for at least one correct attempt at one of S_{xx}, S_{yy} or S_{xy} and then using in the correct formula</p> <p>1st A1ft for a fully correct expression. (ft their $\sum x$ and their $\sum y$) or 3 correct expressions for S_{xx}, S_{xy}, and S_{yy} but possibly incorrect values for these placed correctly in r.</p>	
(b)	<p>2nd A1 for awrt 0.155</p> <p>If $r > 0.5$ they can score B1g in (b) for saying that it (skills test) is not a good guide to performance but B0h since a second acceptable comment about both tests is not possible.</p> <p>Give B1 for one correct line, B1B1 for any 2. If the only comment is the test(s) <u>are</u> a good guide: scores B0B0 If the only comment is the tests are not good: scores B1B0 (second line)</p> <p>The third line is for a comment that suggests that the interview test is OK but the skills test is not since one is positive and the other is negative.</p> <p>Treat 1st B1 as B1g and 2nd as B1h</p> <p>An answer of “no” alone scores B0B0</p>	

Question Number	Scheme	Marks
2.		
(a)	<p>mean is $\frac{2757}{12}, = 229.75$ AWRT 230</p> <p>sd is $\sqrt{\frac{724961}{12} - (229.75)^2}, = 87.34045$ AWRT 87.3</p> <p style="text-align: right;">[Accept $s =$ AWRT 91.2]</p>	M1, A1 M1, A1 (4)
(b)	<p>Ordered list is: 125, 160, 169, 171, 175, 186, 210, 243, 250, 258, 390, 420</p> <p>$Q_2 = \frac{1}{2}(186 + 210) = 198$</p> <p>$Q_1 = \frac{1}{2}(169 + 171) = 170$</p> <p>$Q_3 = \frac{1}{2}(250 + 258) = 254$</p>	B1 B1 B1 (3)
(c)	<p>$Q_3 + 1.5(Q_3 - Q_1) = 254 + 1.5(254 - 170), = 380$ Accept AWRT (370-392)</p> <p>Patients F (420) and B (390) are outliers.</p>	M1, A1 B1ft B1ft (4)
(d)	<p>$\frac{Q_1 - 2Q_2 + Q_3}{Q_3 - Q_1} = \frac{170 - 2 \times 198 + 254}{254 - 170}, = 0.\dot{3}$ AWRT 0.33</p> <p>Positive skew.</p>	M1, A1 A1ft (3)
Total 14 marks		
(a)	<p>1st M1 for using $\frac{\sum x}{n}$ with a credible numerator and $n = 12$.</p> <p>2nd M1 for using a correct formula, root required but can ft their mean</p> <p>NB Use of $s = \sqrt{8321.84...} = 91.22...$ is OK for M1A1 here.</p> <p>Answers only from a calculator in (a) can score full marks</p>	
(b)	<p>1st B1 for median= 198 only, 2nd B1 for lower quartile 3rd B1 for upper quartile</p>	
S.C.	<p>If all Q_1 and Q_3 are incorrect but an ordered list (with ≥ 6 correctly placed) is seen and used then award B0B1 as a special case for these last two marks.</p>	
(c)	<p>M1 for a clear attempt using their quartiles in given formula,</p> <p>A1 for any value in the range 370 - 392</p> <p>1st B1ft for any one correct decision about B or F - ft their limit in range (258, 420)</p> <p>2nd B1ft for correct decision about both F and B - ft their limit in range (258, 420)</p> <p>If more points are given score B0 here for the second B mark. (Can score M0A0B1B1 here)</p>	
(d)	<p>M1 for an attempt to use their figures in the correct formula – must be seen (≥ 2 correct substitutions)</p> <p>1st A1 for AWRT 0.33</p> <p>2nd A1ft for positive skew. Follow through their value/sign of skewness . Ignore any further calculations. “positive correlation” scores A0</p>	

3.	<table border="1" data-bbox="287 190 1257 268"> <tr> <td>Width</td> <td>1</td> <td>1</td> <td>4</td> <td>2</td> <td>3</td> <td>5</td> <td>3</td> <td>12</td> </tr> <tr> <td>Freq. Density</td> <td>6</td> <td>7</td> <td>2</td> <td>6</td> <td>5.5</td> <td>2</td> <td>1.5</td> <td>0.5</td> </tr> </table> <p data-bbox="1037 268 1228 302" style="text-align: right;">0.5×12 or 6</p> <p data-bbox="268 342 857 380">Total area is $(1 \times 6) + (1 \times 7) + (4 \times 2) + \dots = 70$</p> <p data-bbox="268 392 614 465">$(90.5 - 78.5) \times \frac{1}{2} \times \frac{140}{\text{their } 70}$</p> <p data-bbox="268 506 587 539">Number of runners is 12</p>	Width	1	1	4	2	3	5	3	12	Freq. Density	6	7	2	6	5.5	2	1.5	0.5	<p data-bbox="1353 206 1404 237">M1</p> <p data-bbox="1340 271 1383 302">A1</p> <p data-bbox="1340 409 1390 441">M1</p> <p data-bbox="1042 472 1302 504" style="text-align: right;">“70 seen anywhere”</p> <p data-bbox="1340 472 1383 504">B1</p> <p data-bbox="1340 510 1383 542">A1</p> <p data-bbox="1476 546 1513 577" style="text-align: right;">(5)</p> <p data-bbox="1337 580 1516 611">Total 5 marks</p>
Width	1	1	4	2	3	5	3	12												
Freq. Density	6	7	2	6	5.5	2	1.5	0.5												
	<p data-bbox="212 649 1021 723">1st M1 for attempt at width of the correct bar (90.5 - 78.5) [Maybe on histogram or in table]</p> <p data-bbox="212 723 1337 797">1st A1 for 0.5×12 or 6 (may be seen on the histogram. Must be related to the area of the bar above 78.5 - 90.5.</p> <p data-bbox="212 797 925 873">2nd M1 for attempting area of correct bar $\times \frac{140}{\text{their } 70}$</p> <p data-bbox="212 880 861 913">B1 for 70 seen anywhere in their working</p> <p data-bbox="212 913 686 947">2nd A1 for correct answer of 12.</p> <p data-bbox="268 992 1284 1068">Minimum working required is $2 \times 0.5 \times 12$ where the 2 should come from $\frac{140}{70}$</p> <p data-bbox="268 1070 962 1106">Beware $90.5 - 78.5 = 12$ (this scores M1A0M0B0A0)</p> <p data-bbox="268 1144 1050 1180">Common answer is $0.5 \times 12 = 6$ (this scores M1A1M0B0A0)</p> <p data-bbox="268 1218 1150 1254">If unsure send to review e.g. $2 \times 0.5 \times 12 = 12$ without 70 being seen</p>																			

<p>4.</p> <p>(a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p>	$S_{xy} = 1818.5 - \frac{41 \times 406}{10}, = 153.9$ <p>(could be seen in (b))</p> <p>AWRT 154</p> $S_{xx} = 188 - \frac{41^2}{10} = 19.9$ <p>(could be seen in (b))</p> $b = \frac{153.9}{19.9}, = 7.733668....$ $a = 40.6 - b \times 4.1 (= 8.89796....)$ $y = 8.89 + 7.73x$ <p>A typical car will travel 7700 miles every year</p> $x = 5, y = 8.89 + 7.73 \times 5 (= 47.5 - 47.6)$ <p>So mileage predicted is</p> <p>AWRT 48000</p>	<p>M1, A1</p> <p>A1</p> <p>(3)</p> <p>M1, A1</p> <p>M1</p> <p>A1</p> <p>(4)</p> <p>B1ft</p> <p>(1)</p> <p>M1</p> <p>A1</p> <p>(2)</p> <p>Total 10 marks</p>
<p>(a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p>	<p style="text-align: center;">Accept calculations for S_{xx} and S_{xy} in (a) or (b)</p> <p>M1 for correct attempt or expression for either</p> <p>1st A1 for one correct</p> <p>2nd A1 for both correct</p> <p>Ignore the open marks for part (b) they should be awarded as per this scheme</p> <p>1st M1 for $\frac{\text{their } S_{xy}}{\text{their } S_{xx}}$</p> <p>1st A1 for AWRT 7.73</p> <p>2nd M1 for attempt at correct formula for a (minus required). Ft their b. Quoting a correct formula but making one slip in sub.eg. $\bar{y} = 406$ is OK</p> <p>2nd A1 for correct equation with 2dp accuracy. Accept $a = 8.89$, and $b = 7.73$ even if not written as final equation.</p> <p>Correct answers only (from calc) score 4/4 if correct to 2dp or 3/4 if AWRT 2dp</p> <p>B1ft for their $b \times 1000$ to at least 2 sf. Accept “7.7 thousand” but value is needed</p> <p>M1 for substituting $x = 5$ into their final answer to (b).</p> <p>A1 for AWRT 48000 (Accept “48 thousands”)</p>	

<p>5. (a)</p>	<p>Diagram may be drawn with $B \subset (A \cup C)$ or with the 0 for $B \cap (A \cup C)$' simply left blank</p> <div style="text-align: center;"> </div> <p>Accept decimals or probs. in Venn diagram</p> <p>3cc 90,3,2,1 1,(0),2 1 outside Box</p> <p>(b) P(none)=0.01</p> <p>(c) P(A but not B)=0.04</p> <p>(d) P(any wine but C)=0.03</p> <p>(e) P(exactly two)=0.06</p> <p>(f) $P(C A) = \frac{P(C \cap A)}{P(A)} = \frac{93}{96}$ or $\frac{31}{32}$ or AWRT 0.969</p>	<p>M1 A1 M1A1 A1 B1</p> <p>(6)</p> <p>B1ft</p> <p>(1)</p> <p>M1 A1ft</p> <p>(2)</p> <p>M1A1ft</p> <p>(2)</p> <p>M1A1ft</p> <p>(2)</p> <p>M1A1ft,A1</p> <p>(3)</p> <p>Total 16 marks</p>
<p>(a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p> <p>(e)</p> <p>(f)</p>	<p>1st M1 for 3 closed, labelled curves that overlap. A1 for the 90, 3, 2 and 1</p> <p>2nd M1 for one of 1, 0 or 2 correct <u>or</u> a correct sum of 4 values for A, B or C</p> <p>2nd A1 for all 7 values correct. Accept a blank instead of 0.</p> <p>NB final mark is a B1 for the box not an A mark as on EPEN</p> <p>In parts (b) to (f) full marks can be scored for correct answers or correct ft</p> <p>B1ft Follow through their '1' from outside divided by 100</p> <p>M1 for correct expression eg $P(A \cup B) - P(B)$ or calculation e.g. 3 + 1 or 4 on top</p> <p>A1 for a correct probability, follow through with their '3+1' from diagram</p> <p>M1 for correct expression or calculation e.g. 1+2+0 or 99-96 or 3 on top</p> <p>A1 for a correct probability, follow through their '2+1+0' from diagram</p> <p>M1 for a correct expression or calculation e.g. 3+2+1 or 6 on top</p> <p>M1 for a correct expression upto " , " and <u>some</u> correct substitution, ft their values. One of these probabilities must be correct or correct ft. If P(C) on bottom M0</p> <p>1st A1ft follow through their $A \cap C$ and their A but the ratio must be in (0, 1)</p> <p>2nd A1 for correct answer only. Answer only scores 3/3, but check working $P(A \cap C) / P(C)$ is M0</p>	<div style="border: 1px solid black; padding: 5px; width: fit-content;"> <p>For M marks in (c) to (e) they must have a fraction</p> </div>

<p>6. (a)</p> <p>(b)</p> <p>(c)</p>	<p>200 or 200g</p> <p>$P(190 < X < 210) = 0.6$ or $P(X < 210) = 0.8$ or $P(X > 210) = 0.2$ or diagram (o.e.) Correct use of 0.8 or 0.2</p> $Z = (\pm) \frac{210 - 200}{\sigma}$ $\frac{10}{\sigma} = 0.8416$ $\sigma = 11.882129\dots$ <p>0.8416</p> <p>AWRT 11.9</p> $P(X < 180) = P\left(Z < \frac{180 - 200}{\sigma}\right)$ $= P(Z < -1.6832)$ $= 1 - 0.9535$ $= 0.0465 \text{ or AWRT } 0.046$	<p>B1</p> <p>(1)</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>B1</p> <p>A1</p> <p>(5)</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>(3)</p> <p>Total 9 marks</p>
<p>(a)</p> <p>(b)</p> <p>(c)</p>	<p>“mean = 200g” is B0 but “median = 200” or just “200” alone is B1</p> <p>Standardization in (b) and (c). They must use σ not σ^2 or $\sqrt{\sigma}$.</p> <p>1st M1 for a correct probability statement (as given or eg $P(200 < X < 210) = 0.3$ o.e.) or shaded diagram - must have values on z-axis and probability areas shown</p> <p>1st A1 for correct use of 0.8 or $p = 0.2$. Need a correct probability statement. May be implied by a suitable value for z seen (e.g. $z = 0.84$)</p> <p>2nd M1 for attempting to standardise. Values for x and μ used in formula. Don't need $z =$ for this M1 nor a z-value, just mark standardization.</p> <p>B1 for $z = 0.8416$ (or better) [$z = 0.84$ usually just loses this mark in (a)]</p> <p>2nd A1 for AWRT 11.9</p> <p>1st M1 for attempting to Standardise with 200 and their sd(>0) e.g. $(\pm) \frac{180 - 200}{\text{their } \sigma}$</p> <p>2nd M1 NB on open this is an A mark ignore and treat it as 2nd M1 for 1 – a probability from tables provided compatible with their probability statement.</p> <p>A1 for 0.0465 or AWRT 0.046 (Dependent on both Ms in part (c))</p>	

7.(a)	$P(R = 3 \cap B = 0) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$	M1, A1 (2)																															
(b)	<table border="1" style="margin-left: auto; margin-right: auto;"> <tbody> <tr><td style="background-color: #cccccc;">3</td><td>0</td><td>3</td><td>6</td><td>9</td></tr> <tr><td style="background-color: #cccccc;">2</td><td>0</td><td style="background-color: #cccccc;">2</td><td>4</td><td>6</td></tr> <tr><td style="background-color: #cccccc;">1</td><td style="background-color: #cccccc;">0</td><td>1</td><td>2</td><td>3</td></tr> <tr><td style="background-color: #cccccc;">0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr> <tr><td style="background-color: #cccccc;"><i>B</i></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td style="background-color: #cccccc;"><i>R</i></td><td style="background-color: #cccccc;">0</td><td style="background-color: #cccccc;">1</td><td style="background-color: #cccccc;">2</td><td style="background-color: #cccccc;">3</td></tr> </tbody> </table>	3	0	3	6	9	2	0	2	4	6	1	0	1	2	3	0	0	0	0	0	<i>B</i>						<i>R</i>	0	1	2	3	All 0s All 1,2,3s All 4,6,9s B1 B1 B1 (3)
3	0	3	6	9																													
2	0	2	4	6																													
1	0	1	2	3																													
0	0	0	0	0																													
<i>B</i>																																	
	<i>R</i>	0	1	2	3																												
(c)	$a = \frac{7}{16}, b = c = d = \frac{1}{16}$	B1, B1 B1 (3)																															
(d)	$E(T) = \left(1 \times \frac{1}{16}\right) + \left(2 \times \frac{1}{8}\right) + \left(3 \times \frac{1}{8}\right) + \left(4 \times \frac{1}{16}\right) + \dots$ $= 2\frac{1}{4} \text{ or exact equivalent e.g. } 2.25, \frac{9}{4}$	M1 A1 (2)																															
(e)	$\text{Var}(T) = \left(1^2 \times \frac{1}{16}\right) + \left(2^2 \times \frac{1}{8}\right) + \left(3^2 \times \frac{1}{8}\right) + \left(4^2 \times \frac{1}{16}\right) + \dots - \left(\frac{9}{4}\right)^2$ $= \frac{49}{4} - \frac{81}{16} = 7\frac{3}{16} \text{ or } \frac{115}{16} \quad (\text{o.e.})$	M1A1, M1 AWRT 7.19 A1 (4)																															
Total 14 marks																																	
(a)	M1 for $\frac{1}{4} \times \frac{1}{4}$																																
(c)	1st B1 for $\frac{7}{16}$,																																
	2nd B1 for only one error in b, c, d ($b = c = d \neq \frac{1}{16}$ or $b = c = \frac{1}{16} \neq d$ etc), 3rd B1 all of $b, c, d = \frac{1}{16}$																																
(d)	M1 for attempting $\sum tP(T = t)$, 3 or more terms correct or correct ft. Must Attempt to sum.																																
	NB calculating $E(T)$ and then dividing by a number other than 1 scores M0.																																
(e)	1st M1 for attempt at $E(T^2)$, 3 or more terms correct or correct ft.																																
	1st A1 for $\frac{49}{4}$ (o.e.) or a fully correct expression (all non-zero terms must be seen)																																
	2nd M1 for subtracting their $[E(T)]^2$, Must be some attempt to square $-\frac{9}{4}$ is M0 but $-\frac{9}{16}$ could be M1																																
	2nd A1 for correct fraction or AWRT 7.19 Full marks can still be scored in (d) and (e) if a is incorrect																																