Mark Scheme (Final)

January 2008

GCE

GCE Mathematics (6663/01)

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January 2008
6663 Core Mathematics C1
Mark Scheme

Question number	Scheme	Marks
1.	$3x^2 \to kx^3$ or $4x^5 \to kx^6$ or $-7 \to kx$ (*k* a non-zero constant)	M1
	$\frac{3x^3}{3}$ or $\frac{4x^6}{6}$ (Either of these, simplified or unsimplified)	A1
	$x^3 + \frac{2x^6}{3} - 7x$ or equivalent unsimplified, such as $\frac{3x^3}{3} + \frac{4x^6}{6} - 7x^1$	A1
	$+ C$ (or any other constant, e.g. + *K*)	B1 (4)
		4
	M: Given for increasing by one the power of *x* in one of the three terms. A marks: 'Ignore subsequent working' after a correct unsimplified version of a term is seen. B: Allow the mark (independently) for an integration constant appearing at any stage (even if it appears, then disappears from the final answer). This B mark can be allowed even when no other marks are scored.	

Question number	Scheme	Marks	
2.	(a) 2	B1	(1)
	(b) x^9 seen, or $(\text{answer to (a)})^3$ seen, or $(2x^3)^3$ seen.	M1	
	$8x^9$	A1	(2)
			3
	(b) M: Look for x^9 first… if seen, this is M1. If not seen, look for $(\text{answer to (a)})^3$, e.g. 2^3… this would score M1 even if it does not subsequently become 8. (Similarly for other answers to (a)). In $(2x^3)^3$, the 2^3 is implied, so this scores the M mark. <u>Negative answers</u>: (a) Allow -2. Allow ± 2. Allow '2 or -2'. (b) Allow $\pm 8x^9$. Allow '$8x^9$ or $-8x^9$'. N.B. If part (a) is wrong, it is possible to 'restart' in part (b) and to score full marks in part (b).		

Question number	Scheme	Marks
3.	$\frac{(5-\sqrt{3})}{(2+\sqrt{3})}\times\frac{(2-\sqrt{3})}{(2-\sqrt{3})}$	M1
	$=\frac{10-2\sqrt{3}-5\sqrt{3}+(\sqrt{3})^2}{\dots}$ $\left(=\frac{10-7\sqrt{3}+3}{\dots}\right)$	M1
	$(=13-7\sqrt{3})$ $\left(\text{Allow } \frac{13-7\sqrt{3}}{1}\right)$ 13 (a = 13)	A1
	$-7\sqrt{3}$ (b = –7)	A1 (4)
		4

1st M: Multiplying top and bottom by $(2-\sqrt{3})$. (As shown above is sufficient).

2nd M: Attempt to multiply out numerator $(5-\sqrt{3})(2-\sqrt{3})$. Must have at least 3 terms correct.

Final answer: Although 'denominator = 1' may be implied, the $13-7\sqrt{3}$ must obviously be the final answer (not an intermediate step), to score full marks. (Also M0 M1 A1 A1 is not an option).

The A marks cannot be scored unless the 1st M mark has been scored, but this 1st M mark could be implied by correct expansions of both numerator and denominator.

It is possible to score M1 M0 A1 A0 or M1 M0 A0 A1 (after 2 correct terms in the numerator).

Special case: If numerator is multiplied by $(2+\sqrt{3})$ instead of $(2-\sqrt{3})$, the 2nd M can still be scored for at least 3 of these terms correct:
$10-2\sqrt{3}+5\sqrt{3}-(\sqrt{3})^2$.
The maximum score in the special case is 1 mark: M0 M1 A0 A0.

Answer only: Scores no marks.

Alternative method:

$5-\sqrt{3}=(a+b\sqrt{3})(2+\sqrt{3})$

$(a+b\sqrt{3})(2+\sqrt{3})=2a+a\sqrt{3}+2b\sqrt{3}+3$ M1: At least 3 terms correct.

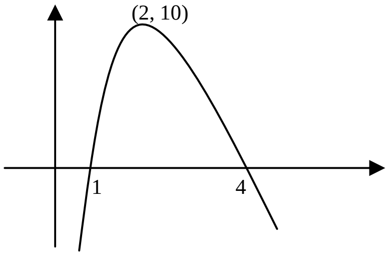
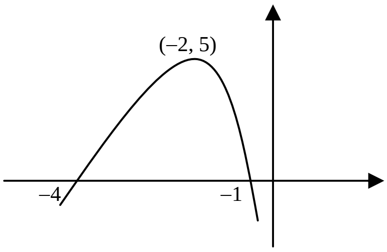
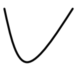
$5=2a+3b$

$-1=a+2b$ $a=\dots$ or $b=\dots$ M1: Form and attempt to solve simultaneous equations.

$a=13$, $b=-7$ A1, A1

Question number	Scheme	Marks	
4.	(a) $m=\frac{4-(-3)}{-6-8}$ or $\frac{-3-4}{8-(-6)}$, $=\frac{7}{-14}$ or $\frac{-7}{14}$ $\left(=-\frac{1}{2}\right)$	M1, A1	
	Equation: $y-4=-\frac{1}{2}(x-(-6))$ or $y-(-3)=-\frac{1}{2}(x-8)$	M1	
	$x+2y-2=0$ (or equiv. with <u>integer</u> coefficients… must have '= 0')	A1	(4)
	(e.g. $14y+7x-14=0$ and $14-7x-14y=0$ are acceptable)		
	(b) $(-6-8)^2+(4-(-3))^2$	M1	
	14^2+7^2 or $(-14)^2+7^2$ or $14^2+(-7)^2$ (M1 A1 may be implied by 245)	A1	
	$AB=\sqrt{14^2+7^2}$ or $\sqrt{7^2(2^2+1^2)}$ or $\sqrt{245}$		
	$7\sqrt{5}$	A1cso	(3)
			7
	(a) 1st M: Attempt to use $m=\frac{y_2-y_1}{x_2-x_1}$ (may be implicit in an equation of *L*). 2nd M: Attempting straight line equation in any form, e.g $y-y_1=m(x-x_1)$, $\frac{y-y_1}{x-x_1}=m$, with any value of *m* (except 0 or ∞) and either (–6, 4) or (8, –3). N.B. It is also possible to use a different point which lies on the line, such as the midpoint of *AB* (1, 0.5). Alternatively, the 2nd M may be scored by using $y=mx+c$ with a numerical gradient and substituting (–6, 4) or (8, –3) to find the value of *c*. Having coords the <u>wrong way round</u>, e.g. $y-(-6)=-\frac{1}{2}(x-4)$, loses the 2nd M mark <u>unless</u> a correct general formula is seen, e.g. $y-y_1=m(x-x_1)$. (b) M: Attempting to use $(x_2-x_1)^2+(y_2-y_1)^2$. <u>Missing bracket</u>, e.g. -14^2+7^2 implies M1 if no earlier version is seen. -14^2+7^2 with no further work would be M1 A0. -14^2+7^2 followed by 'recovery' can score full marks.		

Question number	Scheme	Marks
5.	(a) $\left(2x^{-\frac{1}{2}}+3x^{-1}\right)$ $p=-\frac{1}{2}$, $q=-1$	B1, B1 (2)
	(b) $\left(y=5x-7+2x^{-\frac{1}{2}}+3x^{-1}\right)$	
	$\left(\frac{dy}{dx}=\right)$ 5 $\left(\text{or } 5x^0\right)$ ($5x-7$ correctly differentiated)	B1
	Attempt to differentiate either $2x^p$ with a fractional *p*, giving kx^{p-1} $(k\neq 0)$, (the fraction *p* could be in decimal form)	
	or $3x^q$ with a negative *q*, giving kx^{q-1} $(k\neq 0)$.	M1
	$\left(-\frac{1}{2}\times 2x^{-\frac{3}{2}}-1\times 3x^{-2}=\right)$ $-x^{-\frac{3}{2}}$, $-3x^{-2}$	A1ft, A1ft (4)
		6
	(b): N.B. It is possible to 'start again' in (b), so the *p* and *q* may be different from those seen in (a), but note that the M mark is for the attempt to differentiate $\underline{\underline{2}}x^p$ or $\underline{\underline{3}}x^q$. However, marks for part (a) <u>cannot</u> be earned in part (b). 1st A1ft: ft their $2x^p$, but *p* must be a fraction and coefficient must be simplified (the fraction *p* could be in decimal form). 2nd A1ft: ft their $3x^q$, but *q* must be negative and coefficient must be simplified. 'Simplified' coefficient means $\frac{a}{b}$ where *a* and *b* are integers with no common factors. Only a single + or – sign is allowed (e.g. – – must be replaced by +). Having +*C* loses the B mark.	

Question number	Scheme		Marks	
6.	(a) 	Shape: Max in 1st quadrant and 2 intersections on positive *x*-axis	B1	
		1 and 4 labelled (in correct place) or clearly stated as coordinates	B1	
		(2, 10) labelled or clearly stated	B1	(3)
	(b) 	Shape: Max in 2nd quadrant and 2 intersections on negative *x*-axis	B1	
		–1 and –4 labelled (in correct place) or clearly stated as coordinates	B1	
		(–2, 5) labelled or clearly stated	B1	(3)
	(c) $(a =)\ 2$	May be implicit, i.e. $\mathrm{f}(x+2)$	B1	(1)
	Beware: The answer to part (c) may be seen on the first page.			**7**
	(a) and (b): 1st B: 'Shape' is generous, providing the conditions are satisfied. 2nd and 3rd B marks are dependent upon a sketch having been drawn. 2nd B marks: Allow (0, 1), etc. (coordinates the wrong way round) <u>if</u> the sketch is correct. Points must be labelled correctly and be in appropriate place (e.g. (–2, 5) in the first quadrant is B0). (b) <u>Special case</u>: If the graph is reflected in the *x*-axis (instead of the *y*-axis), B1 B0 B0 can be scored. This requires shape and coordinates to be <u>fully correct</u>, i.e. Shape:  Minimum in 4th quadrant and 2 intersections on positive *x*-axis, 1 and 4 labelled (in correct place) or clearly stated as coordinates, (2, –5) labelled or clearly stated.			

Question number	Scheme	Marks	
7.	(a) $1(p+1)$ or $p+1$	B1	(1)
	(b) $((a))(p+(a))$ [(a) must be a function of *p*]. $[(p+1)(p+p+1)]$	M1	
	$=1+3p+2p^2$ (*)	A1cso	(2)
	(c) $1+3p+2p^2=1$	M1	
	$p(2p+3)=0$ $p=...$	M1	
	$p=-\frac{3}{2}$ (ignore p = 0, if seen, even if 'chosen' as the answer)	A1	(3)
	(d) Noting that even terms are the same.	M1	
	This M mark can be implied by listing at least 4 terms, e.g. $1, -\frac{1}{2}, 1, -\frac{1}{2}, ...$		
	$x_{2008}=-\frac{1}{2}$	A1	(2)
			8
	(b) M: Valid attempt to use the given recurrence relation to find x_3. Missing brackets, e.g. $p+1(p+p+1)$ Condone for the M1, then if all terms in the expansion are correct, with working fully shown, M1 A1 is still allowed. Beware 'working back from the answer', e.g. $1+3p+2p^2=(1+p)(1+2p)$ scores no marks unless the recurrence relation is justified. (c) 2nd M: Attempt to solve a quadratic equation in *p* (e.g. quadratic formula or completing the square). The equation must be based on $x_3=1$. The attempt must lead to a non-zero solution, so just stating the zero solution *p* = 0 is M0. A: The A mark is dependent on both M marks. (d) M: Can be implied by a correct answer for their *p* (answer is *p* + 1), and can also be implied if the working is 'obscure'). Trivialising, e.g. *p* = 0, so every term = 1, is M0. If the additional answer $x_{2008}=1$ (from *p* = 0) is seen, ignore this (isw).		

Question number	Scheme		Marks	
8.	(a) $x^2 + kx + (8-k) \quad (=0)$ $8-k$ need not be bracketed		M1	
	$b^2 - 4ac = k^2 - 4(8-k)$		M1	
	$b^2 - 4ac < 0 \Rightarrow k^2 + 4k - 32 < 0$	(*)	A1cso	(3)
	(b) $(k+8)(k-4) = 0 \quad k = ...$		M1	
	$k = -8 \quad\quad k = 4$		A1	
	Choosing 'inside' region (between the two *k* values)		M1	
	$-8 < k < 4$ or $4 > k > -8$		A1	(4)
				7

(a) 1st M: Using the *k* from the right hand side to form 3-term quadratic in *x* ('= 0' can be implied), or…

attempting to complete the square $\left(x + \frac{k}{2}\right)^2 - \frac{k^2}{4} + 8 - k \; (=0)$ or equiv., using the *k* from the right hand side.

For either approach, condone sign errors.

1st M may be implied when candidate moves straight to the discriminant.

2nd M: Dependent on the 1st M.

Forming expressions in *k* (with no *x*'s) by using b^2 and $4ac$. (Usually seen as the discriminant $b^2 - 4ac$, but separate expressions are fine, and also allow the use of $b^2 + 4ac$.

(For 'completing the square' approach, the expression must be clearly separated from the equation in *x*).

If b^2 and $4ac$ are used in the quadratic formula, they must be clearly separated from the formula to score this mark.

For any approach, condone sign errors.

If the wrong statement $\sqrt{b^2 - 4ac} < 0$ is seen, maximum score is M1 M1 A0.

(b) Condone the use of *x* (instead of *k*) in part (b).

1st M: Attempt to solve a 3-term quadratic equation in *k*.

It might be different from the given quadratic in part (a).

Ignore the use of < in solving the equation. The 1st M1 A1 can be scored if –8 and 4 are achieved, even if stated as $k < -8$, $k < 4$.

Allow the first M1 A1 to be scored in part (a).

N.B. '$k > -8$, $k < 4$' scores 2nd M1 A0

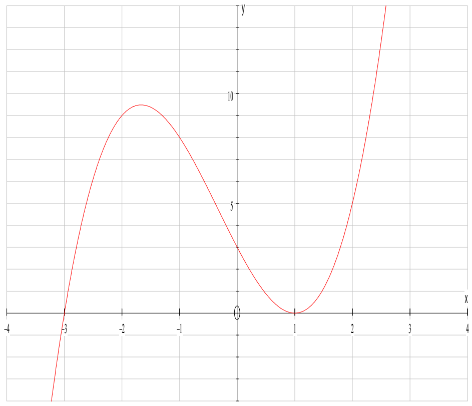
'$k > -8$ or $k < 4$' scores 2nd M1 A0

'$k > -8$ and $k < 4$' scores 2nd M1 A1

'$k = -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3$' scores 2nd M0 A0

Use of ≤ (in the answer) loses the final mark.

Question number	Scheme	Marks
9.	(a) $4x \to kx^2$ or $6\sqrt{x} \to kx^{3/2}$ or $\frac{8}{x^2} \to kx^{-1}$ (k a non-zero constant)	M1
	$\text{f}(x) = 2x^2,\ -4x^{3/2},\ -8x^{-1}$ (+ C) (+ C not required)	A1, A1, A1
	At $x = 4$, $y = 1$: $1 = (2\times16) - \left(4\times4^{3/2}\right) - \left(8\times4^{-1}\right) + C$ Must be in part (a)	M1
	$C = 3$	A1 (6)
	(b) $\text{f}'(4) = 16 - (6\times2) + \frac{8}{16} = \frac{9}{2}\ (= m)$ [M: Attempt $\text{f}'(4)$ with the given f'. Must be in part (b)]	M1
	Gradient of normal is $-\frac{2}{9}\left(= -\frac{1}{m}\right)$ [M: Attempt perp. grad. rule. Dependent on the use of their $\text{f}'(x)$]	M1
	Eqn. of normal: $y - 1 = -\frac{2}{9}(x-4)$ (or any equiv. form, e.g. $\frac{y-1}{x-4} = -\frac{2}{9}$)	M1 A1 (4)
	Typical answers for A1: $\left(y = -\frac{2}{9}x + \frac{17}{9}\right)$ $(2x + 9y - 17 = 0)$ $(y = -0.\dot{2}x + 1.\dot{8})$	
	Final answer: gradient $-\frac{1}{(9/2)}$ or $-\frac{1}{4.5}$ is A0 (but all M marks are available).	
		10
	(a) The first 3 A marks are awarded in the order shown, and the terms must be simplified. 'Simplified' coefficient means $\frac{a}{b}$ where a and b are integers with no common factors. Only a single + or – sign is allowed (e.g. + – must be replaced by –). 2nd M: Using $x = 4$ and $y = 1$ (not $y = 0$) to form an eqn in C. (No C is M0) (b) 2nd M: Dependent upon use of their $\text{f}'(x)$. 3rd M: eqn. of a straight line through (4, 1) with any gradient except 0 or ∞. Alternative for 3rd M: Using (4, 1) in $y = mx + c$ to find a value of c, but an equation (general or specific) must be seen. Having coords the wrong way round, e.g. $y - 4 = -\frac{2}{9}(x-1)$, loses the 3rd M mark unless a correct general formula is seen, e.g. $y - y_1 = m(x - x_1)$. N.B. The A mark is scored for any form of the correct equation… be prepared to apply isw if necessary.	

Question number	Scheme	Marks
10.	(a) 	
	Shape  (drawn anywhere)	B1
	Minimum at (1, 0) (perhaps labelled 1 on x-axis)	B1
	$(-3, 0)$ (or -3 shown on –ve x-axis)	B1
	(0, 3) (or 3 shown on +ve y-axis)	B1 (4)
	N.B. The max. can be anywhere.	
	(b) $y = (x+3)(x^2-2x+1)$	M1
	$= x^3 + x^2 - 5x + 3$ ($k = 3$) [Marks can be awarded if this is seen in part (a)]	A1cso (2)
	(c) $\frac{dy}{dx} = 3x^2 + 2x - 5$	M1 A1
	$3x^2 + 2x - 5 = 3$ or $3x^2 + 2x - 8 = 0$	M1
	$(3x-4)(x+2) = 0$ $x = ...$	M1
	$x = \frac{4}{3}$ (or exact equiv.) , $x = -2$	A1, A1 (6)
		12

(a) The individual marks are independent, <u>but</u> the 2nd, 3rd and 4th B's are dependent upon a sketch having been attempted.

B marks for coordinates: Allow (0, 1), etc. (coordinates the wrong way round) <u>if</u> marked in the correct place on the sketch.

(b) M: Attempt to multiply out $(x-1)^2$ and write as a product with $(x+3)$,
or attempt to multiply out $(x+3)(x-1)$ and write as a product with $(x-1)$,
or attempt to expand $(x+3)(x-1)(x-1)$ directly (at least 7 terms).
The $(x-1)^2$ or $(x+3)(x-1)$ expansion must have 3 (or 4) terms, so should not, for example, be just x^2+1.

A: It is not necessary to state explicitly 'k = 3'.
Condone missing brackets if the intention seems clear and a fully correct expansion is seen.

(c) 1st M: Attempt to differentiate (correct power of x in at least one term).

2nd M: Setting their derivative equal to 3.

3rd M: Attempt to solve a 3-term quadratic based on their derivative.

The equation <u>could</u> come from $\frac{dy}{dx} = 0$.

N.B. After an incorrect k value in (b), full marks are still possible in (c).

Question number	Scheme	Marks
11.	(a) $u_{25} = a + 24d = 30 + 24 \times (-1.5)$	M1
	$= -6$	A1 (2)
	(b) $a + (n-1)d = 30 - 1.5(r-1) = 0$	M1
	$r = 21$	A1 (2)
	(c) $S_{20} = \frac{20}{2}\{60 + 19(-1.5)\}$ or $S_{21} = \frac{21}{2}\{60 + 20(-1.5)\}$ or $S_{21} = \frac{21}{2}\{30 + 0\}$	M1 A1ft
	$= 315$	A1 (3)
		7
	(a) M: Substitution of $a = 30$ and $d = \pm 1.5$ into ($a + 24d$). Use of $a + 25d$ (or any other variations on 24) scores M0. (b) M: Attempting to use the term formula, equated to 0, to form an equation in r (with no other unknowns). Allow this to be called n instead of r. Here, being 'one off' (e.g. equivalent to $a + nd$), scores M1. (c) M: Attempting to use the correct sum formula to obtain S_{20}, S_{21}, or, with their r from part (b), S_{r-1} or S_r. 1st A(ft): A correct numerical expression for S_{20}, S_{21}, or, with their r from part (b), S_{r-1} or S_r …. but the ft is dependent on an <u>integer</u> value of r. Methods such as calculus to find a maximum only begin to score marks <u>after</u> establishing a value of r at which the maximum sum occurs. This value of r can be used for the M1 A1ft, but must be a positive integer to score A marks, so evaluation with, say, $n = 20.5$ would score M1 A0 A0. <u>'Listing' and other methods</u> (a) M: Listing terms (found by a correct method), and picking the <u>25th</u> term. (There may be numerical slips). (b) M: Listing terms (found by a correct method), until the zero term is seen. (There may be numerical slips). 'Trial and error' approaches (or where working is unclear or non-existent) score M1 A1 for 21, M1 A0 for 20 or 22, and M0 A0 otherwise. (c) M: Listing sums, or listing and adding terms (found by a correct method), at least as far as the 20th term. (There may be numerical slips). A2 (scored as A1 A1) for 315 (clearly selected as the answer). 'Trial and error' approaches essentially follow the main scheme, beginning to score marks when trying S_{20}, S_{21}, or, with their r from part (b), S_{r-1} or S_r. If no working (or no legitimate working) is seen, but the answer 315 is given, allow one mark (scored as M1 A0 A0). <u>For reference</u>: Sums: 30, 58.5, 85.5, 111, 135, 157.5, 178.5, 198, 216, 232.5, 247.5, 261, 273, 283.5, 292.5, 300, 306, 310.5, 313.5, 315, ……..	

GENERAL PRINCIPLES FOR C1 MARKING

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm p)^2 \pm q \pm c$, $p \neq 0$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values. There must be some correct substitution.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but will be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may gain no credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Misreads

A misread must be consistent for the whole question to be interpreted as such.

These are not common. In clear cases, please deduct the first 2 A (or B) marks which would have been lost by following the scheme. (Note that 2 marks is the maximum misread penalty, but that misreads which alter the nature or difficulty of the question cannot be treated so generously and it will usually be necessary here to follow the scheme as written).

Sometimes following the scheme as written is more generous to the candidate than applying the misread rule, so in this case use the scheme as written.