

1. A medical student is investigating two methods of taking a person's blood pressure. He takes a random sample of 10 people and measures their blood pressure using an arm cuff and a finger monitor. The table below shows the blood pressure for each person, measured by each method.

Person	A	B	C	D	E	F	G	H	I	J
Arm cuff	140	110	138	127	142	112	122	128	132	160
Finger monitor	154	112	156	152	142	104	126	132	144	180

- (a) Use a paired t-test to determine, at the 10% level of significance, whether or not there is a difference in the mean blood pressure measured using the two methods. State your hypotheses clearly. (8)
- (b) State an assumption about the underlying distribution of measured blood pressure required for this test. (1)
2. The value of orders, in £, made to a firm over the internet has distribution $N(\mu, \sigma^2)$. A random sample of n orders is taken and \bar{X} denotes the sample mean.

- (a) Write down the mean and variance of \bar{X} in terms of μ and σ^2 . (2)

A second sample of m orders is taken and \bar{Y} denotes the mean of this sample. An estimator of the population mean is given by

$$U = \frac{n\bar{X} + m\bar{Y}}{n + m}.$$

- (b) Show that U is an unbiased estimator for μ . (3)
- (c) Show that the variance of U is $\frac{\sigma^2}{n + m}$. (4)
- (d) State which of \bar{X} or U is a better estimator for μ . Give a reason for your answer. (2)

3. The lengths, x mm, of the forewings of a random sample of male and female adult butterflies are measured. The following statistics are obtained from the data.

	No. of butterflies	Sample mean \bar{x}	$\sum x^2$
Females	7	50.6	17 956.5
Males	10	53.2	28 335.1

- (a) Assuming the lengths of the forewings are normally distributed test, at the 10% level of significance, whether or not the variances of the two distributions are the same. State your hypotheses clearly. (7)
- (b) Stating your hypotheses clearly test, at the 5% level of significance, whether the mean length of the forewings of the female butterflies is less than the mean length of the forewings of the male butterflies. (6)
4. The length X mm of a spring made by a machine is normally distributed $N(\mu, \sigma^2)$. A random sample of 20 springs is selected and their lengths measured in mm. Using this sample the unbiased estimates of μ and σ^2 are

$$\bar{x} = 100.6 \qquad s^2 = 1.5$$

Stating your hypotheses clearly test, at the 10% level of significance,

- (a) whether or not the variance of the lengths of springs is different from 0.9, (6)
- (b) whether or not the mean length of the springs is greater than 100 mm. (6)
5. The number of tornadoes per year to hit a particular town follows a Poisson distribution with mean λ . A weatherman claims that due to climate changes the mean number of tornadoes per year has decreased. He records the number of tornadoes x to hit the town last year.

To test the hypotheses $H_0: \lambda = 7$ and $H_1: \lambda < 7$, a critical region of $x \leq 3$ is used.

- (a) Find, in terms λ the power function of this test. (3)
- (b) Find the size of this test. (2)
- (c) Find the probability of a Type II error when $\lambda = 4$. (2)

6. A butter packing machine cuts butter into blocks. The weight of a block of butter is normally distributed with a mean weight of 250 g and a standard deviation of 4 g. A random sample of 15 blocks is taken to monitor any change in the mean weight of the blocks of butter.

(a) Find the critical region of a suitable test using a 2% level of significance. (3)

(b) Assuming the mean weight of a block of butter has increased to 254 g, find the probability of a Type II error. (5)

7. A doctor wishes to study the level of blood glucose in males. The level of blood glucose is normally distributed. The doctor measured the blood glucose of 10 randomly selected male students from a school. The results, in mmol/litre, are given below.

4.7 3.6 3.8 4.7 4.1 2.2 3.6 4.0 4.4 5.0

(a) Calculate a 95% confidence interval for the mean. (7)

(b) Calculate a 95% confidence interval for the variance. (4)

A blood glucose reading of more than 7 mmol/litre is counted as high.

(c) Use appropriate confidence limits from parts (a) and (b) to find the highest estimate of the proportion of male students in the school with a high blood glucose level. (4)