

Mark Scheme (Results)

Summer 2007

GCE

GCE Mathematics

Statistics S4 (6686)

Question Number	Scheme	Marks
2. a)	$E(\bar{X}) = \mu$ $\text{Var}(\bar{X}) = \text{Var}\left(\frac{X_1 + X_2 + X_3 + \dots + X_n}{n}\right)$ $= \frac{\sigma^2}{n}$	B1 B1 (2)
b)	$E(U) = \frac{1}{n+m}(nE(\bar{X}) + mE(\bar{Y}))$ $= \frac{1}{n+m}(n\mu + m\mu)$ $= \mu \Rightarrow U \text{ is unbiased}$	M1 A1 state unbiased A1 (3)
c)	$\text{Var}(\bar{Y}) = \frac{\sigma^2}{m}$ $\text{Var}(U) = \frac{n^2 \text{Var}(\bar{X}) + m^2 \text{Var}(\bar{Y})}{(n+m)^2}$ $= \frac{n^2 \frac{\sigma^2}{n} + m^2 \frac{\sigma^2}{m}}{(n+m)^2}$ $= \frac{n\sigma^2 + m\sigma^2}{(n+m)^2}$ $= \frac{\sigma^2}{n+m} \quad *$	B1 M1 A1 A1 csso (4)
d)	$\frac{n\bar{X} + m\bar{Y}}{n+m}$ is a better estimate since variance is smaller.	B1 B1 (2)

Question Number	Scheme	Marks
3. a	$H_0 : \sigma_F^2 = \sigma_M^2 \quad H_1 : \sigma_F^2 \neq \sigma_M^2$ $s_F^2 = \frac{1}{6}(17956.5 - 7 \times 50.6^2) = \frac{33.98}{6} = 5.66333\dots$ $s_M^2 = \frac{1}{9}(28335.1 - 10 \times 53.2^2) = \frac{32.7}{9} = 3.63333\dots$ $\frac{s_F^2}{s_M^2} = 1.5587\dots \quad (\text{Reciprocal } 0.6415)$ $F_{6,9} = 3.37 \text{ (or } 0.24)$ <p>Not in critical region. <u>Variances</u> of the two distributions <u>are the same</u></p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>M1 A1</p> <p>B1</p> <p>A1</p> <p style="text-align: right;">(7)</p>
b.	$H_0 : \mu_F = \mu_M \quad H_1 : \mu_F < \mu_M$ $\text{Pooled estimate } s^2 = \frac{6 \times 5.66333\dots + 9 \times 3.63333}{15}$ $= 4.44533$ $s = 2.11$ $t = \frac{50.6 - 53.2}{2.11 \sqrt{\frac{1}{7} + \frac{1}{10}}} = \pm 2.50$ $\text{C.V. } t_{15}(5\%) = \pm 1.753$ <p>Significant. The mean length of the <u>females forewing</u> is less than the length of the <u>males forewing</u></p> <p>Notes</p> <p>(a) need to have <u>variance</u> and <u>the same</u> o.e</p> <p>(b) need female and forewing(wing)</p>	<p>B1</p> <p>M1</p> <p>M1 A1</p> <p>B1</p> <p>A1</p> <p style="text-align: right;">(6)</p>

Question Number	Scheme	Marks
4.a)	$H_0: \sigma^2 = 0.9 \quad H_1: \sigma^2 \neq 0.9$ $\nu = 19$ <p>CR (Lower tail 10.117) Upper tail 30.144</p> <p>Test statistic = $\frac{19 \times 1.5}{0.9} = 31.6666$, significant</p> <p>There is sufficient evidence that the <u>variance</u> of the length of spring is <u>different to 0.9</u></p>	<p>B1</p> <p>B1 B1</p> <p>M1 A1 A1</p> <p style="text-align: right;">(6)</p>
b)	$H_0: \mu = 100 \quad H_1: \mu > 100$ $t_{19} = 1.328$ $t = \frac{100.6 - 100}{\sqrt{\frac{1.5}{20}}} = 2.19$ <p>Significant. The mean <u>length of spring</u> is <u>greater than 100</u></p> <p>Notes (a) only need to see 30.144 need variance in conclusion (b) conclusion must be in context. Length of spring needed</p>	<p>B1</p> <p>B1</p> <p>M1 A1 A1</p> <p>B1</p> <p style="text-align: right;">(6)</p>

Question Number	Scheme	Marks
5.a)	Power = $P(X \leq 3 / \lambda)$ $= e^{-\lambda} + e^{-\lambda}\lambda + \frac{e^{-\lambda}\lambda^2}{2} + \frac{e^{-\lambda}\lambda^3}{6}$ $= \frac{e^{-\lambda}}{6}(6 + 6\lambda + 3\lambda^2 + \lambda^3)$	M1 A1 A1 (3)
b)	CR is $X \leq 3$ Size = $P[X \leq 3 / \lambda = 7]$ $= 0.0818$	M1 A1 (2)
c)	P(Type II error) = $1 - \text{power}$ $= 1 - \frac{e^{-4}}{6}(6 + 6 \times 4 + 3 \times 4^2 + 4^3)$ $= 0.5665..$	M1 A1 (2)
6.a)	$\frac{\bar{X} - 250}{\frac{4}{\sqrt{15}}} > 2.3263 \quad \text{or} \quad \frac{\bar{X} - 250}{\frac{4}{\sqrt{15}}} < -2.3263$ <p style="text-align: center;">2.3262</p> $\bar{X} > 252.40... \quad \text{or} \quad \bar{X} < 247.6...$ <p style="text-align: center;">252 and 248</p>	\pm B1 M1 awrt A1 (3)
b)	$P(\bar{X} < 252.4 / \mu = 254) - P(\bar{X} < 247.6 / \mu = 254)$ $= P\left(Z < \frac{252.4 - 254}{\frac{4}{\sqrt{15}}}\right) - P\left(Z < \frac{247.6 - 254}{\frac{4}{\sqrt{15}}}\right)$ $= P(Z < -1.5492) - P(Z < -6.20)$ $= (1 - 0.9394) - (1 - 1)$ $= 0.0606$	using their '252.4' and '247.6' M1 stand using $4/\sqrt{15}$, 254 their '252.4' or '247.6' M1 -1.5492 and -6.20 o.e. A1 M1 A1 (5)
Notes (a) only needs to try and find one side for M1 (b) only need to see one of the standardisation for second M1 if consider only 252.4 and get 0.0606 they get M0 M1 A0 M1 A1 ie they can get 3/5		

Question Number	Scheme	Marks
7.	$\bar{x} = 4.01$ $s = 0.7992\dots$	B1 M1 A1
(a)	$4.01 \pm t_9(2.5\%) \frac{0.7992\dots}{\sqrt{10}} = 4.01 \pm 2.262 \frac{0.7992\dots}{\sqrt{10}}$ <p style="text-align: right; margin-right: 100px;">2.262</p> <p style="text-align: right; margin-right: 100px;">their \bar{x} and s and $\sqrt{10}$</p> $= 4.5816\dots \text{ and } 3.4383\dots$ <p style="text-align: right; margin-right: 100px;">awrt 4.58 and 3.44</p>	B1 M1 A1√ A1 (7)
(b)	$2.700 < \frac{9 \times 0.7992\dots^2}{s^2} < 19.023$ <p style="text-align: right; margin-right: 100px;">2.7, 19.023</p> <p style="text-align: right; margin-right: 100px;">$9 \times s^2 / \sigma^2$</p> $\sigma^2 < 2.13, \quad \sigma^2 > 0.302$ <p style="text-align: right; margin-right: 100px;">both awrt 2.13, 0.302</p>	B1 B1 M1 A1 (4)
(c)	$P(X > 7) = P\left(Z > \frac{7 - \mu}{\sigma}\right)$ <p style="text-align: right; margin-right: 100px;">needs to be as high as possible</p> <p>Therefore μ and σ must be as big as possible</p> $= P\left(Z > \frac{7 - 4.581}{\sqrt{2.13}}\right)$ $= 1 - 0.9515$ $= 0.0485$ $= 4.85\%$ <p style="text-align: right; margin-right: 100px;">4.8 to 4.9</p>	M1 M1 A1√ A1 (4)
	<p>Notes</p> <p>(a) $s^2 = 0.63877\dots$</p> <p>(c) M1 may be implied by them using their highest μ and σ.</p>	