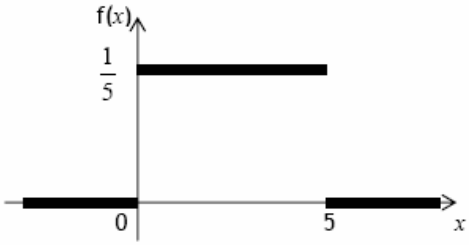
June 2007
6684 Statistics S2
Mark Scheme

Question Number	Scheme	Marks
1(a)	Continuous uniform distribution *or* rectangular distribution.	B1
	 1/5,(0),5 0 may be implied by start at y axis	B1
		B1
		(3)
(b)	$E(X) = 2.5$ ft from their a and b, must be a number	B1ft
	$\text{Var}(X) = \frac{1}{12}(5-0)^2$ or attempt to use $\int_0^5 f(x)x^2dx - \mu^2$ use their f(x)	M1
	$= \frac{25}{12}$ or 2.08 o.e awrt 2.08	A1
		(3)
(c)	$P(X > 3) = \frac{2}{5} = 0.4$ 2 times their 1/5 from diagram	B1ft
		(1)
(d)	$P(X = 3) = 0$	B1
		(1)
		(Total 8)

Question Number	Scheme	Marks
2	One tail test Method 1 $H_0 : \lambda = 5$ ($\lambda = 2.5$) may use λ or μ $H_1 : \lambda > 5$ ($\lambda > 2.5$) $X \sim$ Po (2.5) may be implied $P(X \geq 7) = 1 - P(X \leq 6)$ | [$P(X \geq 5) = 1 - 0.8912 = 0.1088$] att $P(X \geq 7)$ $P(X \geq 6)$ $= 1 - 0.9858$ | $P(X \geq 6) = 1 - 0.9580 = 0.0420$ $= 0.0142$ | CR $X \geq 6$ awrt 0.0142 $0.0142 < 0.05$ | $7 \geq 6$ or 7 is in critical region or 7 is significant (Reject H_0.) There is significant evidence at the 5% significance level that the factory is polluting the river with bacteria. **or** The scientists claim is justified	B1 B1 M1 M1 A1 M1 B1 (7) Total 7
	Method 2 $H_0 : \lambda = 5$ ($\lambda = 2.5$) may use λ or μ $H_1 : \lambda > 5$ ($\lambda > 2.5$) $X \sim$ Po (2.5) may be implied $P(X < 7)$ | [$P(X < 5) = 0.8912$] att $P(X < 7)$ $P(X < 6)$ | $P(X < 6) = 0.9580$ $= 0.9858$ | CR $X \geq 6$ wrt 0.986 $0.9858 > 0.95$ | $7 \geq 6$ or 7 is in critical region or 7 is significant (Reject H_0.) There is significant evidence at the 5% significance level that the factory is polluting the river with bacteria. **or** The scientists claim is justified	B1 B1 M1 M1 A1 M1 B1 (7)

	Two tail test Method 1 $H_o : \lambda = 5$ ($\lambda = 2.5$) may use λ or μ $H_1 : \lambda \neq 5$ ($\lambda \neq 2.5$) $X \sim$ Po (2.5) $P(X \geq 7) = 1 - P(X \leq 6)$ | [$P(X \geq 6) = 1 - 0.9580 = 0.0420$] att $P(X \geq 7)$ | $P(X \geq 7)$ $= 1 - 0.9858$ | $P(X \geq 7) = 1 - 0.9858 = 0.0142$ $= 0.0142$ | CR $X \geq 7$ awrt 0.0142 $0.0142 < 0.025$ | $7 \geq 7$ or 7 is in critical region or 7 is significant (Reject H_0.) There is significant evidence at the 5% significance level that the factory is polluting the river with bacteria. **or** The scientists claim is justified	B1 B0 M1 M1 A1 M1 B1 (7)
	Method 2 $H_o : \lambda = 5$ ($\lambda = 2.5$) may use λ or μ $H_1 : \lambda \neq 5$ ($\lambda \neq 2.5$) $X \sim$ Po (2.5) $P(X < 7)$ | [$P(X < 6) = 0.9580$] att $P(X < 7)$ | $P(X < 7)$ | $P(X < 7) = 0.9858$ $= 0.9858$ | CR $X \geq 7$ awrt 0.986 $0.9858 > 0.975$ | $7 \geq 7$ or 7 is in critical region or 7 is significant (Reject H_0.) There is significant evidence at the 5% significance level that the factory is polluting the river with bacteria. **or** The scientists claim is justified	B1 B0 M1 M1A1 M1 B1 (7)

Question Number	Scheme		Marks
3(a)	$X \sim$ Po (1.5)	need Po and 1.5	B1 (1)
(b)	Faulty components occur at a constant rate. Faulty components occur independently or randomly. Faulty components occur singly.	any two of the 3 only need faulty once	B1 B1 (2)
(c)	$P(X = 2) = P(X \leq 2) - P(X \leq 1)$ or $\frac{e^{-1.5}(1.5)^2}{2}$		M1
	$= 0.8088 - 0.5578$		
	$= 0.251$	awrt 0.251	A1 (2)
(d)	$X \sim$ Po(4.5)	4.5 may be implied	B1
	$P(X \geq 1) = 1 - P(X = 0)$ $= 1 - e^{-4.5}$ $= 1 - 0.0111$ $= 0.9889$	awrt 0.989	M1 A1 (3)
			Total 8

Question Number	Scheme	Marks
4	Attempt to write down combinations — at least one seen	M1
	(5,5,5), (5,5,10) any order (10,10,5) any order, (10,10,10)	A1
	(5,10,5), (10,5,5), (10,5,10), (5,10,10), — all 8 cases considered. May be implied by 3 * (10,5,10) and 3 * (5,5,10)	A1
	median 5 and 10	B1
	Median = 5 $P(M = m) = \left(\frac{1}{4}\right)^3 + 3\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right) = \frac{10}{64} = 0.15625$ — add at least two prob using ¼ and ¾. identified by having same median of 5 or 10 Allow no 3 for M	M1 A1
	Median = 10 $P(M = m) = \left(\frac{3}{4}\right)^3 + 3\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right) = \frac{54}{64} = 0.84375$	A1
		(7) Total 7

Question Number	Scheme		Marks
5(a)	If $X \sim B(n,p)$ and		
	n is large, n > 50		B1
	p is small, p < 0.2		B1
	then X can be approximated by Po(np)		
			(2)
(b)	P(2 consecutive calls) = 0.01^2		M1
	= 0.0001		A1
			(2)
(c)	$X \sim B(5, 0.01)$	may be implied	B1
	$P(X>1) = 1 - P(X = 1) - P(X = 0)$		M1
	$= 1 - 5(0.01)(0.99)^4 - (0.99)^5$		
	$= 1 - 0.0480298\ldots - 0.95099\ldots$		
	= 0.00098	awrt 0.00098	A1
			(3)
(d)	$X \sim B(1000, 0.01)$	may be implied by correct mean and variance	B1
	Mean = np = 10		B1
	Variance = $np(1 - p)$ = 9.9		B1
			(3)
(e)	$X \sim$ Po(10)		
	$P(X > 6) = 1 - P(X \leq 6)$		M1
	= 1 – 0.1301		
	= 0.8699	awrt 0.870	A1
			(2)
			Total 12

Question Number	Scheme	Marks
6	One tail test Method 1 $H_o : p = 0.2$ $H_1 : p > 0.2$ $X \sim B(5, 0.2)$ may be implied $P(X \geq 3) = 1 - P(X \leq 2)$ | $[P(X \geq 3) = 1 - 0.9421 = 0.0579]$ att $P(X \geq 3)$ $P(X \geq 4)$ $= 1 - 0.9421$ | $P(X \geq 4) = 1 - 0.9933 = 0.0067$ $= 0.0579$ | CR $X \geq 4$ awrt 0.0579 $0.0579 > 0.05$ | $3 \leq 4$ or 3 is not in critical region or 3 is not significant (Do not reject H_0.) There is insufficient evidence at the 5% significance level that there is an increase in the number of times the taxi/driver is late. **Or** Linda's claim is not justified	B1 B1 M1 M1 A1 M1 B1 (7) Total 7
	Method 2 $H_o : p = 0.2$ $H_1 : p > 0.2$ $X \sim B(5, 0.2)$ may be implied $P(X < 3) =$ | $[P(X < 3) = 0.9421]$ att $P(X < 3)$ $P(X < 4)$ $P(X < 4) = 0.9933$ 0.9421 | CR $X \geq 4$ awrt 0.942 $0.9421 < 0.95$ | $3 \leq 4$ or 3 is not in critical region or 3 is not significant (Do not reject H_0.) There is insufficient evidence at the 5% significance level that there is an increase in the number of times the taxi/driver is late. **Or** Linda's claim is not justified	B1 B1 M1 M1A1 M1 B1 (7)

Two tail test

Method 1

$H_o : p = 0.2$ B1

$H_1 : p \neq 0.2$ B0

M1

$X \sim X \sim B(5, 0.2)$ may be implied

M1

$P(X \geq 3) = 1 - P(X \leq 2)$ $= 1 - 0.9421$	$[P(X \geq 3) = 1 - 0.9421 = 0.0579]$ $P(X \geq 4) = 1 - 0.9933 = 0.0067$	att $P(X \geq 3)$	$P(X \geq 4)$
$= 0.0579$	CR $X \geq 4$	awrt 0.0579	

A1

M1

$0.0579 > 0.025$ | $3 \leq 4$ or 3 is not in critical region or 3 is not significant

B1

(Do not reject H_0.) There is insufficient evidence at the 5% significance level that there is an increase in the number of times the taxi/driver is late.

Or Linda's claim is not justified (7)

B1

Method 2

$H_o : p = 0.2$ B0

$H_1 : p \neq 0.2$ M1

$X \sim X \sim B(5, 0.2)$ may be implied

$P(X < 3) =$	$[P(X < 3) = 0.9421]$ $P(X < 4) = 0.9933$	att $P(X < 3)$	$P(X < 4)$
0.9421	CR $X \geq 4$	awrt 0.942	

M1A1

$0.9421 < 0.975$ | $3 \leq 4$ or 3 is not in critical region or 3 is not significant M1

B1

Do not reject H_0. There is insufficient evidence at the 5% significance level that there is an increase in the number of times the taxi/driver is late.

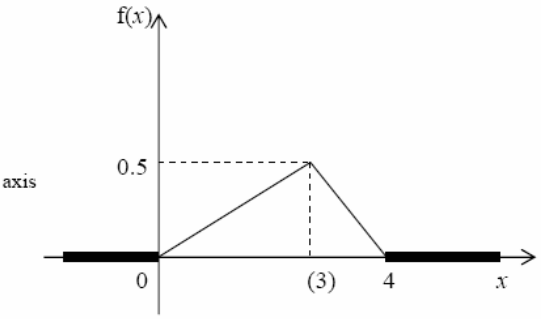
Or Linda's claim is not justified (7)

Special Case

If they use a probability of $\frac{1}{7}$ throughout the question they may gain B1 B1 M0 M1 A0 M1 B1.

NB they must attempt to work out the probabilities using $\frac{1}{7}$

Question Number	Scheme	Marks
7(a) i	If $X \sim B(n,p)$ and n is large or $n > 10$ **or** $np > 5$ **or** $nq > 5$ p is close to 0.5 **or** $nq > 5$ **and** $np > 5$ then X can be approximated by $N(np, np(1-p))$	B1 B1 (2)
ii	mean = np	B1
	variance = $np(1-p)$ must be in terms of p	B1 (2)
(b)	$X \sim N(60, 58.2)$ or $X \sim N(60, 7.63^2)$ 60, 58.2	B1, B1
	$P(X \geq 40) = P(X > 39.5)$ using 39.5 or 40.5	M1
	$= 1 - P\left(z < \pm\left(\frac{39.5-60}{\sqrt{58.2}}\right)\right)$ standardising 39.5 or 40 or 40.5 and their μ and σ	M1
	$= 1 - P(z < -2.68715\ldots)$	
	$= 0.9965$ allow answers in range 0.996 – 0.997	A1dep on both M (5)
(c)	$E(X) = 60$ may be implied or ft from part (b)	B1ft
	Expected profit = $(2000 - 60) \times 11 - 2000 \times 0.70$ = £19 940.	M1 A1 (3) Total 12

Question Number	Scheme	Marks
8(a)	 (0), 4, 0.5 0 may be implied by start at y both patio must be straight	B1 B1 B1 (3)
(b)	Mode is $x = 3$	B1 (1)
(c)	$F(x) = \int_0^x \frac{1}{6}t \text{ dt}$ (for $0 \le x \le 3$) ignore limits for M $= \frac{1}{12}x^2$ must use limit of 0 $F(x) = \int_3^x 2 - \frac{1}{2}t \text{ dt}; + \int_0^3 \frac{1}{6}t \text{ dt}$ (for $3 < x \le 4$) need limit of 3 and variable upper limit; need limit 0 and 3 $= 2x - \frac{1}{4}x^2 - 3$ $F(x)\begin{cases} 0 & x < 0 \\ \frac{1}{12}x^2 & 0 \le x \le 3 \\ 2x - \frac{1}{4}x^2 - 3 & 3 < x \le 4 \\ 1 & x > 4 \end{cases}$ middle pair ends	M1 A1 M1; M1 A1 B1 ft B1 (7)
(d)	F(m) = 0.5 either eq $\frac{1}{12}x^2 = 0.5$ eq for their $0 \le x \le 3$ $x = \sqrt{6} = 2.45$ $\sqrt{6}$ or awrt 2.45	M1 A1ft A1 (3) Total 14