Mark Scheme (Results)

Summer 2007

GCE

GCE Mathematics

Further Pure Mathematics FP2 (6675)

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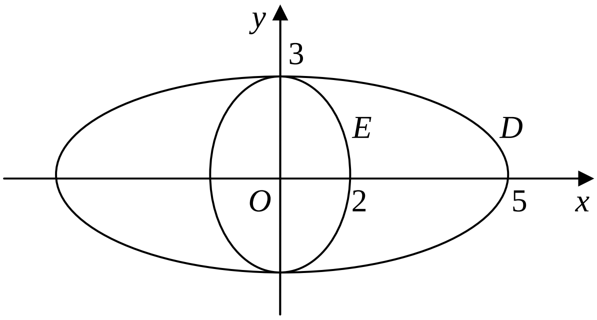
Question Number	Scheme	Marks
1.	$x^2+4x-5=(x+2)^2-9$	B1
	$\displaystyle\int \frac{1}{\sqrt{\left((x+2)^2-9\right)}}\,dx = \text{arcosh}\,\frac{x+2}{3}$	M1 A1ft
	ft their completing the square, requires arcosh	
	$\left[\text{arcosh}\,\frac{x+2}{3}\right]_1^3 = \text{arcosh}\,\frac{5}{3}\ \left(-\text{arcosh}\,1\right)$	
	$=\ln\left(\frac{5}{3}+\sqrt{\frac{25}{9}-1}\right)=\ln\left(\frac{5}{3}+\frac{4}{3}\right)=\ln 3$	M1 A1 **(5)**
		[5]
	Alternative	
	$x^2+4x-5=(x+2)^2-9$	B1
	Let $x+2=3\sec\theta$, $\quad \frac{dx}{d\theta}=3\sec\theta\tan\theta$	
	$\displaystyle\int \frac{1}{\sqrt{\left((x+2)^2-9\right)}}\,dx = \int \frac{3\sec\theta\tan\theta}{\sqrt{\left(9\sec^2\theta-9\right)}}\,d\theta$	M1
	$=\displaystyle\int \sec\theta\,d\theta$	A1ft
	$\left[\ln\left(\sec\theta+\tan\theta\right)\right]_{\text{arcsec}1}^{\text{arcsec}\frac{5}{3}}=\ln\left(\frac{5}{3}+\frac{4}{3}\right)=\ln 3$	M1 A1 **(5)**

Question Number	Scheme	Marks
2.	(a) 	
	One ellipse centred at *O*	B1
	Another ellipse, centred at *O*, touching on *y*-axis	B1
	Intersections: At least 2, 5, and 3 shown correctly	B1 **(3)**
	(b) Using $b^2 = a^2(1-e^2)$, or equivalent, to find *e* or *ae* for *D* or *E*.	M1
	For *S*: $a = 5$ and $b = 3$, $e = \frac{4}{5}$, $ae = 4$ ignore sign with *ae*	A1
	For *T*: $a' = 3$ and $b' = 2$, $e' = \frac{\sqrt{5}}{3}$, $a'e' = \sqrt{5}$ ignore sign with $a'e'$	A1
	$ST = \sqrt{(16+5)} = \sqrt{21}$	M1 A1 **(5)**
		[8]

Question Number	Scheme	Marks
3.	$\dfrac{dy}{dx} = \dfrac{1}{4}\left(4x - \dfrac{1}{x}\right)$	B1
	$\displaystyle\int\left(1+\left(\frac{dy}{dx}\right)^2\right)^{\frac{1}{2}} dx = \int\left(1+\left(x-\frac{1}{4x}\right)^2\right)^{\frac{1}{2}} dx$	M1
	$\displaystyle = \int\left(1+x^2+\frac{1}{16x^2}-\frac{1}{2}\right)^{\frac{1}{2}} dx = \int\left(\left(x+\frac{1}{4x}\right)^2\right)^{\frac{1}{2}} dx = \int\left(x+\frac{1}{4x}\right)dx$	M1 A1
	$= \dfrac{x^2}{2} + \dfrac{\ln x}{4}$	A1
	$\left[\dfrac{x^2}{2} + \dfrac{\ln x}{4}\right]_{0.5}^{2} = 2 + \dfrac{\ln 2}{4} - \dfrac{1}{8} - \dfrac{\ln 0.5}{4} = \dfrac{15}{8} + \dfrac{1}{2}\ln 2$	M1 A1 **(7)**
	$\left(a = \dfrac{15}{8},\ b = \dfrac{1}{2}\right)$	**[7]**

Question Number	Scheme	Marks
4.	(a)	
	$\cosh A\cosh B-\sinh A\sinh B=\left(\dfrac{e^{A}+e^{-A}}{2}\right)\left(\dfrac{e^{B}+e^{-B}}{2}\right)-\left(\dfrac{e^{A}-e^{-A}}{2}\right)\left(\dfrac{e^{B}-e^{-B}}{2}\right)$	M1
	$=\dfrac{1}{4}\left(e^{A+B}+e^{-A+B}+e^{A-B}+e^{-A-B}-e^{A+B}+e^{-A+B}+e^{A-B}-e^{-A-B}\right)$	
	$=\dfrac{1}{4}\left(2e^{-A+B}+2e^{A-B}\right)=\dfrac{e^{A-B}+e^{-(A-B)}}{2}=\cosh(A-B)$ ∗ cso	M1 A1 **(3)**
	(b) $\cosh x\cosh 1-\sinh x\sinh 1=\sinh x$	M1
	$\cosh x\cosh 1=\sinh x(1+\sinh 1)\quad\Rightarrow\quad \tanh x=\dfrac{\cosh 1}{1+\sinh 1}$	M1
	$\tanh x=\dfrac{\dfrac{e+e^{-1}}{2}}{1+\dfrac{e-e^{-1}}{2}}=\dfrac{e+e^{-1}}{2+e-e^{-1}}=\dfrac{e^{2}+1}{e^{2}+2e-1}$ ∗	M1 A1 **(4)**
	cso	
		[7]
	Alternative for (b)	
	$\dfrac{e^{x-1}+e^{-(x-1)}}{2}=\dfrac{e^{x}-e^{-x}}{2}$	M1
	Leading to $e^{2x}=\dfrac{e^{2}+e}{e-1}$	M1
	$\tanh x=\dfrac{e^{x}-e^{-x}}{e^{x}+e^{-x}}=\dfrac{e^{2x}-1}{e^{2x}+1}=\dfrac{e^{2}+e-(e-1)}{e^{2}+e+(e-1)}=\dfrac{e^{2}+1}{e^{2}+2e-1}$ ∗ cso	M1 A1 **(4)**

Question Number	Scheme	Marks
5.	(a) $x = t - \sin 2t \;\Rightarrow\; \dot{x} = 1 - 2\cos 2t \;\Rightarrow\; \ddot{x} = 4\sin 2t$ either	M1
	$y = \cos 2t \;\Rightarrow\; \dot{y} = -2\sin 2t \;\Rightarrow\; \ddot{y} = -4\cos 2t$ both	A1
	Obtaining $\dot{x}^2 + \dot{y}^2$ and $\dot{x}\ddot{y} - \ddot{x}\dot{y}$ in terms of t	M1
	$\dot{x}^2 + \dot{y}^2 = 1 + 4\cos^2 2t - 4\cos 2t + 4\sin^2 2t = 5 - 4\cos 2t$	A1
	$\dot{x}\ddot{y} - \ddot{x}\dot{y} = -4\cos 2t(1 - 2\cos 2t) - 4\sin 2t(-2\sin 2t) = 8 - 4\cos 2t$	A1
	$\rho = \dfrac{(5 - 4\cos 2t)^{3/2}}{8 - 4\cos 2t}$	A1 **(6)**
	(b) The least value of y $(\cos 2t)$ is -1	B1
	$\rho = \dfrac{(5+4)^{3/2}}{8+4} = \dfrac{9}{4}$ accept equivalent fractions or 2.25	B1 **(2)**
		[8]

Question Number	Scheme	Marks
6.	(a) $I_n = -\frac{3}{4}\left[x^n(8-x)^{4/3}\right]_0^8 + \frac{3}{4}\int nx^{n-1}(8-x)^{4/3}\,dx$	M1 A1
	$= \frac{3}{4}\int nx^{n-1}(8-x)^{4/3}\,dx$ ft numeric constants only	A1ft
	$\int nx^{n-1}(8-x)(8-x)^{1/3}\,dx = \int nx^{n-1}8(8-x)^{1/3}\,dx - \int nx^{n-1}x(8-x)^{1/3}\,dx$	M1 A1
	$I_n = 6nI_{n-1} - \frac{3}{4}nI_n \Rightarrow I_n = \frac{24n}{3n+4}I_{n-1}$ * cso	A1 **(6)**
	(b) $I_0 = \int_0^8 (8-x)^{1/3}\,dx = \left[-\frac{3}{4}(8-x)^{4/3}\right]_0^8 = \frac{3}{4}\times 8^{4/3} = 12$	M1 A1
	$I = \int_0^8 x(x+5)(8-x)^{1/3}\,dx = I_2 + 5I_1$	M1
	$I_1 = \frac{24}{7}I_0, \quad I_2 = \frac{48}{10}I_1 = \frac{48}{10}\times\frac{24}{7}I_0 \left(= \frac{576}{35}I_0\right)$	M1 A1
	(The previous line can be implied by $I = I_2 + 5I_1 = \frac{168}{5}I_0$)	
	$I = \left(\frac{576}{35} + 5\times\frac{24}{7}\right)\times 12 = \frac{2016}{5}\ (= 403.2)$	A1 **(6)**
		[12]

Question Number	Scheme	Marks
7.	(a) $\dfrac{d}{dx}\left(\text{arsinh}\, x^{\frac{1}{2}}\right) = \dfrac{1}{\sqrt{(1+x)}} \times \dfrac{1}{2} x^{-\frac{1}{2}} \quad \left(= \dfrac{1}{2\sqrt{x}\sqrt{(1+x)}} \right)$	M1 A1
	At $x = 4$, $\dfrac{dy}{dx} = \dfrac{1}{4\sqrt{5}}$ accept equivalents	A1 **(3)**
	(b) $x = \sinh^2 \theta$, $\dfrac{dx}{d\theta} = 2\sinh\theta\cosh\theta$	
	$\int \text{arsinh}\sqrt{x}\, dx = \int \theta \times 2\sinh\theta\cosh\theta\, d\theta$	M1 A1
	$= \int \theta \sinh 2\theta\, d\theta = \dfrac{\theta\cosh 2\theta}{2} - \int \dfrac{\cosh 2\theta}{2} d\theta$	M1 A1 + A1
	$= \ \ldots \ - \dfrac{\sinh 2\theta}{4}$	M1
	$\left[\dfrac{\theta\cosh 2\theta}{2} - \dfrac{\sinh 2\theta}{4} \right]_0^{\text{arsinh}\, 2} = \ \ldots$ attempt at substitution	M1
	$= \left[\dfrac{\theta\left(1 + 2\sinh^2\theta\right)}{2} - \dfrac{2\sinh\theta\cosh\theta}{4} \right] = \dfrac{1}{2}\text{arsinh}\, 2 \times (1+8) - \dfrac{4\sqrt{5}}{4}$	M1 A1
	$= \dfrac{9}{2}\ln\left(2+\sqrt{5}\right) - \sqrt{5}$	A1 **(10)**
		[13]
	Alternative for (a)	
	$x = \sinh^2 y$, $\quad 2\sinh y\cosh y \dfrac{dy}{dx} = 1$	M1
	$\dfrac{dy}{dx} = \dfrac{1}{2\sinh y\cosh y} = \dfrac{1}{2\sinh y\sqrt{\left(\sinh^2 y + 1\right)}} \left(= \dfrac{1}{2\sqrt{x}\sqrt{(1+x)}} \right)$	A1
	At $x = 4$, $\dfrac{dy}{dx} = \dfrac{1}{4\sqrt{5}}$ accept equivalents	A1 **(3)**
	An alternative for (b) *is given on the next page*	

Question Number	Scheme	Marks
7.	*Alternative for* (b)	
	$\int 1\times \text{arsinh}\sqrt{x}\,dx = x\,\text{arsinh}\sqrt{x} - \int x \times \frac{1}{2\sqrt{x}\sqrt{(1+x)}}\,dx$	M1 A1 + A1
	$= x\,\text{arsinh}\sqrt{x} - \int \frac{\sqrt{x}}{2\sqrt{(1+x)}}\,dx$	
	Let $x = \sinh^2\theta, \quad \frac{dx}{d\theta} = 2\sinh\theta\cosh\theta$	
	$\int \frac{\sqrt{x}}{\sqrt{(1+x)}}\,dx = \int \frac{\sinh\theta}{\cosh\theta}\times 2\sinh\theta\cosh\theta\,d\theta$	M1 A1
	$= 2\int \sinh^2\theta\,d\theta = 2\int \frac{\cosh 2\theta - 1}{2}\,d\theta, \quad = \frac{\sinh 2\theta}{2} - \theta$	M1, M1
	$\left[\frac{\sinh 2\theta}{2} - \theta\right]_0^{\text{arsinh}\,2} = \left[\frac{2\sinh\theta\cosh\theta}{2} - \theta\right]_0^{\text{arsinh}\,2} = \frac{2\times 2\times\sqrt{5}}{2} - \text{arsinh}\,2$	M1 A1
	$\int_0^4 \text{arsinh}\sqrt{x}\,dx = 4\,\text{arsinh}\,2 - \frac{1}{2}\left(\frac{2\times 2\times\sqrt{5}}{2} - \text{arsinh}\,2\right) = \frac{9}{2}\ln\left(2+\sqrt{5}\right) - \sqrt{5}$	A1 **(10)**
	The last 7 marks of the alternative solution can be gained as follows	
	Let $x = \tan^2\theta, \quad \frac{dx}{d\theta} = 2\tan\theta\sec^2\theta$	
	$\int \frac{\sqrt{x}}{\sqrt{(1+x)}}\,dx = \int \frac{\tan\theta}{\sec\theta}\times 2\tan\theta\sec^2\theta\,d\theta$ dependent on first M1	M1 A1
	$= \int 2\sec\theta\tan^2\theta\,d\theta$	
	$\int (\sec\theta\tan\theta)\tan\theta\,d\theta = \sec\theta\tan\theta - \int \sec^3\theta\,d\theta$	M1
	$= \sec\theta\tan\theta - \int \sec\theta\left(1+\tan^2\theta\right)d\theta$	
	Hence $\int \sec\theta\tan^2\theta\,d\theta = \frac{1}{2}\sec\theta\tan\theta - \frac{1}{2}\int \sec\theta\,d\theta$	
	$= \frac{1}{2}\sec\theta\tan\theta - \frac{1}{2}\ln\left(\sec\theta + \tan\theta\right)$	M1
	$\left[\ \dots\ \right]_0^{\arctan 2} = \frac{1}{2}\times\sqrt{5}\times 2 - \frac{1}{2}\ln\left(\sqrt{5}+2\right)$	M1 A1
	$\int_0^4 \text{arsinh}\sqrt{x}\,dx = 4\,\text{arsinh}\,2 - \sqrt{5} + \frac{1}{2}\ln\left(2+\sqrt{5}\right) = \frac{9}{2}\ln\left(2+\sqrt{5}\right) - \sqrt{5}$	A1

Question Number	Scheme	Marks	
8.	(a) Gradient of $PQ = \dfrac{2ap - 2aq}{ap^2 - aq^2} = \dfrac{2}{p+q}$ Can be implied	B1	
	Use of any correct method or formula to obtain an equation of PQ in any form.	M1	
	Leading to $(p+q)y = 2(x + apq)$ ✱	A1	**(3)**
	(b) Gradient of normal at P is $-p$. Given or implied at any stage	B1	
	Obtaining any correct form for normal at either point. Allow if just written down. $y + px = 2ap + ap^3$ $y + qx = 2aq + aq^3$	M1 A1	
	Using both normal equations and eliminating x or y. Allow in any unsimplified form.	M1	
	$(p-q)x = 2a(p-q) + a(p^3 - q^3)$ Any correct form for x or y	A1	
	Leading to $x = a(p^2 + q^2 + pq + 2)$ ✱ cso	A1	
	$y = -apq(p+q)$ ✱ cso	A1	**(7)**
	(c) $0 = 2(5a + apq) \Rightarrow pq = -5$	B1	
	Using $pq = -5$ in both $x = a(p^2 + q^2 + pq + 2)$ and $y = -apq(p+q)$. $x = a(p^2 + q^2 - 3)$ $y = 5a(p+q)$	M1	
	Any complete method for relating x and y, independently of p and q.	M1	
	A correct equation in any form. $x = a\left((p+q)^2 - 2pq - 3\right) = a\left(\left(\dfrac{y}{5a}\right)^2 + 10 - 3\right)$	A1	
	Leading to $y^2 = 25a(x - 7a)$ Accept equivalent forms of $\mathrm{f}(x)$	A1	**(5)**
			[15]
	The algebra above can be written in many alternative equivalent forms.		