

# Mark Scheme (Results)

## January 2007

GCE

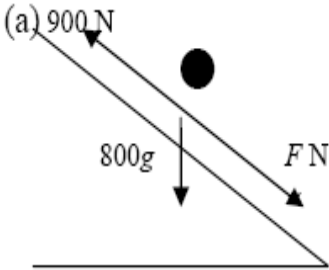
GCE Mathematics

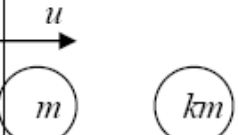
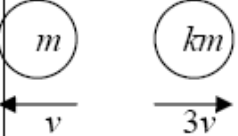
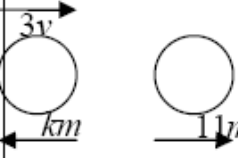
Mechanics M2 (6678)

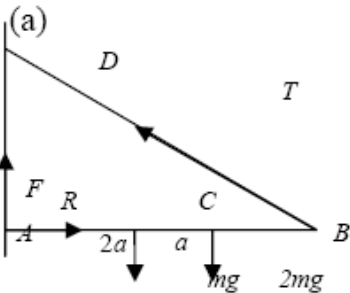


January 2007  
6678 Mechanics M2  
Mark Scheme

Question Number	Scheme	Marks
1.	<p>(a) <math>\frac{1}{2}0.8(15^2 - 10^2) = 50 \text{ (J)}</math></p> <p>(b) <math>F = \mu R = \mu 0.8g</math>            Work-energy <math>\mu 0.8g \times 20 = 50</math>  <math>\mu \approx 0.32</math></p>	<p>M1 A1 <u>2</u></p> <p>M1 M1 A1ft A1 <u>4</u> <b>6</b></p>
	<p><i>Alternative for (b)</i></p> $v^2 = u^2 + 2as \Rightarrow a = \frac{15^2 - 10^2}{2 \times 20} = 3.125$ <p>N2L <math>F = \mu mg = ma = 3.125m</math>  <math>\mu \approx 0.32</math></p>	<p>M1 M1 A1ft A1 <u>4</u></p>
	<p><i>Alternative for (b)</i></p> <p>WE <math>F = \frac{50}{20} (= 2.5)</math></p> $F = \mu R \Rightarrow \frac{50}{20} = \mu 0.8g$ $\mu \approx 0.32$	<p>M1 M1 A1 ft A1 <b>4</b></p>
	<p>The first M1 for (b) could be scored in (a):</p> $v^2 = u^2 + 2as \Rightarrow 10^2 = 15^2 - 2 \times 20 \times (-)a \Rightarrow a = (-)\frac{125}{40}$ $F = ma \Rightarrow F = 2.5$ $WD = F \times d \Rightarrow 2.5 \times 20 = 50J$	<p>(b)M1 (a)M1A1</p>

2.		$F + 800g \sin \alpha = 900$ $F = 573\frac{1}{3}$ $W = 573\frac{1}{3} \times 15 = 8600$ $= 8.6 \text{ kW}$	M1 A1 M1 A1 <u>4</u>
NB. Going up hill is an error, not a Misread			
(b)	N2L	$800 \times 9.8 \times \frac{1}{24} - 900 = 800a \quad *$ $a = -\frac{43}{60} \quad \text{awrt } -0.72$ $0 = 15 - \frac{43}{60}T$ $T \approx 21 \quad \text{accept } 20.9$	M1 A1 M1 Alcso <u>4</u> <b>8</b>
* If they are using their F from (a) then they need to have scored the M1 in (a) in order to score the M1 here.			
Alternative for (b)			
WD: $573\frac{1}{3}s = \frac{1}{2} \times 800 \times 15^2$			
$s = 157$			
Use of $v^2 = u^2 + 2as$ M1 for getting as far as an equation in $a$ .			
$a = 0.72$ A1 finish as above.			
2 <sup>nd</sup> Alternative for (b)			
$Ft = \text{Change in momentum:}$			
M1 Using the correct $F$			
M1 Use of the method to form an equation			
A1 Equation correct unsimplified but fully substituted			
A1 $T \approx 21$			

Question Number	Scheme	Marks
3.	<p>(a) Large Small Template            Mass Ratios <math>24^2</math> <math>8^2</math>, <math>512</math> anything in ratio 9 : 1 : 8            (c.1810 c.200 c.1610)  <math>M(A)</math> <math>9 \times 24 = 16 \times 1 + 8\bar{x}</math>  <math>\bar{x} = 25</math> (cm) exact</p> <p>(b) M(axis) <math>11M = 12 \times \frac{1}{4}m</math> ft their <math>\bar{x}</math>  <math>(36 - \bar{x})M = 12 \times \frac{1}{4}m</math>  <math>M = \frac{3}{11}m</math> (o.e.e.)</p>	B1, B1ft M1* A1 DM1* A1 <u>6</u> M1 † A1ft DM1 † A1 <u>4</u> <b>10</b>
4. (a)	 <p>NEL <math>3v - (-v) = eu</math>  <math>u = 8v</math></p>	M1 A1 A1 <u>3</u>
(b)	 <p>LM <math>8mv = -mv + 3kmv</math> ft their <math>u</math>  <math>(m \times (u) = -mv + 3kmv)</math>  <math>k = 3</math></p>	M1 A1ft A1 <u>3</u>
(c)	 <p>LM <math>9mv = -3my + 11my</math> ft their <math>k</math>            NEL <math>2y = e \times 3v</math>  <math>y = \frac{9}{8}v \Rightarrow e = \frac{3}{4}</math> * cso</p>	M1 A1ft M1 A1 <u>4</u>
(d)	$y = \frac{9}{8}v > v \Rightarrow$ further collision between $P$ and $Q$ A1 is cso – watch out for incorrect statements re. velocity	M1 A1 <u>2</u> <b>12</b>

Question Number	Scheme	Marks
5.	<p>(a) </p> <p>M(A) <math>T \sin \theta \times 4a = mg \times 2a + 2mg \times 3a</math></p> $T = \frac{8mg}{4} \times \frac{5}{3} = \frac{10}{3}mg$ <p>Accept 32.7m, 33m</p> <p>(b) <math>\rightarrow R = T \cos \theta = \frac{10}{3}mg \times \frac{4}{5}; = \frac{8}{3}mg</math> * cs0</p> <p>(c) <math>\uparrow F + T \sin \theta = 3mg \Rightarrow F = mg</math> ft their T  Or: M(B) <math>F \times 4a = mg \times 2a + 2mg \times a \Rightarrow F = mg</math></p> $F = \mu R \Rightarrow \mu = \frac{3}{8}$ <p>(a) Alternative approach:  <math>\rightarrow R = T \cos \theta</math>  <math>\uparrow F + T \sin \theta = 3mg</math>  M(B) <math>F \times 4a = mg \times 2a + 2mg \times a (\Rightarrow F = mg)</math>  <math>\Rightarrow mg + T \sin \theta = 3mg \Rightarrow T = \frac{2mg}{\sin \theta} = \frac{10mg}{3}</math></p> <p>If they use this method, watch out for F=mg just quoted in (c): M1A1</p>	<p>M1* A1=A1  DM1* A1 <u>5</u></p> <p>M1 A1ft; A1  <u>3</u></p> <p>M1  A1ft</p> <p>M1 A1 <u>4</u> <b>12</b></p>

6.	<p>(a) N2L <math>(1.5t^2 - 3)\mathbf{i} + 2t\mathbf{j} = 0.5\mathbf{a}</math>  <math>\mathbf{a} = (3t^2 - 6)\mathbf{i} + 4t\mathbf{j}</math></p> <p>(b) <math>\mathbf{v} = (t^3 - 6t)\mathbf{i} + 2t^2\mathbf{j} \quad (+\mathbf{c})</math>  <math>t = 2 \quad -4\mathbf{i} + 5\mathbf{j} = -4\mathbf{i} + 8\mathbf{j} + \mathbf{c} \quad (\mathbf{c} = -3\mathbf{j})</math>  <math>\mathbf{v} = (t^3 - 6t)\mathbf{i} + (2t^2 - 3)\mathbf{j} \quad (\text{ms}^{-1})</math>  <math>t = 3 \quad \mathbf{v} = 9\mathbf{i} + 15\mathbf{j} \quad (\text{ms}^{-1}) \quad *</math></p> <p>(c) <math>\mathbf{Q} = 0.5(-3\mathbf{i} + 20\mathbf{j} - (9\mathbf{i} + 15\mathbf{j})) \quad (= 0.5(-12\mathbf{i} + 5\mathbf{j}))</math>  <math> \mathbf{Q}  = 0.5\sqrt{(5^2 + 12^2)} = 6.5</math></p> <p>(d) acute angle is <math>\arctan \frac{5}{12} \approx 23^\circ</math>  or required angle is <math>\arctan \frac{-5}{12}</math>  or acute angle is <math>\arccos \frac{12}{13} \approx 23^\circ</math>  or required angle is <math>\arccos \frac{-12}{13}</math>    required angle is <math>157^\circ</math>      awrt <math>157^\circ, 203^\circ</math></p>	<p>M1 A1    <u>2</u></p> <p>M1 A1 M1 A1 A1    <u>5</u>      cso</p> <p>M1 M1 A1    <u>3</u></p> <p>M1 A1</p> <p>A1    <u>3</u>    <b>13</b></p>
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Question Number	Scheme	Marks
7.	<p>(a) Energy <math>\frac{1}{2}m(24.5^2 - u^2) = mg \times 15</math>  <math>u^2 = 24.5^2 - 30g = 306.25</math>  <math>u = \sqrt{306.25} = 17.5</math> ★</p> <p>(b) <math>\rightarrow u_x = u \cos \theta = 17.5 \times 0.8 = 14</math>  <math>\psi = \arccos \frac{14}{24.5} \approx 55^\circ</math> accept <math>55.2^\circ</math>  <small>(0.96 rads, or 0.963 rads)</small></p> <p>(c) <math>\uparrow u_y = u \sin \theta = 17.5 \times 0.6 = 10.5</math>  <math>s = ut + \frac{1}{2}at^2 \Rightarrow -45 = 10.5t - 4.9t^2</math>  leading to <math>t = 4.3</math>, awrt <math>t = 4.3</math> or <math>t = 4\frac{2}{7}</math>  <math>\rightarrow BD = 14 \times 4\frac{2}{7}</math> (14 x t) ft their t  = 60 (m) only</p>	M1 A1=A1 A1 <u>4</u> B1 M1 A1 <u>3</u> B1 M1 A1 A1 M1 A1ft A1 <u>7</u> <b>14</b>
	<p><i>Alternative for (a)</i>  <math>\rightarrow u_x = u \cos \theta = 0.8u</math>, <math>\uparrow u_y = u \sin \theta = 0.6u</math>  <math>v_y^2 = 0.36u^2 + 2 \times 9.8 \times 15 = 0.36u^2 + 294</math>  <math>24.5^2 = u_x^2 + v_y^2 = 0.64u^2 + 0.36u^2 + 294</math>  <math>u^2 = 306.25 \Rightarrow u = 17.5</math> ★ cso</p> <p><i>Alternative for (b)</i> <math>\rightarrow u_x = u \cos \theta = 17.5 \times 0.8 = 14</math>  <math>\uparrow v_y^2 = u^2 \sin^2 \theta + 2 \times 9.8 \times 15 = 404.25</math>  <math>\psi = \arctan \frac{\sqrt{404.25}}{14} \approx 55^\circ</math> accept <math>55.2^\circ</math></p> <p><i>Alternative for (c)</i> Use of <math>y = x \tan \theta - \frac{g \sec^2 \theta}{2u^2} x^2</math>  <math>-45 = \frac{3}{4}x</math>, <math>-\frac{g}{2 \times 17.5^2} \times \frac{25}{16} x^2</math>  <math>x^2 - 30x - 1800 = 0</math> o.e.  Factors or quadratic formula  BD = 60 (m)</p>	M1 A1,A1 A1 <u>4</u> B1 M1 A1 <u>3</u> M1 B1,A1 A1 M1 A1ft A1