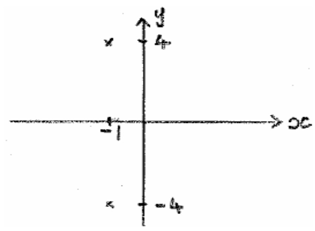
January 2007
6674 Further Pure Mathematics FP1
Mark Scheme

Question Number	Scheme	Marks
1.	(a) Method for finding z : $z = \frac{-2 \pm \sqrt{4-68}}{2}$, $= \frac{-2 \pm \sqrt{64}\, i}{2}$	M1, A1
	[Completing the square: $(z+1)^2 + 16 = 0$, $z = -1 \pm \sqrt{16}\, i$ M1,A1	
	$z = -1 \pm 4i$ $(a = -1,\ b = \pm 4)$	A1 (3)
	(b) 	B1 √ (1)
		[4]
	Notes (a) First A1 is unsimplified but requires i $-1 \pm 8i$ only scores M1 unless intermediate step seen when M1A1 possible Correct answer with no working is full marks SC: If M0 awarded, $k \pm 4i$, $k + 4i$, $k - 4i$ scores B1 (Epen M0A0A1) Use of $z = a + ib$ (i) $z^2 - 2az + a^2 + b^2 = z^2 + 2z + 17 = 0$ and compare coefficients M1 $a^2 + b^2 = 17$ and a $= -1$; $z = -1 \pm 4i$ A1, A1 (ii) $(a + ib)^2 + 2(a + ib) + 17 = 0$ and compare coefficients M1 $2b(a+1) = 0$ *and* $a^2 - b^2 + 2a = -17$, $a = -1$ *and* $b = \pm 4$ A1, A1 (b) Must be a conjugate pair. Allow: Coords marked at points or "correct" numbers on axes.(allow "graduations") (Ignore any lines drawn)	

2.	Attempt to arrange in correct form $\frac{dy}{dx} + \frac{2}{x}y = \frac{\cos x}{x}$	M1
	Integrating Factor: $= e^{\int \frac{2}{x}dx}$, $[(= e^{2\ln x} = e^{\ln x^2}) = x^2$	M1,A1
	[$x^2\frac{dy}{dx} + 2xy = x\cos x$ implies M1M1A1]	
	$\therefore \quad x^2 y = \int x^2 . \frac{\cos x}{x} dx$ or equiv. [I.F. y $= \int I.F.(candidate's RHS)dx$]	M1√
	By Parts: $(x^2 y) = x\sin x - \int \sin x\, dx$	M1
	i.e. $(x^2 y) = x\sin x, + \cos x \ (+c)$	A1, A1cao
	$y = \frac{\sin x}{x} + \frac{\cos x}{x^2} + \frac{c}{x^2}$	A1√ **[8]**
	Notes: First M: At least two terms divided by x. "By parts" M: Must be complete method, e.g $\int x^{"2} \cos x\, dx$ requires **two** applications Because of functions involved**, be generous with sign,** but $x\sin x \pm \int \cos x\, dx$ is M0 (S.C. "Loop" integral like $\int e^x \cos x\, dx$, *allow M1 if two applications of "by parts", despite incomplete method*) Final A f.t. for dividing all terms by candidates I.F., providing "c" used.	

3.	(a) $\dfrac{z_2}{z_1} = \dfrac{1+pi}{5+3i}\cdot\dfrac{(5-3i))}{(5-3i)}$	M1
	$= \dfrac{5+5pi-3i+3p}{(34)}$ [Multiply out and attempt use of $i^2 = -1$]	M1
	$= \dfrac{5+3p}{34} + \dfrac{5p-3}{34}i$ or $\dfrac{5+3p}{34} - \dfrac{3-3p}{34}i$	A1 **(3)**
	(b) For $\dfrac{z_2}{z_1} = c + id$ using $\dfrac{d}{c} = \tan\dfrac{\pi}{4}$:	M1
	$[\ 5p - 3 = 5 + 3p\ ]$ $\Rightarrow p = 4$	A1 **(2)**
		[5]
	Notes: In (a) if $\dfrac{z_1}{z_2}$ used treat as MR. Can score (a)M1M1A0 (b)M1A0 $\left[(a)\dfrac{5+3p}{1+p^2} + \dfrac{3-5p}{1+p^2}i \quad (b)\ -\dfrac{1}{4} \right]$ Allow A1 if answer "all over" 34, real and imag. collected up) $1 + pi = (a+ib)(5+3i)$: M1 compare real and imag. is first M mark If denominator in (a) incorrect, both marks in (b) still available In (b), if use $\arg z_2 - \arg z_1 = \dfrac{\pi}{4}$: M1 for $\arctan p - \arctan\dfrac{3}{5} = \dfrac{\pi}{4}$ $\left[\arctan p = \dfrac{\pi}{4} + 0.5404.... = 1.3258\right]$ Allow A1 for p = 4 without further work or for that shown in brackets, i.e. assume values retained on calculator (no penalty because it looks as though not exact)	

4.	Working from RHS:	
	(a) Combining $\frac{1}{r} - \frac{1}{r+1}$ $\left[\frac{1}{r(r+1)} \right]$	M1
	Forming single fraction : $\frac{r(r-1)(r+1)+(r+1)-r}{r(r+1)}$	M1
	$= \frac{r(r^2-1)+1}{r(r+1)} = \frac{r^3-r+1}{r(r+1)}$ **AG**	A1cso **(3)**
	Note: For A1, must be intermediate step, as shown	
	Working from LHS:	
	(a) $\frac{r(r^2-1)+1}{r(r+1)} = \frac{r(r+1)(r-1)+1}{r(r+1)} = r-1+\frac{1}{r(r+1)}$ M1	
	Splitting $\frac{1}{r(r+1)}$ into partial fractions M1	
	Showing $= \frac{r(r^2-1)+1}{r(r+1)} = r-1+\frac{1}{r}-\frac{1}{r+1}$ no incorrect working seen A1	
	Notes: In first method, second M needs all necessary terms, allowing for sign errors In second method first M is for division: Second method mark is for method shown (allow “cover up” rule stated) If long division, allow reasonable attempt which has remainder constant or linear function of r. Setting $\frac{r(r^2-1)+1}{r(r+1)} = \frac{A}{r} + \frac{B}{r+1}$ is M0 If 3 or 4 constants used in a correct initial statement, M1 for finding 2 constants; M1 for complete method to find remaining constant(s)	

	(b) $\sum_1^n r - \sum_1^n 1 + \sum_1^n \left(\frac{1}{r} - \frac{1}{r+1}\right)$	
	$= \frac{n(n+1)}{2}$, (–) n, + ………….. ……. +	B1, B1
	$\left[(1 - \frac{1}{2}) + (\frac{1}{2} - \frac{1}{3}) + \text{...........} + (\frac{1}{n} - \frac{1}{n+1})\right] =$	M1
	Simplification of method of differences: $1 - \frac{1}{n+1}$	A1
	$\{ = \frac{n(n-1)}{2} + [1 - \frac{1}{(n+1)}]\}$	
	Attempt single fraction: $= \frac{n(n+1)(n-1)+2n}{2(n+1)}$ (dep. prev. M1)	depM1
	$= \frac{n(n^2+1)}{2(n+1)}$ *or* $\frac{n^3+n}{2(n+1)}$	A1 **(6)**
		[9]
	Alternative: Using Difference method on whole expression:	
	$[0 + 1 - \frac{1}{2}] + [1 + \frac{1}{2} - \frac{1}{3}] + [2 + \frac{1}{3} - \frac{1}{4}]$ …….. $[n - 1 + \frac{1}{n} - \frac{1}{n+1}]$	M1
	$= (1 + 2 + 3$ ………. $+ n - 1)$, $+ [(1 - \frac{1}{n+1})]$ any form	B1, + [A1]
	$= \frac{n(n-1)}{2}$, $\{+ \frac{n}{n+1}\}$	B1,
	$= \frac{n(n+1)(n-1)+2n}{2(n+1)}$ [Attempt single fraction]	depM1
	$= \frac{n(n^2+1)}{2(n+1)}$ *or* $\frac{n^3+n}{2(n+1)}$	A1
	Notes: First M mark is for use of method of differences and attempt at some simplification First A mark is for simplified result of this method (no more than 2 terms) Second M mark for attempt at forming single fraction, dependent on first M mark In alternative first B1 need not be added but need to see 1 2 …….. (n – 1)	

Question Number	Scheme	Marks
5.	(a) $[(x > -2)]$: Attempt to solve $x^2 - 1 = 3(1-x)(x+2)$	M1
	$[4x^2 + 3x - 7 = 0]$	
	$x = 1$, *or* $-\frac{7}{4}$	B1, A1
	$[(x < -2)]$: Attempt to solve $x^2 - 1 = -3(1-x)(x+2)$	M1
	Solving $x + 1 = 3x + 6$ $(2x^2 + 3x - 5 = 0)$	M1dep
	$x = -\frac{5}{2}$	A1 **(6)**
	(b) $-\frac{7}{4} < x < 1$ One part	M1
	Both correct and enclosed	A1
	$x < -\frac{5}{2}$ { Must be for x < –2 and only one value}	B1 √ **(3)** **[9]**
	Notes: "Squaring" in (a) If candidates do not notice the factor of $(x-1)^2$ they have quartic to solve; Squaring and finding quartic = 0 $[8x^4 + 18x^3 - 25x^2 - 36x + 35 = 0]$ Finding one factor and factorising $(x-1)(8x^3 + 26x^2 + x - 35) = 0$ M1 Finding one other factor and reducing other factor to quadratic, likely to be $(x-1)^2(8x^2 + 34x + 35) = 0$ M1 Complete factorisation $(x-1)^2(2x+5)(4x+7) = 0$ M1 [SecondM1 implies the first, if candidate starts there or cancels $(x-1)^2$] $x = 1$ B1, $x = -7/4$ A1, $x = -5/2$ A1 $x = 1$ allowed anywhere, no penalty in (b) In (b) correct answers seen with no working is independent of (a) (graphical calculator) mark as scheme. Only allow the accuracy mark if no other interval, in both parts *≤ used penalise first time used*	

6.	(a) f(2.0) = – 0.30685……. = – 0.3069 AWRT 3 d.p.	M1
	f(2.5) = 0.41629……… = 0.4163 both correct 4 d.p.	A1
	States change of sign, so root (between 2 and 2.5)	B1 **(3)**
	Note: B1 gained if candidate's 2 values do show a change of sign and statement made	
	(b) $\alpha = (2) + \frac{\lvert f(2)\rvert}{\lvert f(2)\rvert + \lvert f(2.5)\rvert} \text{ x } 0.5$ or $\frac{\alpha - 2}{2.5 - \alpha} = \frac{\lvert f(2.0)\rvert}{\lvert f(2.5)\rvert}$ or equivalent	M1
	Or $\frac{x}{\lvert f(2)\rvert} = \frac{0.5 - x}{\lvert f(2.5)\rvert}$ and x found	
	= 2.212 AWRT	A1 **(2)**
	(c) f(2.25) = 0.06093……. (≥ 3 d.p.) [Allow ln.2.25 + 2.25 – 3]	B1
	$f'(x) = \frac{1}{x} + 1$, $f'(2.25) = 1.\dot{4}$ *or* $1\frac{4}{9}$ *or* $\frac{13}{9}$ (*allow* 1.444)	M1,A1
	$\alpha = 2.25 - \frac{f(2.25)}{f'(2.25)}$, = 2.20781.... = 2.208 AWRT	M1A1 **(5)**
	(d) f(2.2075) =, { –6.3…. x 10^{-4} } f(2.2085) = , { 8.1…. x 10^{-4} }	M1
	∴ Correct values (≥ 1 s.f.) , (root in interval) so root is 2.208 to 3 d.p.	A1 **(2)** **[12]**
	Notes: c) First M in (c) is just for $\frac{1}{x} + 1$ If no intermediate values seen B1M1A1M1A0 is possible for 2.209 or 2.21, otherwise as scheme (B1 eased to award this if not evaluated) (d) A1 requires values correct (≥ 1 s.f.) and statement (need not say change of sign) M can be given for candidate's f(2.2075) and f(2.2085) Allow N-R applied at least twice more, but A1 requires 2.20794 or better and statement MR in (c) 2.5 instead of 2.25 (Answer 2.203) award on ePen B0M1A0M1A1	

7.	(a) $y = x^{-2} \Rightarrow \frac{dy}{dt} = -2x^{-3}\frac{dx}{dt}$ [Use of chain rule; need $\frac{dx}{dt}$]	M1
	$\Rightarrow \frac{d^2y}{dt^2} = -2x^{-3}\frac{d^2x}{dt^2}, \quad +6x^{-4}\left(\frac{dx}{dt}\right)^2$	A1√, M1A1
	(÷ given d.e. by x^4) $\quad \frac{2}{x^3}\frac{d^2x}{dt^2} - \frac{6}{x^4}\left(\frac{dx}{dt}\right)^2 = \frac{1}{x^2} - 3$	
	becomes $\left(-\frac{d^2y}{dt^2} = y - 3\right) \quad \frac{d^2y}{dt^2} + y = 3$ **AG**	A1 cso **(5)**
	(b) Auxiliary equation: $m^2 + 1 = 0$ and produce Complementary Function $y = \ldots$	M1
	$(y) = A\cos t + B\sin t$	A1cao
	Particular integral: $y = 3$	B1
	∴ General solution: $(y) = A\cos t + B\sin t + 3$	A1√ **(4)**
	(c) $\frac{1}{x^2} = A\cos t + B\sin t + 3$	
	$x = \frac{1}{2},\ t = 0 \Rightarrow (4 = A + 3) \quad A = 1$	B1
	Differentiating (to include $\frac{dx}{dt}$): $\quad -2x^{-3}\frac{dx}{dt} = -A\sin t + B\cos t$	M1
	$\frac{dx}{dt} = 0,\ t = 0 \Rightarrow (0 = 0 + B) \quad B = 0$	M1
	$\therefore \frac{1}{x^2} = 3 + \cos t$ so $x = \frac{1}{\sqrt{3 + \cos t}}$	A1 cao **(4)**
	(d) (Max. value of x when $\cos t = -1$) so max $x = \frac{1}{\sqrt{2}}$ or AWRT 0.707	B1 **(1)** **[14]**
	Notes: (See separate sheet for several variations) (a) Second M1 is for attempt at product rule. (be generous) Final A1 requires all working correct and sufficient "substitution" work (b) Answer can be stated; M1 is implied by correct C.F. stated (allow θ for t) A1 f.t. for candidates CF + PI Allow $m^2 + m = 0$ and $m^2 - 1 = 0$ for M1. Marks for (b) can be gained in (c) (b) Second M : complete method to find other constant (This may involve solving two equations in A and B)	

Question	Scheme	Marks
8.	(a) $x = r\cos\theta = 4\sin\theta\cos^3\theta$	M1
	$\frac{dx}{d\theta} = 4\cos^4\theta - 12\cos^2\theta\sin^2\theta$ any correct expression	M1A1
	Solving $\frac{dx}{d\theta} = 0$ $[\frac{dx}{d\theta} = 0 \Rightarrow 4\cos^2\theta\,(\cos^2\theta - 3\sin^2\theta) = 0]$	M1
	$\sin\theta = \frac{1}{2}$ *or* $\cos\theta = \frac{\sqrt{3}}{2}$ *or* $\tan\theta = \frac{1}{\sqrt{3}}$ $\Rightarrow \theta = \frac{\pi}{6}$ AG	A1 cso
	$r = 4\sin\frac{\pi}{6}\cos^2\frac{\pi}{6} = \frac{3}{2}$ AG	A1 cso **(6)**
	(b) $A = \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} r^2\,d\theta = \frac{1}{2}.16\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sin^2\theta\cos^4\theta\,d\theta$	
	$8\sin^2\theta\cos^4\theta = 2\cos^2\theta(4\sin^2\theta\cos^2\theta) = 2\cos^2\theta\sin^2 2\theta$	M1
	$= (\cos 2\theta + 1)\sin^2 2\theta$	M1
	$= \cos 2\theta\sin^2 2\theta + \frac{1-\cos 4\theta}{2}$ =Answer AG	A1 cso **(3)**
	(c) Area $= \left[\frac{1}{6}\sin^3 2\theta + \frac{\theta}{2} - \frac{\sin 4\theta}{8}\right]_{\left(\frac{\pi}{6}\right)}^{\left(\frac{\pi}{4}\right)}$ (ignore limits)	M1A1
	$= \left(\frac{1}{6}\sin^3\frac{\pi}{2} + \frac{\pi}{8} - \frac{\sin\pi}{8}\right) - \left(\frac{1}{6}\sin^3\frac{\pi}{3} + \frac{\pi}{12} - \frac{\sin\frac{2\pi}{3}}{8}\right)$ (sub. limits)	M1
	$= \left(\frac{1}{6} + \frac{\pi}{8}\right) - \left(\frac{\sqrt{3}}{16} + \frac{\pi}{12} - \frac{\sqrt{3}}{16}\right) = \frac{1}{6}, + \frac{\pi}{24}$ both cao	A1,A1 **(5)** **[14]**
	Notes: (a) So many ways x may be expressing e.g $2\sin 2\theta\cos^2\theta,\ \sin 2\theta(1+\cos 2\theta),\ \sin 2\theta + (1/2)\sin 4\theta$ leading to many results for $\frac{dx}{d\theta}$ Some relevant equations in solving $[(1-4\sin^2\theta)=0,\quad (4\cos^2\theta - 3) = 0,\quad (1-3\tan^2\theta) = 0,\ \cos 3\theta = 0]$ Showing that $\theta = \frac{\pi}{6}$ satisfies $\frac{dx}{d\theta} = 0$, allow M1A1 providing $\frac{dx}{d\theta}$ correct Starting with x = r sin θ can gain M0M1M1 in (a) (b) First M1 for use of double angle formula for sin 2A Second M1 for use of $\cos 2A = 2\cos^2 A - 1$ Answer given: must be intermediate step, as shown, and no incorrect work (c) For first M, of the form $a\sin^3 2\theta + \frac{\theta}{2} \pm b\sin 4\theta$ (Allow if two of correct form) On ePen the order of the As in answer is as written	