

January 2007
6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme	Marks
1.	<p>** represents a constant</p> $f(x) = (2 - 5x)^{-2} = \underline{(2)}^{-2} \left(1 - \frac{5x}{2}\right)^{-2} = \underline{\frac{1}{4}} \left(1 - \frac{5x}{2}\right)^{-2}$ $= \frac{1}{4} \left\{ 1 + (-2)(**x) + \frac{(-2)(-3)}{2!} (**x)^2 + \frac{(-2)(-3)(-4)}{3!} (**x)^3 + \dots \right\}$ $= \frac{1}{4} \left\{ 1 + (-2)\left(\frac{-5x}{2}\right) + \frac{(-2)(-3)}{2!} \left(\frac{-5x}{2}\right)^2 + \frac{(-2)(-3)(-4)}{3!} \left(\frac{-5x}{2}\right)^3 + \dots \right\}$ $= \frac{1}{4} \left\{ 1 + 5x + \frac{75x^2}{4} + \frac{125x^3}{2} + \dots \right\}$ $= \frac{1}{4} + \frac{5x}{4} + \frac{75x^2}{16} + \frac{125x^3}{8} + \dots$ $= \frac{1}{4} + 1\frac{1}{4}x + 4\frac{11}{16}x^2 + 15\frac{5}{8}x^3 + \dots$	<p>Takes 2 outside the bracket to give any of $(2)^{-2}$ or $\frac{1}{4}$. B1</p> <p>Expands $(1 + **x)^{-2}$ to give an unsimplified $1 + (-2)(**x)$; M1</p> <p>A correct unsimplified $\{ \dots \}$ expansion with candidate's $(**x)$ A1</p> <p>Anything that cancels to $\frac{1}{4} + \frac{5x}{4}$; A1;</p> <p>Simplified $\frac{75x^2}{16} + \frac{125x^3}{8}$ A1</p> <p style="text-align: right;">[5]</p>
5 marks		

Question Number	Scheme	Marks
<p><i>Aliter</i></p> <p>1.</p> <p>Way 2</p>	$f(x) = (2 - 5x)^{-2}$ $= \left\{ \begin{aligned} &(2)^{-2} + (-2)(2)^{-3}(**x); + \frac{(-2)(-3)}{2!} (2)^{-4} (**x)^2 \\ &+ \frac{(-2)(-3)(-4)}{3!} (2)^{-5} (**x)^3 + \dots \end{aligned} \right\}$ $= \left\{ \begin{aligned} &(2)^{-2} + (-2)(2)^{-3}(-5x); + \frac{(-2)(-3)}{2!} (2)^{-4} (-5x)^2 \\ &+ \frac{(-2)(-3)(-4)}{3!} (2)^{-5} (-5x)^3 + \dots \end{aligned} \right\}$ $= \left\{ \begin{aligned} &\frac{1}{4} + (-2)\left(\frac{1}{8}\right)(-5x); + (3)\left(\frac{1}{16}\right)(25x^2) \\ &+ (-4)\left(\frac{1}{16}\right)(-125x^3) + \dots \end{aligned} \right\}$ $= \frac{1}{4} + \frac{5x}{4}; + \frac{75x^2}{16} + \frac{125x^3}{8} + \dots$ $= \frac{1}{4} + 1\frac{1}{4}x; + 4\frac{11}{16}x^2 + 15\frac{5}{8}x^3 + \dots$	<p>$\frac{1}{4}$ or $(2)^{-2}$ B1</p> <p>Expands $(2 - 5x)^{-2}$ to give an unsimplified $(2)^{-2} + (-2)(2)^{-3}(**x)$; M1</p> <p>A correct unsimplified {.....} expansion with candidate's $(**x)$ A1</p> <p>Anything that cancels to $\frac{1}{4} + \frac{5x}{4}$; A1;</p> <p>Simplified $\frac{75x^2}{16} + \frac{125x^3}{8}$ A1</p> <p style="text-align: right;">[5]</p>
		5 marks

Attempts using Maclaurin expansions need to be referred to your team leader.

Question Number	Scheme	Marks
2. (a)	$\text{Volume} = \pi \int_{-\frac{1}{4}}^{\frac{1}{2}} \left(\frac{1}{3(1+2x)} \right)^2 dx = \frac{\pi}{9} \int_{-\frac{1}{4}}^{\frac{1}{2}} \frac{1}{(1+2x)^2} dx$ $= \left(\frac{\pi}{9} \right) \int_{-\frac{1}{4}}^{\frac{1}{2}} (1+2x)^{-2} dx$ $= \left(\frac{\pi}{9} \right) \left[\frac{(1+2x)^{-1}}{(-1)(2)} \right]_{-\frac{1}{4}}^{\frac{1}{2}}$ $= \left(\frac{\pi}{9} \right) \left[-\frac{1}{2}(1+2x)^{-1} \right]_{-\frac{1}{4}}^{\frac{1}{2}}$ $= \left(\frac{\pi}{9} \right) \left[\left(\frac{-1}{2(2)} \right) - \left(\frac{-1}{2(\frac{1}{2})} \right) \right]$ $= \left(\frac{\pi}{9} \right) \left[-\frac{1}{4} - (-1) \right]$ $= \frac{\pi}{12}$	<p>Use of $V = \pi \int y^2 dx$. Can be implied. Ignore limits. B1</p> <p>Moving their power to the top. (Do not allow power of -1.) Can be implied. M1 Ignore limits and $\frac{\pi}{9}$</p> <p>Integrating to give $\frac{\pm p(1+2x)^{-1}}{-\frac{1}{2}(1+2x)^{-1}}$ M1 A1</p> <p>Use of limits to give exact values of $\frac{\pi}{12}$ or $\frac{3\pi}{36}$ or $\frac{2\pi}{24}$ or aef A1 aef</p>
	(b)	<p>From Fig.1, $AB = \frac{1}{2} - \left(-\frac{1}{4}\right) = \frac{3}{4}$ units</p> <p>As $\frac{3}{4}$ units \equiv 3cm</p> <p>then scale factor $k = \frac{3}{\left(\frac{3}{4}\right)} = 4$.</p> <p>Hence Volume of paperweight = $(4)^3 \left(\frac{\pi}{12}\right)$</p> <p>$V = \frac{16\pi}{3} \text{ cm}^3 = 16.75516\dots \text{ cm}^3$</p>
Note: $\frac{\pi}{9}$ (or implied) is not needed for the middle three marks of question 2(a).		7 marks

Question Number	Scheme	Marks
<p>Aliter</p> <p>2. (a)</p> <p>Way 2</p>	$\text{Volume} = \pi \int_{-\frac{1}{4}}^{\frac{1}{2}} \left(\frac{1}{3(1+2x)} \right)^2 dx = \pi \int_{-\frac{1}{4}}^{\frac{1}{2}} \frac{1}{(3+6x)^2} dx$ $= (\pi) \int_{-\frac{1}{4}}^{\frac{1}{2}} (3+6x)^{-2} dx$ $= (\pi) \left[\frac{(3+6x)^{-1}}{(-1)(6)} \right]_{-\frac{1}{4}}^{\frac{1}{2}}$ $= (\pi) \left[\frac{-1}{6} (3+6x)^{-1} \right]_{-\frac{1}{4}}^{\frac{1}{2}}$ $= (\pi) \left[\left(\frac{-1}{6(6)} \right) - \left(\frac{-1}{6(\frac{3}{2})} \right) \right]$ $= (\pi) \left[-\frac{1}{36} - \left(-\frac{1}{9}\right) \right]$ $= \frac{\pi}{12}$	<p>Use of $V = \pi \int y^2 dx$.</p> <p>Can be implied. Ignore limits.</p> <p>Moving their power to the top. (Do not allow power of -1.) Can be implied. Ignore limits and π</p> <p>Integrating to give $\frac{\pm p(3+6x)^{-1}}{-\frac{1}{6}(3+6x)^{-1}}$</p> <p>Use of limits to give exact values of $\frac{\pi}{12}$ or $\frac{3\pi}{36}$ or $\frac{2\pi}{24}$ or aef</p> <p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1 aef</p> <p>[5]</p>

Note: π is not needed for the middle three marks of question 2(a).

Question Number	Scheme	Marks
3. (a)	$x = 7 \cos t - \cos 7t, \quad y = 7 \sin t - \sin 7t,$ $\frac{dx}{dt} = -7 \sin t + 7 \sin 7t, \quad \frac{dy}{dt} = 7 \cos t - 7 \cos 7t$ $\therefore \frac{dy}{dx} = \frac{7 \cos t - 7 \cos 7t}{-7 \sin t + 7 \sin 7t}$	<p>Attempt to differentiate x and y with respect to t to give $\frac{dx}{dt}$ in the form $\pm A \sin t \pm B \sin 7t$ M1</p> <p>$\frac{dy}{dt}$ in the form $\pm C \cos t \pm D \cos 7t$ A1</p> <p>Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$ A1</p> <p>Candidate's $\frac{dy}{dx}$ B1 $\sqrt{\quad}$</p> <p>[3]</p>
(b)	<p>When $t = \frac{\pi}{6}$, $m(\mathbf{T}) = \frac{dy}{dx} = \frac{7 \cos \frac{\pi}{6} - 7 \cos \frac{7\pi}{6}}{-7 \sin \frac{\pi}{6} + 7 \sin \frac{7\pi}{6}}$,</p> $= \frac{\frac{7\sqrt{3}}{2} - \left(-\frac{7\sqrt{3}}{2}\right)}{-\frac{7}{2} - \frac{7}{2}} = \frac{7\sqrt{3}}{-7} = -\sqrt{3} = \text{awrt } -1.73$ <p>Hence $m(\mathbf{N}) = \frac{-1}{-\sqrt{3}}$ or $\frac{1}{\sqrt{3}} = \text{awrt } 0.58$</p> <p>When $t = \frac{\pi}{6}$,</p> $x = 7 \cos \frac{\pi}{6} - \cos \frac{7\pi}{6} = \frac{7\sqrt{3}}{2} - \left(-\frac{\sqrt{3}}{2}\right) = \frac{8\sqrt{3}}{2} = 4\sqrt{3}$ $y = 7 \sin \frac{\pi}{6} - \sin \frac{7\pi}{6} = \frac{7}{2} - \left(-\frac{1}{2}\right) = \frac{8}{2} = 4$ <p>N: $y - 4 = \frac{1}{\sqrt{3}}(x - 4\sqrt{3})$</p> <p>N: $y = \frac{1}{\sqrt{3}}x$ or $y = \frac{\sqrt{3}}{3}x$ or $3y = \sqrt{3}x$</p> <p>or $4 = \frac{1}{\sqrt{3}}(4\sqrt{3}) + c \Rightarrow c = 4 - 4 = 0$</p> <p>Hence N: $y = \frac{1}{\sqrt{3}}x$ or $y = \frac{\sqrt{3}}{3}x$ or $3y = \sqrt{3}x$</p>	<p>Substitutes $t = \frac{\pi}{6}$ or 30° into their $\frac{dy}{dx}$ expression; M1</p> <p>to give any of the four underlined expressions oe A1 cso</p> <p>(must be correct solution only)</p> <p>Uses $m(\mathbf{T})$ to 'correctly' find $m(\mathbf{N})$. Can be ft from "their tangent gradient". A1 $\sqrt{\quad}$ oe.</p> <p>The point $(4\sqrt{3}, 4)$ or (awrt 6.9, 4) B1</p> <p>Finding an equation of a normal with their point and their normal gradient or finds c by using $y = (\text{their gradient})x + "c"$. M1</p> <p>Correct simplified EXACT equation of <u>normal</u>. This is dependent on candidate using correct $(4\sqrt{3}, 4)$ A1 oe</p> <p>[6] 9 marks</p>

Question Number	Scheme	Marks
<p>Aliter 3. (a) Way 2</p> <p>(b)</p>	<p>$x = 7 \cos t - \cos 7t$, $y = 7 \sin t - \sin 7t$,</p> <p>$\frac{dx}{dt} = -7 \sin t + 7 \sin 7t$, $\frac{dy}{dt} = 7 \cos t - 7 \cos 7t$</p> <p>$\frac{dy}{dx} = \frac{7 \cos t - 7 \cos 7t}{-7 \sin t + 7 \sin 7t} = \frac{-7(-2 \sin 4t \sin 3t)}{-7(2 \cos 4t \sin 3t)} = \tan 4t$</p> <p>When $t = \frac{\pi}{6}$, $m(\mathbf{T}) = \frac{dy}{dx} = \tan \frac{4\pi}{6}$;</p> <p>$= \frac{2\left(\frac{\sqrt{3}}{2}\right)(1)}{2\left(-\frac{1}{2}\right)(1)} = -\sqrt{3} = \underline{\text{awrt } -1.73}$</p> <p>Hence $m(\mathbf{N}) = \frac{-1}{-\sqrt{3}}$ or $\frac{1}{\sqrt{3}} = \underline{\text{awrt } 0.58}$</p> <p>When $t = \frac{\pi}{6}$,</p> <p>$x = 7 \cos \frac{\pi}{6} - \cos \frac{7\pi}{6} = \frac{7\sqrt{3}}{2} - \left(-\frac{\sqrt{3}}{2}\right) = \frac{8\sqrt{3}}{2} = 4\sqrt{3}$</p> <p>$y = 7 \sin \frac{\pi}{6} - \sin \frac{7\pi}{6} = \frac{7}{2} - \left(-\frac{1}{2}\right) = \frac{8}{2} = 4$</p> <p>N: $y - 4 = \frac{1}{\sqrt{3}}(x - 4\sqrt{3})$</p> <p>N: $y = \frac{1}{\sqrt{3}}x$ or $y = \frac{\sqrt{3}}{3}x$ or $3y = \sqrt{3}x$</p> <p>or $4 = \frac{1}{\sqrt{3}}(4\sqrt{3}) + c \Rightarrow c = 4 - 4 = 0$</p> <p>Hence N: $y = \frac{1}{\sqrt{3}}x$ or $y = \frac{\sqrt{3}}{3}x$ or $3y = \sqrt{3}x$</p>	<p>Attempt to differentiate x and y with respect to t to give $\frac{dx}{dt}$ in the form $\pm A \sin t \pm B \sin 7t$</p> <p>$\frac{dy}{dt}$ in the form $\pm C \cos t \pm D \cos 7t$</p> <p>Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$</p> <p>Candidate's $\frac{dy}{dx}$</p> <p>Substitutes $t = \frac{\pi}{6}$ or 30° into their $\frac{dy}{dx}$ expression;</p> <p>to give any of the three underlined expressions oe</p> <p>(must be correct solution only)</p> <p>Uses $m(\mathbf{T})$ to 'correctly' find $m(\mathbf{N})$. Can be ft from "their tangent gradient".</p> <p>The point $(4\sqrt{3}, 4)$ or <u>(awrt 6.9, 4)</u></p> <p>Finding an equation of a normal with their point and their normal gradient or finds c by using $y = (\text{their gradient})x + "c"$.</p> <p>Correct simplified EXACT equation of <u>normal</u>. This is dependent on candidate using correct $(4\sqrt{3}, 4)$</p> <p>[3]</p> <p>M1</p> <p>A1</p> <p>B1 $\sqrt{\quad}$</p> <p>M1</p> <p>A1 cso</p> <p>A1 $\sqrt{\quad}$ oe.</p> <p>B1</p> <p>M1</p> <p>A1 oe</p> <p>[6]</p> <p>9 marks</p>

Beware: A candidate finding an $m(\mathbf{T}) = 0$ can obtain A1 ft for $m(\mathbf{N}) \rightarrow \infty$, but obtains M0 if they write $y - 4 = \infty(x - 4\sqrt{3})$. If they write, however, $\mathbf{N}: x = 4\sqrt{3}$, then they can score M1.

Beware: A candidate finding an $m(\mathbf{T}) = \infty$ can obtain A1 ft for $m(\mathbf{N}) = 0$, and also obtains M1 if they write $y - 4 = 0(x - 4\sqrt{3})$ or $y = 4$.

Question Number	Scheme	Marks
4. (a)	$\frac{2x-1}{(x-1)(2x-3)} \equiv \frac{A}{x-1} + \frac{B}{2x-3}$ $2x-1 \equiv A(2x-3) + B(x-1)$ <p>Let $x = \frac{3}{2}$, $2 = B(\frac{1}{2}) \Rightarrow B = 4$</p> <p>Let $x = 1$, $1 = A(-1) \Rightarrow A = -1$</p> <p>giving $\frac{-1}{x-1} + \frac{4}{2x-3}$</p>	<p>Forming this identity. NB: A & B are not assigned in this question</p> <p>M1</p> <p>either one of $A = -1$ or $B = 4$. both correct for their A, B.</p> <p>A1 A1</p> <p>[3]</p>
(b) & (c)	$\int \frac{dy}{y} = \int \frac{(2x-1)}{(2x-3)(x-1)} dx$ $= \int \frac{-1}{x-1} + \frac{4}{2x-3} dx$ $\therefore \ln y = -\ln(x-1) + 2\ln(2x-3) + c$ <p>$y = 10, x = 2$ gives $c = \ln 10$</p> $\therefore \ln y = -\ln(x-1) + 2\ln(2x-3) + \ln 10$ $\ln y = -\ln(x-1) + \ln(2x-3)^2 + \ln 10$ $\ln y = \ln\left(\frac{(2x-3)^2}{x-1}\right) + \ln 10 \text{ or}$ $\ln y = \ln\left(\frac{10(2x-3)^2}{x-1}\right)$ $y = \frac{10(2x-3)^2}{x-1}$	<p>Separates variables as shown Can be implied</p> <p>B1</p> <p>Replaces RHS with their partial fraction to be integrated.</p> <p>M1 $\sqrt{\quad}$</p> <p><i>At least</i> two terms in ln's <i>At least</i> two ln terms correct All three terms correct and '+ c'</p> <p>M1 A1 $\sqrt{\quad}$ A1</p> <p>[5]</p> <p>$c = \ln 10$</p> <p>B1</p> <p>Using the power law for logarithms</p> <p>M1</p> <p>Using the product and/or quotient laws for logarithms to obtain a single RHS logarithmic term with/without constant c.</p> <p>M1</p> <p>$y = \frac{10(2x-3)^2}{x-1}$ or aef. isw</p> <p>A1 aef</p> <p>[4]</p>
		12 marks

Question Number	Scheme	Marks
<p><i>Aliter</i></p> <p>4. (b) & (c) Way 2</p>	$\int \frac{dy}{y} = \int \frac{(2x-1)}{(2x-3)(x-1)} dx$ $= \int \frac{-1}{(x-1)} + \frac{4}{(2x-3)} dx$ <p>$\therefore \ln y = -\ln(x-1) + 2\ln(2x-3) + c$</p> <p><i>See below for the award of B1</i></p> $\ln y = -\ln(x-1) + \ln(2x-3)^2 + c$ $\ln y = \ln\left(\frac{(2x-3)^2}{x-1}\right) + c$ $\ln y = \ln\left(\frac{A(2x-3)^2}{x-1}\right) \quad \text{where } c = \ln A$ <p>or $e^{\ln y} = e^{\ln\left(\frac{(2x-3)^2}{x-1}\right) + c} = e^{\ln\left(\frac{(2x-3)^2}{x-1}\right)} e^c$</p> $y = \frac{A(2x-3)^2}{(x-1)}$ <p>$y = 10, x = 2$ gives $A = 10$</p> $\underline{y = \frac{10(2x-3)^2}{(x-1)}}$	<p>Separates variables as shown Can be implied</p> <p>B1</p> <p>Replaces RHS with their partial fraction to be integrated.</p> <p>M1 $\sqrt{\quad}$</p> <p><i>At least</i> two terms in ln's <i>At least</i> two ln terms correct All three terms correct and '+ c'</p> <p>M1 A1 $\sqrt{\quad}$ A1</p> <p><i>decide to award B1 here!!</i></p> <p>B1</p> <p>Using the power law for logarithms</p> <p>M1</p> <p>Using the product and/or quotient laws for logarithms to obtain a single RHS logarithmic term with/without constant c.</p> <p>M1</p> <p>$A = 10$ for B1</p> <p>award above</p> <p>$y = \frac{10(2x-3)^2}{(x-1)}$ or aef & isw</p> <p>A1 aef</p> <p>[5] & [4]</p>

Note: The B1 mark (part (c)) should be awarded in the same place on ePEN as in the Way 1 approach.

Question Number	Scheme	Marks
<p><i>Aliter</i></p> <p>(b) & (c)</p> <p>Way 3</p>	$\int \frac{dy}{y} = \int \frac{(2x-1)}{(2x-3)(x-1)} dx$ $= \int \frac{-1}{(x-1)} + \frac{2}{(x-\frac{3}{2})} dx$ $\therefore \ln y = -\ln(x-1) + 2\ln(x-\frac{3}{2}) + c$ <p>$y = 10, x = 2$ gives $c = \ln 10 - 2\ln(\frac{1}{2}) = \ln 40$</p> $\therefore \ln y = -\ln(x-1) + 2\ln(x-\frac{3}{2}) + \ln 40$ $\ln y = -\ln(x-1) + \ln(x-\frac{3}{2})^2 + \ln 40$ $\ln y = \ln\left(\frac{(x-\frac{3}{2})^2}{(x-1)}\right) + \ln 40 \quad \text{or}$ $\ln y = \ln\left(\frac{40(x-\frac{3}{2})^2}{(x-1)}\right)$ $y = \frac{40(x-\frac{3}{2})^2}{(x-1)}$	<p>Separates variables as shown Can be implied B1</p> <p>Replaces RHS with their partial fraction to be integrated. M1 $\sqrt{\quad}$</p> <p><i>At least</i> two terms in ln's M1 <i>At least</i> two ln terms correct A1 $\sqrt{\quad}$ All three terms correct and '+ c' A1</p> <p>[5]</p> <p>$c = \ln 10 - 2\ln(\frac{1}{2})$ or $c = \ln 40$ B1 oe</p> <p>Using the power law for logarithms M1</p> <p>Using the product and/or quotient laws for logarithms to obtain a single RHS logarithmic term with/without constant c. M1</p> <p>$y = \frac{40(x-\frac{3}{2})^2}{(x-1)}$ or aef. isw A1 aef</p> <p>[4]</p>

Note: Please mark parts (b) and (c) together for any of the three ways.

Question Number	Scheme	Marks
5. (a)	<p style="text-align: center;">$\sin x + \cos y = 0.5$ (eqn *)</p> <p>$\left\{ \begin{array}{l} \cancel{\frac{dy}{dx}} \\ \cancel{\frac{dy}{dx}} \end{array} \right\} \times \cos x - \sin y \frac{dy}{dx} = 0$ (eqn #)</p> <p style="text-align: center;">$\frac{dy}{dx} = \frac{\cos x}{\sin y}$</p> <p>(b) $\frac{dy}{dx} = 0 \Rightarrow \frac{\cos x}{\sin y} = 0 \Rightarrow \cos x = 0$</p> <p>giving $x = -\frac{\pi}{2}$ or $x = \frac{\pi}{2}$</p> <p>When $x = -\frac{\pi}{2}$, $\sin(-\frac{\pi}{2}) + \cos y = 0.5$ When $x = \frac{\pi}{2}$, $\sin(\frac{\pi}{2}) + \cos y = 0.5$</p> <p>$\Rightarrow \cos y = 1.5 \Rightarrow y$ has no solutions $\Rightarrow \cos y = -0.5 \Rightarrow y = \frac{2\pi}{3}$ or $-\frac{2\pi}{3}$</p> <p>In specified range $(x, y) = (\frac{\pi}{2}, \frac{2\pi}{3})$ and $(\frac{\pi}{2}, -\frac{2\pi}{3})$</p>	<p>Differentiates implicitly to include $\pm \sin y \frac{dy}{dx}$. (Ignore $(\frac{dy}{dx} =)$.)</p> <p>M1</p> <p>A1 cso</p> <p>[2]</p> <p>Candidate realises that they need to solve 'their numerator' = 0 ...or candidate sets $\frac{dy}{dx} = 0$ in their (eqn #) and attempts to solve the resulting equation.</p> <p>M1 \sqrt</p> <p>A1</p> <p>both $x = -\frac{\pi}{2}, \frac{\pi}{2}$ or $x = \pm 90^\circ$ or awrt $x = \pm 1.57$ required here</p> <p>Substitutes either their $x = \frac{\pi}{2}$ or $x = -\frac{\pi}{2}$ into eqn *</p> <p>M1</p> <p>A1</p> <p>Only one of $y = \frac{2\pi}{3}$ or $-\frac{2\pi}{3}$ or 120° or -120° or awrt -2.09 or awrt 2.09</p> <p>A1</p> <p>Only exact coordinates of $(\frac{\pi}{2}, \frac{2\pi}{3})$ and $(\frac{\pi}{2}, -\frac{2\pi}{3})$</p> <p>A1</p> <p>Do not award this mark if candidate states other coordinates inside the required range.</p> <p>[5]</p> <p>7 marks</p>

Question Number	Scheme	Marks
<p>6.</p> <p>(a)</p> <p>Way 1</p> <p><i>Aliter</i></p> <p>(a)</p> <p>Way 2</p>	$y = 2^x = e^{x \ln 2}$ $\frac{dy}{dx} = \ln 2 \cdot e^{x \ln 2}$ <p>Hence $\frac{dy}{dx} = \ln 2 \cdot (2^x) = 2^x \ln 2$ AG</p>	$\frac{dy}{dx} = \ln 2 \cdot e^{x \ln 2}$ $2^x \ln 2$ AG A1 cso [2]
	<p>$\ln y = \ln(2^x)$ leads to $\ln y = x \ln 2$</p> $\frac{1}{y} \frac{dy}{dx} = \ln 2$ <p>Hence $\frac{dy}{dx} = y \ln 2 = 2^x \ln 2$ AG</p>	<div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <p>Takes logs of both sides, then uses the power law of logarithms...</p> <p>... and differentiates implicitly to give $\frac{1}{y} \frac{dy}{dx} = \ln 2$</p> </div> $2^x \ln 2$ AG A1 cso [2]
<p>(b)</p>	$y = 2^{(x^2)} \Rightarrow \frac{dy}{dx} = 2x \cdot 2^{(x^2)} \cdot \ln 2$ <p>When $x = 2$, $\frac{dy}{dx} = 2(2)2^4 \ln 2$</p> $\frac{dy}{dx} = \underline{64 \ln 2} = 44.3614\dots$	$Ax 2^{(x^2)}$ $2x \cdot 2^{(x^2)} \cdot \ln 2$ or $2x \cdot y \cdot \ln 2$ if y is defined Substitutes $x = 2$ into their $\frac{dy}{dx}$ which is of the form $\pm k 2^{(x^2)}$ or $Ax 2^{(x^2)}$ $\underline{64 \ln 2}$ or awrt 44.4 A1 [4]
6 marks		

Question Number	Scheme	Marks
<p><i>Aliter</i></p> <p>6. (b)</p> <p>Way 2</p>	<p>$\ln y = \ln(2^{x^2})$ leads to $\ln y = x^2 \ln 2$</p> <p>$\frac{1}{y} \frac{dy}{dx} = 2x \ln 2$</p> <p>When $x = 2$, $\frac{dy}{dx} = 2(2)2^4 \ln 2$</p> <p>$\frac{dy}{dx} = \underline{64 \ln 2} = 44.3614\dots$</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[4]</p>

Question Number	Scheme	Marks
7.	$\mathbf{a} = \overline{OA} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k} \Rightarrow \overline{OA} = 3$ $\mathbf{b} = \overline{OB} = \mathbf{i} + \mathbf{j} - 4\mathbf{k} \Rightarrow \overline{OB} = \sqrt{18}$ $\overline{BC} = \pm(2\mathbf{i} + 2\mathbf{j} + \mathbf{k}) \Rightarrow \overline{BC} = 3$ $\overline{AC} = \pm(\mathbf{i} + \mathbf{j} - 4\mathbf{k}) \Rightarrow \overline{AC} = \sqrt{18}$	
(a)	$\mathbf{c} = \overline{OC} = \underline{3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}}$	$\underline{3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}}$ <p>B1 cao [1]</p>
(b)	$\overline{OA} \bullet \overline{OB} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 1 \\ -4 \end{pmatrix} = \underline{2+2-4} = 0 \quad \text{or...}$ $\overline{BO} \bullet \overline{BC} = \begin{pmatrix} -1 \\ -1 \\ 4 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} = \underline{-2-2+4} = 0 \quad \text{or...}$ $\overline{AC} \bullet \overline{BC} = \begin{pmatrix} 1 \\ 1 \\ -4 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} = \underline{2+2-4} = 0 \quad \text{or...}$ $\overline{AO} \bullet \overline{AC} = \begin{pmatrix} -2 \\ -2 \\ -1 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 1 \\ -4 \end{pmatrix} = \underline{-2-2+4} = 0$ <p>and therefore OA is perpendicular to OB and hence OACB is a rectangle.</p> $\text{Area} = 3 \times \sqrt{18} = 3\sqrt{18} = 9\sqrt{2}$	<p>An attempt to take the dot product between either \overline{OA} and \overline{OB}, \overline{OA} and \overline{AC}, \overline{AC} and \overline{BC} or \overline{OB} and \overline{BC}</p> <p>M1</p> <p>Showing the result is equal to zero. A1</p> <p><u>perpendicular and OACB is a rectangle</u> A1 cso</p> <p>Using distance formula to find either the correct height or width. M1</p> <p>Multiplying the rectangle's height by its width. M1</p> <p>exact value of $3\sqrt{18}$, $9\sqrt{2}$, $\sqrt{162}$ or aef A1</p> <p>[6]</p>
(c)	$\overline{OD} = \mathbf{d} = \frac{1}{2}(3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k})$	$\underline{\frac{1}{2}(3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k})}$ <p>B1 [1]</p>

Question Number	Scheme	Marks
<p>(d)</p> <p>Way 1</p> <p><i>Aliter</i></p> <p>(d)</p> <p>Way 2</p>	<p><i>using dot product formula</i></p> $\overline{DA} = \pm \left(\frac{1}{2}\mathbf{i} + \frac{1}{2}\mathbf{j} + \frac{5}{2}\mathbf{k} \right) \quad \& \quad \overline{DC} = \pm \left(\frac{3}{2}\mathbf{i} + \frac{3}{2}\mathbf{j} - \frac{3}{2}\mathbf{k} \right)$ <p>or $\overline{BA} = \pm (\mathbf{i} + \mathbf{j} + 5\mathbf{k}) \quad \& \quad \overline{OC} = \pm (3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k})$</p> $\cos D = (\pm) \frac{\begin{pmatrix} 0.5 \\ 0.5 \\ 2.5 \end{pmatrix} \cdot \begin{pmatrix} 1.5 \\ 1.5 \\ -1.5 \end{pmatrix}}{\frac{\sqrt{27}}{2} \cdot \frac{\sqrt{27}}{2}} = (\pm) \frac{\frac{3}{4} + \frac{3}{4} - \frac{15}{4}}{\frac{27}{4}} = (\pm) \frac{1}{3}$ $D = \cos^{-1}\left(-\frac{1}{3}\right)$ $D = 109.47122\dots^\circ$ <p><i>using dot product formula and direction vectors</i></p> $d\overline{BA} = \pm (\mathbf{i} + \mathbf{j} + 5\mathbf{k}) \quad \& \quad d\overline{OC} = \pm (\mathbf{i} + \mathbf{j} - \mathbf{k})$ $\cos D = (\pm) \frac{\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix}}{\sqrt{3} \cdot \sqrt{27}} = (\pm) \frac{1+1-5}{\sqrt{3} \cdot \sqrt{27}} = (\pm) \frac{1}{3}$ $D = \cos^{-1}\left(-\frac{1}{3}\right)$ $D = 109.47122\dots^\circ$	<p>Identifies a set of two relevant vectors Correct vectors \pm</p> <p>Applies dot product formula on multiples of these vectors. <u>Correct ft. application of dot product formula.</u></p> <p>Attempts to find the correct angle D rather than $180^\circ - D$.</p> <p>109.5° or awrt 109° or 1.91°</p> <p>Identifies a set of two direction vectors Correct vectors \pm</p> <p>Applies dot product formula on multiples of these vectors. <u>Correct ft. application of dot product formula.</u></p> <p>Attempts to find the correct angle D rather than $180^\circ - D$.</p> <p>109.5° or awrt 109° or 1.91°</p>
	<p>M1</p> <p>A1</p> <p>dM1</p> <p>A1 $\sqrt{\quad}$</p> <p>ddM1 $\sqrt{\quad}$</p> <p>A1</p> <p>[6]</p> <p>M1</p> <p>A1</p> <p>dM1</p> <p>A1 $\sqrt{\quad}$</p> <p>ddM1 $\sqrt{\quad}$</p> <p>A1</p> <p>[6]</p>	

Question Number	Scheme	Marks
<p><i>Aliter</i></p> <p>(d)</p> <p>Way 3</p>	<p><i>using dot product formula and similar triangles</i></p> <p>$d\overline{OA} = (2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ & $d\overline{OC} = (\mathbf{i} + \mathbf{j} - \mathbf{k})$</p> $\cos\left(\frac{1}{2}D\right) = \frac{\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}}{\sqrt{9} \cdot \sqrt{3}} = \frac{2 + 2 - 1}{\sqrt{9} \cdot \sqrt{3}} = \frac{1}{\sqrt{3}}$ $D = 2 \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ $D = 109.47122\dots^\circ$	<p>Identifies a set of two direction vectors Correct vectors M1 A1</p> <p>Applies dot product formula on multiples of these vectors. <u>Correct ft. application of dot product formula.</u> dM1 A1 $\sqrt{}$</p> <p>Attempts to find the correct angle D by doubling their angle for $\frac{1}{2}D$. 109.5° or awrt 109° or 1.91° ddM1 $\sqrt{}$ A1</p> <p>[6]</p>
<p><i>Aliter</i></p> <p>(d)</p> <p>Way 4</p>	<p><i>using cosine rule</i></p> <p>$\overline{DA} = \frac{1}{2}\mathbf{i} + \frac{1}{2}\mathbf{j} + \frac{5}{2}\mathbf{k}$, $\overline{DC} = \frac{3}{2}\mathbf{i} + \frac{3}{2}\mathbf{j} - \frac{3}{2}\mathbf{k}$, $\overline{AC} = \mathbf{i} + \mathbf{j} - 4\mathbf{k}$</p> $ \overline{DA} = \frac{\sqrt{27}}{2}, \quad \overline{DC} = \frac{\sqrt{27}}{2}, \quad \overline{AC} = \sqrt{18}$ $\cos D = \frac{\left(\frac{\sqrt{27}}{2}\right)^2 + \left(\frac{\sqrt{27}}{2}\right)^2 - (\sqrt{18})^2}{2\left(\frac{\sqrt{27}}{2}\right)\left(\frac{\sqrt{27}}{2}\right)} = -\frac{1}{3}$ $D = \cos^{-1}\left(-\frac{1}{3}\right)$ $D = 109.47122\dots^\circ$	<p>Attempts to find all the lengths of all three edges of $\triangle ADC$ All Correct M1 A1</p> <p>Using the cosine rule formula with correct 'subtraction'. <u>Correct ft application of the cosine rule formula</u> dM1 A1 $\sqrt{}$</p> <p>Attempts to find the correct angle D rather than $180^\circ - D$. 109.5° or awrt 109° or 1.91° ddM1 $\sqrt{}$ A1</p> <p>[6]</p>

Question Number	Scheme	Marks
<p>Aliter (d) Way 5</p>	<p><i>using trigonometry on a right angled triangle</i> $\overline{DA} = \frac{1}{2}\mathbf{i} + \frac{1}{2}\mathbf{j} + \frac{5}{2}\mathbf{k}$ $\overline{OA} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ $\overline{AC} = \mathbf{i} + \mathbf{j} - 4\mathbf{k}$</p> <p>Let X be the midpoint of AC</p> <p>$\overline{DA} = \frac{\sqrt{27}}{2}$, $\overline{DX} = \frac{1}{2} \overline{OA} = \frac{3}{2}$, $\overline{AX} = \frac{1}{2} \overline{AC} = \frac{1}{2}\sqrt{18}$ (hypotenuse), (adjacent) , (opposite)</p> <p>$\sin(\frac{1}{2}D) = \frac{\frac{\sqrt{18}}{2}}{\frac{\sqrt{27}}{2}}$, $\cos(\frac{1}{2}D) = \frac{\frac{3}{2}}{\frac{\sqrt{27}}{2}}$ or $\tan(\frac{1}{2}D) = \frac{\frac{\sqrt{18}}{2}}{\frac{3}{2}}$</p> <p>eg. $D = 2 \tan^{-1}\left(\frac{\frac{\sqrt{18}}{2}}{\frac{3}{2}}\right)$</p> <p>$D = 109.47122\dots^\circ$</p>	<p>Attempts to find two out of the three lengths in $\triangle ADX$ M1</p> <p>Any two correct A1</p> <p>Uses correct sohcahtoa to find $\frac{1}{2}D$ dM1</p> <p>Correct ft application of sohcahtoa A1 $\sqrt{}$</p> <p>Attempts to find the correct angle D by doubling their angle for $\frac{1}{2}D$. ddM1 $\sqrt{}$</p> <p>109.5° or awrt 109° or 1.91° A1</p> <p>[6]</p>
<p>Aliter (d) Way 6</p>	<p><i>using trigonometry on a right angled similar triangle OAC</i> $\overline{OC} = 3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$ $\overline{OA} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ $\overline{AC} = \mathbf{i} + \mathbf{j} - 4\mathbf{k}$</p> <p>$\overline{OC} = \sqrt{27}$, $\overline{OA} = 3$, $\overline{AC} = \sqrt{18}$ (hypotenuse), (adjacent), (opposite)</p> <p>$\sin(\frac{1}{2}D) = \frac{\sqrt{18}}{\sqrt{27}}$, $\cos(\frac{1}{2}D) = \frac{3}{\sqrt{27}}$ or $\tan(\frac{1}{2}D) = \frac{\sqrt{18}}{3}$</p> <p>eg. $D = 2 \tan^{-1}\left(\frac{\sqrt{18}}{3}\right)$</p> <p>$D = 109.47122\dots^\circ$</p>	<p>Attempts to find two out of the three lengths in $\triangle OAC$ M1</p> <p>Any two correct A1</p> <p>Uses correct sohcahtoa to find $\frac{1}{2}D$ dM1</p> <p>Correct ft application of sohcahtoa A1 $\sqrt{}$</p> <p>Attempts to find the correct angle D by doubling their angle for $\frac{1}{2}D$. ddM1 $\sqrt{}$</p> <p>109.5° or awrt 109° or 1.91° A1</p> <p>[6]</p>

Question Number	Scheme	Marks
<p><i>Aliter</i></p> <p>7. (b) (i)</p> <p>Way 2</p> <p><i>Aliter</i></p> <p>7. (b) (i)</p> <p>Way 3</p>	<p>$\mathbf{c} = \overline{OC} = \pm(3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k})$ $\overline{AB} = \pm(-\mathbf{i} - \mathbf{j} - 5\mathbf{k})$</p> <p>$\overline{OC} = \sqrt{(3)^2 + (3)^2 + (-3)^2} = \sqrt{(1)^2 + (1)^2 + (-5)^2} = \overline{AB}$</p> <p>As $\overline{OC} = \overline{AB} = \sqrt{27}$</p> <p>then the <u>diagonals are equal</u>, and OACB is a <u>rectangle</u>.</p> <p>$\mathbf{a} = \overline{OA} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k} \Rightarrow \overline{OA} = 3$ $\mathbf{b} = \overline{OB} = \mathbf{i} + \mathbf{j} - 4\mathbf{k} \Rightarrow \overline{OB} = \sqrt{18}$ $\overline{BC} = \pm(2\mathbf{i} + 2\mathbf{j} + \mathbf{k}) \Rightarrow \overline{BC} = 3$ $\overline{AC} = \pm(\mathbf{i} + \mathbf{j} - 4\mathbf{k}) \Rightarrow \overline{AC} = \sqrt{18}$ $\mathbf{c} = \overline{OC} = \pm(3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}) \Rightarrow \overline{OC} = \sqrt{27}$ $\overline{AB} = \pm(-\mathbf{i} - \mathbf{j} - 5\mathbf{k}) \Rightarrow \overline{AB} = \sqrt{27}$</p> <p>$(OA)^2 + (AC)^2 = (OC)^2$ or $(BC)^2 + (OB)^2 = (OC)^2$ or equivalent or $(OA)^2 + (OB)^2 = (AB)^2$ or $(BC)^2 + (AC)^2 = (AB)^2$</p> <p>$\Rightarrow \underline{(3)^2 + (\sqrt{18})^2 = (\sqrt{27})^2}$</p> <p>and therefore OA is <u>perpendicular</u> to OB or AC is <u>perpendicular</u> to BC and hence <u>OACB is a rectangle</u>.</p>	<p>A complete method of proving that the diagonals are equal. M1</p> <p>Correct result. A1</p> <p><u>diagonals are equal</u> and <u>OACB is a rectangle</u> A1 cso</p> <p>[3]</p> <p>A complete method of proving that Pythagoras holds using their values. M1</p> <p><u>Correct result</u> A1</p> <p><u>perpendicular</u> and <u>OACB is a rectangle</u> A1 cso</p> <p>[3]</p> <p>14 marks</p>

Question Number	Scheme	Marks																					
8. (a)	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding: 2px 10px;">x</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">1</td> <td style="padding: 2px 10px;">2</td> <td style="padding: 2px 10px;">3</td> <td style="padding: 2px 10px;">4</td> <td style="padding: 2px 10px;">5</td> </tr> <tr> <td style="padding: 2px 10px;">y</td> <td style="padding: 2px 10px;">e^1</td> <td style="padding: 2px 10px;">e^2</td> <td style="padding: 2px 10px;">$e^{\sqrt{7}}$</td> <td style="padding: 2px 10px;">$e^{\sqrt{10}}$</td> <td style="padding: 2px 10px;">$e^{\sqrt{13}}$</td> <td style="padding: 2px 10px;">e^4</td> </tr> <tr> <td style="padding: 2px 10px;">or y</td> <td style="padding: 2px 10px;">2.71828...</td> <td style="padding: 2px 10px;">7.38906...</td> <td style="padding: 2px 10px;">14.09403...</td> <td style="padding: 2px 10px;">23.62434...</td> <td style="padding: 2px 10px;">36.80197...</td> <td style="padding: 2px 10px;">54.59815...</td> </tr> </table> <p style="text-align: right; margin-right: 20px;"> Either $e^{\sqrt{7}}$, $e^{\sqrt{10}}$ and $e^{\sqrt{13}}$ or awrt 14.1, 23.6 and 36.8 or e to the power awrt 2.65, 3.16, 3.61 (or mixture of decimals and e's) At least two correct All three correct </p>	x	0	1	2	3	4	5	y	e^1	e^2	$e^{\sqrt{7}}$	$e^{\sqrt{10}}$	$e^{\sqrt{13}}$	e^4	or y	2.71828...	7.38906...	14.09403...	23.62434...	36.80197...	54.59815...	B1 B1 [2]
x	0	1	2	3	4	5																	
y	e^1	e^2	$e^{\sqrt{7}}$	$e^{\sqrt{10}}$	$e^{\sqrt{13}}$	e^4																	
or y	2.71828...	7.38906...	14.09403...	23.62434...	36.80197...	54.59815...																	
8. (b)	$I \approx \frac{1}{2} \times 1; \times \left\{ e^1 + 2(e^2 + e^{\sqrt{7}} + e^{\sqrt{10}} + e^{\sqrt{13}}) + e^4 \right\}$ $= \frac{1}{2} \times 221.1352227... = 110.5676113... = \underline{110.6} \text{ (4sf)}$	<p style="text-align: right;"> Outside brackets $\frac{1}{2} \times 1$ <u>For structure of trapezium</u> <u>rule {.....} ;</u> M1 $\sqrt{\quad}$ A1 cao [3] </p>																					

Beware: In part (b) candidates can add up the individual trapezia:

$$(b) I \approx \frac{1}{2} \cdot 1(e^1 + e^2) + \frac{1}{2} \cdot 1(e^2 + e^{\sqrt{7}}) + \frac{1}{2} \cdot 1(e^{\sqrt{7}} + e^{\sqrt{10}}) + \frac{1}{2} \cdot 1(e^{\sqrt{10}} + e^{\sqrt{13}}) + \frac{1}{2} \cdot 1(e^{\sqrt{13}} + e^4)$$

Question Number	Scheme	Marks
(c)	$t = (3x + 1)^{\frac{1}{2}} \Rightarrow \frac{dt}{dx} = \frac{1}{2} \cdot 3 \cdot (3x + 1)^{-\frac{1}{2}}$ $\dots \text{ or } t^2 = 3x + 1 \Rightarrow \underline{2t \frac{dt}{dx} = 3}$ $\text{so } \frac{dt}{dx} = \frac{3}{2 \cdot (3x + 1)^{\frac{1}{2}}} = \frac{3}{2t} \Rightarrow \frac{dx}{dt} = \frac{2t}{3}$ $\therefore I = \int e^{\sqrt{3x+1}} dx = \int e^t \frac{dx}{dt} \cdot dt = \int e^t \cdot \frac{2t}{3} \cdot dt$ $\therefore I = \int \frac{2}{3} t e^t dt$ <p>change limits: when $x = 0$, $t = 1$ & when $x = 5$, $t = 4$</p> <p>Hence $I = \int_1^4 \frac{2}{3} t e^t dt$; where $a = 1$, $b = 4$, $k = \frac{2}{3}$</p>	<p>A1</p> <p>A1</p> <div style="border: 1px solid black; padding: 5px; margin: 5px 0;"> <p>Candidate obtains either $\frac{dt}{dx}$ or $\frac{dx}{dt}$ in terms of t ...</p> <p>... and moves on to substitute this into I to convert an integral wrt x to an integral wrt t.</p> </div> <p>dM1</p> <p>A1</p> <p>B1</p> <p>[5]</p>
(d)	$\left\{ \begin{array}{l} u = t \Rightarrow \frac{du}{dt} = 1 \\ \frac{dv}{dt} = e^t \Rightarrow v = e^t \end{array} \right\}$ $k \int t e^t dt = k \left(t e^t - \int e^t \cdot 1 dt \right)$ $= k \left(t e^t - e^t \right) + c$ $\therefore \int_1^4 \frac{2}{3} t e^t dt = \frac{2}{3} \left\{ (4e^4 - e^4) - (e^1 - e^1) \right\}$ $= \frac{2}{3} (3e^4) = \underline{2e^4} = 109.1963\dots$	<p>Let k be any constant for the first three marks of this part.</p> <p>Use of 'integration by parts' formula in the correct direction.</p> <p>Correct expression with a constant factor k.</p> <p><u>Correct integration</u> with/without a constant factor k</p> <p>Substitutes their changed limits into the integrand and subtracts oe.</p> <p>either $2e^4$ or awrt 109.2</p> <p>A1</p> <p>[5]</p> <p>15 marks</p>

- Note: dM1 denotes a method mark which is dependent upon the award of the previous method mark
- ddM1 denotes a method mark which is dependent upon the award of the previous two method marks.