January 2007
6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme		Marks
	** represents a constant		
1.	$f(x) = (2-5x)^{-2} = \underline{(2)^{-2}}\left(1-\frac{5x}{2}\right)^{-2} = \underline{\frac{1}{4}}\left(1-\frac{5x}{2}\right)^{-2}$	Takes 2 outside the bracket to give any of $(2)^{-2}$ or $\frac{1}{4}$.	B1
	$= \frac{1}{4}\left\{\underline{1+(-2)(**x);+\frac{(-2)(-3)}{2!}(**x)^2+\frac{(-2)(-3)(-4)}{3!}(**x)^3+...}\right\}$	Expands $(1+**x)^{-2}$ to give an unsimplified $1+(-2)(**x)$;	M1
		A correct unsimplified $\{\underline{..........}\}$ expansion with candidate's $(**x)$	A1
	$= \frac{1}{4}\left\{\underline{1+(-2)(\tfrac{-5x}{2});+\frac{(-2)(-3)}{2!}(\tfrac{-5x}{2})^2+\frac{(-2)(-3)(-4)}{3!}(\tfrac{-5x}{2})^3+...}\right\}$		
	$= \frac{1}{4}\left\{1+5x;+\frac{75x^2}{4}+\frac{125x^3}{2}+...\right\}$		
	$= \frac{1}{4}+\frac{5x}{4};+\frac{75x^2}{16}+\frac{125x^3}{8}+...$	Anything that cancels to $\frac{1}{4}+\frac{5x}{4}$;	A1;
		Simplified $\frac{75x^2}{16}+\frac{125x^3}{8}$	A1
	$= \frac{1}{4}+1\frac{1}{4}x;+4\frac{11}{16}x^2+15\frac{5}{8}x^3+...$		
			[5]
			5 marks

Question Number	Scheme		Marks
Aliter 1. **Way 2**	$f(x) = (2-5x)^{-2}$		
		$\frac{1}{4}$ or $(2)^{-2}$	B1
	$= \left\{ \begin{array}{l} (2)^{-2} + (-2)(2)^{-3}(**x); + \frac{(-2)(-3)}{2!}(2)^{-4}(**x)^2 \\ + \frac{(-2)(-3)(-4)}{3!}(2)^{-5}(**x)^3 + \ldots \end{array} \right\}$	Expands $(2-5x)^{-2}$ to give an unsimplifed $(2)^{-2} + (-2)(2)^{-3}(**x)$;	M1
		A correct unsimplified $\{\underline{\ldots\ldots\ldots}\}$ expansion with candidate's $(**x)$	A1
	$= \left\{ \begin{array}{l} (2)^{-2} + (-2)(2)^{-3}(-5x); + \frac{(-2)(-3)}{2!}(2)^{-4}(-5x)^2 \\ + \frac{(-2)(-3)(-4)}{3!}(2)^{-5}(-5x)^3 + \ldots \end{array} \right\}$		
	$= \left\{ \begin{array}{l} \frac{1}{4} + (-2)(\frac{1}{8})(-5x); + (3)(\frac{1}{16})(25x^2) \\ + (-4)(\frac{1}{16})(-125x^3) + \ldots \end{array} \right\}$		
	$= \frac{1}{4} + \frac{5x}{4}; + \frac{75x^2}{16} + \frac{125x^3}{8} + \ldots$	Anything that cancels to $\frac{1}{4} + \frac{5x}{4}$;	A1;
		Simplified $\frac{75x^2}{16} + \frac{125x^3}{8}$	A1
	$= \frac{1}{4} + 1\frac{1}{4}x; + 4\frac{11}{16}x^2 + 15\frac{5}{8}x^3 + \ldots$		
			[5]
			5 marks

Attempts using Maclaurin expansions need to be referred to your team leader.

Question Number	Scheme		Marks
2. (a)	Volume $= \pi \int_{-\frac{1}{4}}^{\frac{1}{2}} \left(\frac{1}{3(1+2x)}\right)^2 dx = \frac{\pi}{9}\int_{-\frac{1}{4}}^{\frac{1}{2}} \frac{1}{(1+2x)^2} dx$	**Use** of $V = \pi\int y^2 \, dx$. Can be implied. Ignore limits.	B1
	$= \left(\frac{\pi}{9}\right)\int_{-\frac{1}{4}}^{\frac{1}{2}} (1+2x)^{-2} \, dx$	Moving their power to the top. **(Do not allow power of -1.)** Can be implied. Ignore limits and $\frac{\pi}{9}$	M1
	$= \left(\frac{\pi}{9}\right)\left[\frac{(1+2x)^{-1}}{(-1)(2)}\right]_{-\frac{1}{4}}^{\frac{1}{2}}$	Integrating to give $\pm p(1+2x)^{-1}$ $-\frac{1}{2}(1+2x)^{-1}$	M1 A1
	$= \left(\frac{\pi}{9}\right)\left[-\frac{1}{2}(1+2x)^{-1}\right]_{-\frac{1}{4}}^{\frac{1}{2}}$		
	$= \left(\frac{\pi}{9}\right)\left[\left(\frac{-1}{2(2)}\right) - \left(\frac{-1}{2(\frac{1}{2})}\right)\right]$		
	$= \left(\frac{\pi}{9}\right)\left[-\frac{1}{4} - (-1)\right]$		
	$= \frac{\pi}{12}$	Use of limits to give exact values of $\frac{\pi}{12}$ or $\frac{3\pi}{36}$ or $\frac{2\pi}{24}$ or aef	A1 aef
			[5]
(b)	From Fig.1, AB $= \frac{1}{2} - \left(-\frac{1}{4}\right) = \frac{3}{4}$ units		
	As $\frac{3}{4}$ units $\equiv 3$cm		
	then scale factor $k = \frac{3}{\left(\frac{3}{4}\right)} = 4$.		
	Hence Volume of paperweight $= (4)^3\left(\frac{\pi}{12}\right)$	$(4)^3 \times$ (their answer to part (a))	M1
	$V = \frac{16\pi}{3}$ cm^3 $= 16.75516...$ cm^3	$\frac{16\pi}{3}$ or awrt 16.8 or $\frac{64\pi}{12}$ or aef	A1
			[2]
			7 marks

Note: $\frac{\pi}{9}$ (or implied) is not needed for the middle three marks of question 2(a).

Question Number	Scheme		Marks
Aliter 2. (a) **Way 2**	Volume $= \pi \underline{\int_{-\frac{1}{4}}^{\frac{1}{2}} \left(\frac{1}{3(1+2x)}\right)^2 dx} = \pi \int_{-\frac{1}{4}}^{\frac{1}{2}} \frac{1}{(3+6x)^2} dx$	**Use** of $V = \pi \underline{\int y^2\ dx}$. Can be implied. Ignore limits.	B1
	$= (\pi) \int_{-\frac{1}{4}}^{\frac{1}{2}} (3+6x)^{-2}\ dx$	Moving their power to the top. **(Do not allow power of -1.)** Can be implied. Ignore limits and π	M1
	$= (\pi)\left[\frac{(3+6x)^{-1}}{(-1)(6)}\right]_{-\frac{1}{4}}^{\frac{1}{2}}$	Integrating to give $\underline{\pm p(3+6x)^{-1}}$	M1
		$\underline{-\frac{1}{6}(3+6x)^{-1}}$	A1
	$= (\pi)\left[\underline{-\frac{1}{6}(3+6x)^{-1}}\right]_{-\frac{1}{4}}^{\frac{1}{2}}$		
	$= (\pi)\left[\left(\frac{-1}{6(6)}\right) - \left(\frac{-1}{6(\frac{3}{2})}\right)\right]$		
	$= (\pi)\left[-\frac{1}{36} - (-\frac{1}{9})\right]$		
	$= \frac{\pi}{12}$	Use of limits to give exact values of $\frac{\pi}{12}$ or $\frac{3\pi}{36}$ or $\frac{2\pi}{24}$ or aef	A1 aef
			[5]

Note: π is not needed for the middle three marks of question 2(a).

Question Number	Scheme		Marks
3. (a)	$x = 7\cos t - \cos 7t\,,\quad y = 7\sin t - \sin 7t\,,$		
	$\frac{dx}{dt} = -7\sin t + 7\sin 7t\,,\quad \frac{dy}{dt} = 7\cos t - 7\cos 7t$	Attempt to differentiate x **and** y with respect to t to give $\frac{dx}{dt}$ in the form $\pm A\sin t \pm B\sin 7t$ $\frac{dy}{dt}$ in the form $\pm C\cos t \pm D\cos 7t$	M1
		Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$	A1
	$\therefore \frac{dy}{dx} = \frac{7\cos t - 7\cos 7t}{-7\sin t + 7\sin 7t}$	Candidate's $\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$	B1 $\sqrt{}$
			[3]
(b)	When $t = \frac{\pi}{6}$, m(**T**) $= \frac{dy}{dx} = \frac{7\cos\frac{\pi}{6} - 7\cos\frac{7\pi}{6}}{-7\sin\frac{\pi}{6} + 7\sin\frac{7\pi}{6}}$;	Substitutes $t = \frac{\pi}{6}$ or 30° into their $\frac{dy}{dx}$ expression;	M1
	$= \underline{\frac{\frac{7\sqrt{3}}{2} - \left(-\frac{7\sqrt{3}}{2}\right)}{-\frac{7}{2} - \frac{7}{2}}} = \underline{\frac{7\sqrt{3}}{-7}} = \underline{-\sqrt{3}} = \underline{\text{awrt } -1.73}$	to give any of the four underlined expressions oe **(must be correct solution only)**	A1 **cso**
	Hence m(**N**) $= \frac{-1}{-\sqrt{3}}$ or $\frac{1}{\sqrt{3}}$ = awrt 0.58	Uses m(**T**) to 'correctly' find m(**N**). Can be ft from "their tangent gradient".	A1$\sqrt{}$ oe.
	When $t = \frac{\pi}{6}$, $x = 7\cos\frac{\pi}{6} - \cos\frac{7\pi}{6} = \frac{7\sqrt{3}}{2} - \left(-\frac{\sqrt{3}}{2}\right) = \frac{8\sqrt{3}}{2} = 4\sqrt{3}$ $y = 7\sin\frac{\pi}{6} - \sin\frac{7\pi}{6} = \frac{7}{2} - \left(-\frac{1}{2}\right) = \frac{8}{2} = 4$	The point $\underline{\left(4\sqrt{3},\, 4\right)}$ or $\underline{\left(\text{awrt } 6.9,\, 4\right)}$	B1
	N: $y - 4 = \frac{1}{\sqrt{3}}\left(x - 4\sqrt{3}\right)$	Finding an equation of a normal with their point and their normal gradient or finds c by using $y = (\text{their gradient})x + "c"$.	M1
	N: $\underline{y = \frac{1}{\sqrt{3}}x}$ or $\underline{y = \frac{\sqrt{3}}{3}x}$ or $\underline{3y = \sqrt{3}x}$	Correct simplified EXACT equation of <u>normal</u>. This is dependent on candidate using correct $\left(4\sqrt{3},\, 4\right)$	<u>A1</u> oe
	or $4 = \frac{1}{\sqrt{3}}\left(4\sqrt{3}\right) + c \Rightarrow c = 4 - 4 = 0$		
	Hence **N:** $\underline{y = \frac{1}{\sqrt{3}}x}$ or $\underline{y = \frac{\sqrt{3}}{3}x}$ or $\underline{3y = \sqrt{3}x}$		
			[6] **9 marks**

Question Number	Scheme		Marks
Aliter 3. (a) **Way 2**	$x = 7\cos t - \cos 7t\,,\ \ y = 7\sin t - \sin 7t\,,$		
	$\frac{dx}{dt} = -7\sin t + 7\sin 7t\,,\ \ \frac{dy}{dt} = 7\cos t - 7\cos 7t$	Attempt to differentiate x **and** y with respect to t to give $\frac{dx}{dt}$ in the form $\pm A\sin t \pm B\sin 7t$ $\frac{dy}{dt}$ in the form $\pm C\cos t \pm D\cos 7t$	M1
		Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$	A1
	$\frac{dy}{dx} = \frac{7\cos t - 7\cos 7t}{-7\sin t + 7\sin 7t} = \frac{-7(-2\sin 4t\sin 3t)}{-7(2\cos 4t\sin 3t)} = \tan 4t$	Candidate's $\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$	B1 √
			[3]
(b)	When $t = \frac{\pi}{6},\ \ m(\mathbf{T}) = \frac{dy}{dx} = \tan\frac{4\pi}{6}$;	Substitutes $t = \frac{\pi}{6}$ or 30° into their $\frac{dy}{dx}$ expression;	M1
	$= \underline{\frac{2\left(\frac{\sqrt{3}}{2}\right)(1)}{2\left(-\frac{1}{2}\right)(1)}} = \underline{-\sqrt{3}} = \underline{\text{awrt } -1.73}$	to give any of the three underlined expressions oe **(must be correct solution only)**	A1 **cso**
	Hence $m(\mathbf{N}) = \frac{-1}{-\sqrt{3}}$ or $\frac{1}{\sqrt{3}}$ = awrt 0.58	Uses $m(\mathbf{T})$ to 'correctly' find $m(\mathbf{N})$. Can be ft from "their tangent gradient".	A1√ oe.
	When $t = \frac{\pi}{6}$, $x = 7\cos\frac{\pi}{6} - \cos\frac{7\pi}{6} = \frac{7\sqrt{3}}{2} - \left(-\frac{\sqrt{3}}{2}\right) = \frac{8\sqrt{3}}{2} = 4\sqrt{3}$ $y = 7\sin\frac{\pi}{6} - \sin\frac{7\pi}{6} = \frac{7}{2} - \left(-\frac{1}{2}\right) = \frac{8}{2} = 4$	The point $\underline{\left(4\sqrt{3},\ 4\right)}$ or $\underline{(\text{awrt } 6.9,\ 4)}$	B1
	N: $y - 4 = \frac{1}{\sqrt{3}}\left(x - 4\sqrt{3}\right)$	Finding an equation of a normal with their point and their normal gradient or finds c by using $y = (\text{their gradient})x + "c"$.	M1
	N: $\underline{y = \frac{1}{\sqrt{3}}x}$ or $\underline{y = \frac{\sqrt{3}}{3}x}$ or $\underline{3y = \sqrt{3}x}$	Correct simplified EXACT equation of <u>normal</u>. This is dependent on candidate using correct $\left(4\sqrt{3},\ 4\right)$	<u>A1</u> oe
	or $4 = \frac{1}{\sqrt{3}}\left(4\sqrt{3}\right) + c \ \Rightarrow\ c = 4 - 4 = 0$		
	Hence **N:** $\underline{y = \frac{1}{\sqrt{3}}x}$ or $\underline{y = \frac{\sqrt{3}}{3}x}$ or $\underline{3y = \sqrt{3}x}$		
			[6]
			9 marks

Beware: A candidate finding an m(**T**) = 0 can obtain A1ft for m(**N**) $\rightarrow \infty$, but obtains M0 if they write $y - 4 = \infty(x - 4\sqrt{3})$. If they write, however, **N**: $x = 4\sqrt{3}$, then they can score M1.

Beware: A candidate finding an m(**T**) = ∞ can obtain A1ft for m(**N**) = 0, and also obtains M1 if they write $y - 4 = 0(x - 4\sqrt{3})$ or y = 4.

Question Number	Scheme		Marks
4. (a)	$\frac{2x-1}{(x-1)(2x-3)} \equiv \frac{A}{(x-1)} + \frac{B}{(2x-3)}$		
	$2x-1 \equiv A(2x-3)+B(x-1)$	Forming this identity. **NB**: A & B are not assigned in this question	M1
	Let $x=\frac{3}{2}$, $2 = B\left(\frac{1}{2}\right) \Rightarrow B=4$		
	Let $x=1$, $1 = A(-1) \Rightarrow A=-1$	either one of $A=-1$ or $B=4$. both correct for their A, B.	A1 A1
	giving $\frac{-1}{(x-1)} + \frac{4}{(2x-3)}$		**[3]**
(b) & (c)	$\int \frac{dy}{y} = \int \frac{(2x-1)}{(2x-3)(x-1)}\,dx$	Separates variables as shown Can be implied	B1
	$= \int \frac{-1}{(x-1)} + \frac{4}{(2x-3)}\,dx$	Replaces RHS with their partial fraction to be integrated.	M1 $\sqrt{}$
	$\therefore \ln y = -\ln(x-1) + 2\ln(2x-3) + c$	***At least*** two terms in ln's ***At least*** two ln terms correct All three terms correct and '+ c'	M1 A1 $\sqrt{}$ A1 **[5]**
	$y=10,\ x=2$ gives $c=\ln 10$	$c=\ln 10$	B1
	$\therefore \ln y = -\ln(x-1) + 2\ln(2x-3) + \ln 10$		
	$\ln y = -\ln(x-1) + \ln(2x-3)^2 + \ln 10$	Using the power law for logarithms	M1
	$\ln y = \ln\left(\frac{(2x-3)^2}{(x-1)}\right) + \ln 10$ or $\ln y = \ln\left(\frac{10(2x-3)^2}{(x-1)}\right)$	Using the product and/or quotient laws for logarithms to obtain a single RHS logarithmic term with/without constant c.	M1
	$y = \frac{10(2x-3)^2}{(x-1)}$	$y = \frac{10(2x-3)^2}{(x-1)}$ or aef. isw	A1 aef
			[4]
			12 marks

Question Number	Scheme		Marks
Aliter 4. (b) & (c) **Way 2**	$\int \frac{dy}{y} = \int \frac{(2x-1)}{(2x-3)(x-1)}\,dx$	Separates variables as shown Can be implied	B1
	$= \int \frac{-1}{(x-1)} + \frac{4}{(2x-3)}\,dx$	Replaces RHS with their partial fraction to be integrated.	M1 $\sqrt{}$
	$\therefore \ln y = -\ln(x-1) + 2\ln(2x-3) + c$	***At least*** two terms in ln's ***At least*** two ln terms correct All three terms correct and '+ c'	M1 A1 $\sqrt{}$ A1
	See below for the award of B1	***decide to award B1 here!!***	B1
	$\ln y = -\ln(x-1) + \ln(2x-3)^2 + c$	Using the power law for logarithms	M1
	$\ln y = \ln\left(\frac{(2x-3)^2}{x-1}\right) + c$	Using the product and/or quotient laws for logarithms to obtain a single RHS logarithmic term with/without constant c.	M1
	$\ln y = \ln\left(\frac{A(2x-3)^2}{x-1}\right)$ where $c = \ln A$		
	or $e^{\ln y} = e^{\ln\left(\frac{(2x-3)^2}{x-1}\right) + c} = e^{\ln\left(\frac{(2x-3)^2}{x-1}\right)} e^c$		
	$y = \frac{A(2x-3)^2}{(x-1)}$		
	$y = 10,\ x = 2$ gives $A = 10$	$A = 10$ for B1	award above
	$y = \frac{10(2x-3)^2}{(x-1)}$	$y = \frac{10(2x-3)^2}{(x-1)}$ or aef & isw	A1 aef
			[5] & [4]

Note: The B1 mark (part (c)) should be awarded in the same place on ePEN as in the Way 1 approach.

Question Number	Scheme		Marks
Aliter (b) & (c) **Way 3**	$\int \frac{dy}{y} = \int \frac{(2x-1)}{(2x-3)(x-1)}\,dx$	Separates variables as shown Can be implied	B1
	$= \int \frac{-1}{(x-1)} + \frac{2}{(x-\frac{3}{2})}\,dx$	Replaces RHS with their partial fraction to be integrated.	M1 $\sqrt{}$
	$\therefore \ln y = -\ln(x-1) + 2\ln(x-\frac{3}{2}) + c$	***At least*** two terms in ln's ***At least*** two ln terms correct All three terms correct and '+ c'	M1 A1 $\sqrt{}$ A1 **[5]**
	$y = 10,\ x = 2$ gives $c = \underline{\ln 10 - 2\ln\left(\frac{1}{2}\right)} = \underline{\ln 40}$	$c = \ln 10 - 2\ln\left(\frac{1}{2}\right)$ or $c = \ln 40$	B1 oe
	$\therefore \ln y = -\ln(x-1) + 2\ln(x-\frac{3}{2}) + \ln 40$		
	$\ln y = -\ln(x-1) + \ln(x-\frac{3}{2})^2 + \ln 10$	Using the power law for logarithms	M1
	$\ln y = \ln\left(\frac{(x-\frac{3}{2})^2}{(x-1)}\right) + \ln 40$ or $\ln y = \ln\left(\frac{40(x-\frac{3}{2})^2}{(x-1)}\right)$	Using the product and/or quotient laws for logarithms to obtain a single RHS logarithmic term with/without constant c.	M1
	$\underline{y = \frac{40(x-\frac{3}{2})^2}{(x-1)}}$	$\underline{y = \frac{40(x-\frac{3}{2})^2}{(x-1)}}$ or aef. isw	A1 aef **[4]**

Note: Please mark parts (b) and (c) together for any of the three ways.

Question Number	Scheme		Marks
5. (a)	$\sin x + \cos y = 0.5 \quad (\text{eqn } *)$		
	$\left\{\frac{dy}{dx} \times\right\} \quad \cos x - \sin y\frac{dy}{dx} = 0 \quad (\text{eqn } \#)$	Differentiates implicitly to include $\pm \sin y\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} =\right)$.)	M1
	$\frac{dy}{dx} = \frac{\cos x}{\sin y}$	$\frac{\cos x}{\sin y}$	A1 **cso**
			[2]
(b)	$\frac{dy}{dx} = 0 \Rightarrow \frac{\cos x}{\sin y} = 0 \Rightarrow \cos x = 0$	Candidate realises that they need to solve 'their numerator' = 0 …or candidate sets $\frac{dy}{dx} = 0$ in their (eqn #) and attempts to solve the resulting equation.	M1 $\sqrt{}$
	giving $x = -\frac{\pi}{2}$ or $x = \frac{\pi}{2}$	both $x = -\frac{\pi}{2}, \frac{\pi}{2}$ or $x = \pm 90^\circ$ or awrt $x = \pm 1.57$ required here	A1
	When $x = -\frac{\pi}{2}$, $\sin\left(-\frac{\pi}{2}\right) + \cos y = 0.5$ When $x = \frac{\pi}{2}$, $\sin\left(\frac{\pi}{2}\right) + \cos y = 0.5$	Substitutes either their $x = \frac{\pi}{2}$ or $x = -\frac{\pi}{2}$ into eqn $*$	M1
	$\Rightarrow \cos y = 1.5 \Rightarrow$ y has no solutions $\Rightarrow \cos y = -0.5 \Rightarrow y = \frac{2\pi}{3}$ or $-\frac{2\pi}{3}$	Only one of $y = \frac{2\pi}{3}$ or $-\frac{2\pi}{3}$ or 120° or -120° or awrt -2.09 or awrt 2.09	A1
	In specified range $(x, y) = \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and $\left(\frac{\pi}{2}, -\frac{2\pi}{3}\right)$	Only exact coordinates of $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and $\left(\frac{\pi}{2}, -\frac{2\pi}{3}\right)$ **Do not award this mark if candidate states other coordinates inside the required range.**	A1
			[5]
			7 marks

Question Number	Scheme		Marks
6.	$y = 2^x = e^{x\ln 2}$		
(a) **Way 1**	$\frac{dy}{dx} = \ln 2 . e^{x\ln 2}$	$\frac{dy}{dx} = \ln 2 . e^{x\ln 2}$	M1
	Hence $\frac{dy}{dx} = \ln 2.(2^x) = 2^x \ln 2$ **AG**	$2^x \ln 2$ **AG**	A1 **cso**
			[2]
Aliter (a) **Way 2**	$\ln y = \ln\left(2^x\right)$ leads to $\ln y = x\ln 2$ $\frac{1}{y}\frac{dy}{dx} = \ln 2$	Takes logs of both sides, then uses the power law of logarithms… … and differentiates implicitly to give $\frac{1}{y}\frac{dy}{dx} = \ln 2$	M1
	Hence $\frac{dy}{dx} = y\ln 2 = 2^x \ln 2$ **AG**	$2^x \ln 2$ **AG**	A1 **cso**
			[2]
(b)	$y = 2^{(x^2)} \Rightarrow \frac{dy}{dx} = 2x.\ 2^{(x^2)}.\ln 2$	$Ax\ 2^{(x^2)}$ $2x.\ 2^{(x^2)}.\ln 2$ or $2x.y.\ln 2$ if y is defined	M1 A1
	When x = 2, $\frac{dy}{dx} = 2(2)2^4 \ln 2$	Substitutes x = 2 into their $\frac{dy}{dx}$ which is of the form $\pm k\,2^{(x^2)}$ or $Ax\ 2^{(x^2)}$	M1
	$\frac{dy}{dx} = \underline{64\ln 2} = 44.3614...$	$\underline{64\ln 2}$ or awrt 44.4	A1
			[4]
			6 marks

Question Number	Scheme		Marks
Aliter 6. (b) **Way 2**	$\ln y = \ln\left(2^{x^2}\right)$ leads to $\ln y = x^2 \ln 2$		
	$\frac{1}{y}\frac{dy}{dx} = 2x.\ln 2$	$\frac{1}{y}\frac{dy}{dx} = Ax.\ln 2$	M1
		$\frac{1}{y}\frac{dy}{dx} = 2x.\ln 2$	A1
	When x = 2, $\frac{dy}{dx} = 2(2)2^4 \ln 2$	Substitutes x = 2 into their $\frac{dy}{dx}$ which is of the form $\pm k\,2^{(x^2)}$ or $Ax\ 2^{(x^2)}$	M1
	$\frac{dy}{dx} = \underline{64\ln 2} = 44.3614...$	$\underline{64\ln 2}$ or awrt 44.4	A1
			[4]

Question Number	Scheme		Marks
7.	$\mathbf{a} = \overrightarrow{OA} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k} \quad \Rightarrow \lvert\overrightarrow{OA}\rvert = 3$ $\mathbf{b} = \overrightarrow{OB} = \mathbf{i} + \mathbf{j} - 4\mathbf{k} \quad \Rightarrow \lvert\overrightarrow{OB}\rvert = \sqrt{18}$ $\overrightarrow{BC} = \pm(2\mathbf{i} + 2\mathbf{j} + \mathbf{k}) \Rightarrow \lvert\overrightarrow{BC}\rvert = 3$ $\overrightarrow{AC} = \pm(\mathbf{i} + \mathbf{j} - 4\mathbf{k}) \Rightarrow \lvert\overrightarrow{AC}\rvert = \sqrt{18}$		
(a)	$\mathbf{c} = \overrightarrow{OC} = \underline{3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}}$	$\underline{3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}}$	B1 **cao** **[1]**
(b)	$\overrightarrow{OA} \bullet \overrightarrow{OB} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 1 \\ -4 \end{pmatrix} = \underline{2 + 2 - 4} = 0$ or… $\overrightarrow{BO} \bullet \overrightarrow{BC} = \begin{pmatrix} -1 \\ -1 \\ 4 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} = \underline{-2 - 2 + 4} = 0$ or… $\overrightarrow{AC} \bullet \overrightarrow{BC} = \begin{pmatrix} 1 \\ 1 \\ -4 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} = \underline{2 + 2 - 4} = 0$ or… $\overrightarrow{AO} \bullet \overrightarrow{AC} = \begin{pmatrix} -2 \\ -2 \\ -1 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 1 \\ -4 \end{pmatrix} = \underline{-2 - 2 + 4} = 0$	An attempt to take the dot product between either $\overrightarrow{OA}$ and $\overrightarrow{OB}$, $\overrightarrow{OA}$ and $\overrightarrow{AC}$, $\overrightarrow{AC}$ and $\overrightarrow{BC}$ or $\overrightarrow{OB}$ and $\overrightarrow{BC}$ Showing the result is equal to zero.	M1 A1
	and therefore OA is perpendicular to OB and hence OACB is a rectangle.	perpendicular and OACB is a rectangle	A1 **cso**
		Using distance formula to find either the correct height or width.	M1
	Area $= 3 \times \sqrt{18} = 3\sqrt{18} = 9\sqrt{2}$	Multiplying the rectangle's height by its width.	M1
		exact value of $3\sqrt{18}$, $9\sqrt{2}$, $\sqrt{162}$ or aef	A1 **[6]**
(c)	$\overrightarrow{OD} = \mathbf{d} = \frac{1}{2}(3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k})$	$\underline{\frac{1}{2}(3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k})}$	B1 **[1]**

Question Number	Scheme		Marks
(d) **Way 1**	*using dot product formula* $\overrightarrow{DA}=\pm\left(\tfrac{1}{2}\mathbf{i}+\tfrac{1}{2}\mathbf{j}+\tfrac{5}{2}\mathbf{k}\right)$ & $\overrightarrow{DC}=\pm\left(\tfrac{3}{2}\mathbf{i}+\tfrac{3}{2}\mathbf{j}-\tfrac{3}{2}\mathbf{k}\right)$ or $\overrightarrow{BA}=\pm(\mathbf{i}+\mathbf{j}+5\mathbf{k})$ & $\overrightarrow{OC}=\pm(3\mathbf{i}+3\mathbf{j}-3\mathbf{k})$	Identifies a set of two relevant vectors Correct vectors ±	M1 A1
	$\cos D=(\pm)\dfrac{\begin{pmatrix}0.5\\0.5\\2.5\end{pmatrix}\bullet\begin{pmatrix}1.5\\1.5\\-1.5\end{pmatrix}}{\frac{\sqrt{27}}{2}.\frac{\sqrt{27}}{2}}=(\pm)\dfrac{\frac{3}{4}+\frac{3}{4}-\frac{15}{4}}{\frac{27}{4}}=(\pm)\dfrac{1}{3}$	Applies dot product formula on multiples of these vectors. Correct ft. application of dot product formula.	dM1 A1 $\sqrt{}$
	$D=\cos^{-1}\left(-\dfrac{1}{3}\right)$	Attempts to find the correct angle D rather than $180^\circ-D$.	ddM1 $\sqrt{}$
	$D=109.47122...^\circ$	109.5° or awrt 109° or 1.91^c	A1
			[6]
Aliter (d) **Way 2**	*using dot product formula and direction vectors* $\mathrm{d}\overrightarrow{BA}=\pm(\mathbf{i}+\mathbf{j}+5\mathbf{k})$ & $\mathrm{d}\overrightarrow{OC}=\pm(\mathbf{i}+\mathbf{j}-\mathbf{k})$	Identifies a set of two direction vectors Correct vectors ±	M1 A1
	$\cos D=(\pm)\dfrac{\begin{pmatrix}1\\1\\-1\end{pmatrix}\bullet\begin{pmatrix}1\\1\\5\end{pmatrix}}{\sqrt{3}.\sqrt{27}}=(\pm)\dfrac{1+1-5}{\sqrt{3}.\sqrt{27}}=(\pm)\dfrac{1}{3}$	Applies dot product formula on multiples of these vectors. Correct ft. application of dot product formula.	dM1 A1 $\sqrt{}$
	$D=\cos^{-1}\left(-\dfrac{1}{3}\right)$	Attempts to find the correct angle D rather than $180^\circ-D$.	ddM1 $\sqrt{}$
	$D=109.47122...^\circ$	109.5° or awrt 109° or 1.91^c	A1
			[6]

Question Number	Scheme		Marks
Aliter	*using dot product formula and similar triangles*		
(d)	$d\overrightarrow{OA} = (2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ & $d\overrightarrow{OC} = (\mathbf{i} + \mathbf{j} - \mathbf{k})$	Identifies a set of two direction vectors	M1
		Correct vectors	A1
Way 3	$\cos\left(\tfrac{1}{2}D\right) = \dfrac{\begin{pmatrix}2\\2\\1\end{pmatrix}\bullet\begin{pmatrix}1\\1\\-1\end{pmatrix}}{\sqrt{9}.\sqrt{3}} = \dfrac{2+2-1}{\sqrt{9}.\sqrt{3}} = \dfrac{1}{\sqrt{3}}$	Applies dot product formula on multiples of these vectors.	dM1
		<u>Correct ft. application of dot product formula.</u>	A1 √
	$D = 2\cos^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$	Attempts to find the correct angle D by doubling their angle for $\tfrac{1}{2}D$.	ddM1 √
	$D = 109.47122...^\circ$	109.5° or awrt109° or 1.91^c	A1
			[6]
Aliter	*using cosine rule*		
(d)	$\overrightarrow{DA} = \tfrac{1}{2}\mathbf{i} + \tfrac{1}{2}\mathbf{j} + \tfrac{5}{2}\mathbf{k}$, $\overrightarrow{DC} = \tfrac{3}{2}\mathbf{i} + \tfrac{3}{2}\mathbf{j} - \tfrac{3}{2}\mathbf{k}$, $\overrightarrow{AC} = \mathbf{i} + \mathbf{j} - 4\mathbf{k}$		
Way 4	$\lvert\overrightarrow{DA}\rvert = \dfrac{\sqrt{27}}{2}$, $\lvert\overrightarrow{DC}\rvert = \dfrac{\sqrt{27}}{2}$, $\lvert\overrightarrow{AC}\rvert = \sqrt{18}$	Attempts to find all the lengths of all three edges of ΔADC	M1
		All Correct	A1
	$\cos D = \dfrac{\left(\dfrac{\sqrt{27}}{2}\right)^2 + \left(\dfrac{\sqrt{27}}{2}\right)^2 - \left(\sqrt{18}\right)^2}{2\left(\dfrac{\sqrt{27}}{2}\right)\left(\dfrac{\sqrt{27}}{2}\right)} = -\dfrac{1}{3}$	Using the cosine rule formula with correct 'subtraction'.	dM1
		<u>Correct ft application of the cosine rule formula</u>	A1 √
	$D = \cos^{-1}\left(-\dfrac{1}{3}\right)$	Attempts to find the correct angle D rather than $180^\circ - D$.	ddM1 √
	$D = 109.47122...^\circ$	109.5° or awrt109° or 1.91^c	A1
			[6]

Question Number	Scheme		Marks
Aliter (d) **Way 5**	*using trigonometry on a right angled triangle* $\overrightarrow{DA} = \frac{1}{2}\mathbf{i} + \frac{1}{2}\mathbf{j} + \frac{5}{2}\mathbf{k}$ $\overrightarrow{OA} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ $\overrightarrow{AC} = \mathbf{i} + \mathbf{j} - 4\mathbf{k}$		
	Let X be the midpoint of AC	Attempts to find two out of the three lengths in ΔADX	M1
	$\lvert\overrightarrow{DA}\rvert = \frac{\sqrt{27}}{2}$, $\lvert\overrightarrow{DX}\rvert = \frac{1}{2}\lvert\overrightarrow{OA}\rvert = \frac{3}{2}$, $\lvert\overrightarrow{AX}\rvert = \frac{1}{2}\lvert\overrightarrow{AC}\rvert = \frac{1}{2}\sqrt{18}$ (hypotenuse), (adjacent) , (opposite)	Any two correct	A1
	$\sin(\frac{1}{2}D) = \frac{\frac{\sqrt{18}}{2}}{\frac{\sqrt{27}}{2}}$, $\cos(\frac{1}{2}D) = \frac{\frac{3}{2}}{\frac{\sqrt{27}}{2}}$ or $\tan(\frac{1}{2}D) = \frac{\frac{\sqrt{18}}{2}}{\frac{3}{2}}$	Uses correct sohcahtoa to find $\frac{1}{2}D$	dM1
		Correct ft application of sohcahtoa	A1 $\sqrt{}$
	eg. $D = 2\tan^{-1}\left(\frac{\frac{\sqrt{18}}{2}}{\frac{3}{2}}\right)$	Attempts to find the correct angle D by doubling their angle for $\frac{1}{2}D$.	ddM1 $\sqrt{}$
	$D = 109.47122...^\circ$	109.5° or awrt 109° or 1.91^c	A1
			[6]
Aliter (d) **Way 6**	*using trigonometry on a right angled similar triangle OAC* $\overrightarrow{OC} = 3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$ $\overrightarrow{OA} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ $\overrightarrow{AC} = \mathbf{i} + \mathbf{j} - 4\mathbf{k}$		
	$\lvert\overrightarrow{OC}\rvert = \sqrt{27}$, $\lvert\overrightarrow{OA}\rvert = 3$, $\lvert\overrightarrow{AC}\rvert = \sqrt{18}$ (hypotenuse), (adjacent), (opposite)	Attempts to find two out of the three lengths in ΔOAC	M1
		Any two correct	A1
	$\sin(\frac{1}{2}D) = \frac{\sqrt{18}}{\sqrt{27}}$, $\cos(\frac{1}{2}D) = \frac{3}{\sqrt{27}}$ or $\tan(\frac{1}{2}D) = \frac{\sqrt{18}}{3}$	Uses correct sohcahtoa to find $\frac{1}{2}D$	dM1
		Correct ft application of sohcahtoa	A1 $\sqrt{}$
	eg. $D = 2\tan^{-1}\left(\frac{\sqrt{18}}{3}\right)$	Attempts to find the correct angle D by doubling their angle for $\frac{1}{2}D$.	ddM1 $\sqrt{}$
	$D = 109.47122...^\circ$	109.5° or awrt 109° or 1.91^c	A1
			[6]

Question Number	Scheme		Marks
Aliter 7. (b) (i) **Way 2**	$\mathbf{c} = \overrightarrow{OC} = \pm(3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k})$ $\overrightarrow{AB} = \pm(-\mathbf{i} - \mathbf{j} - 5\mathbf{k})$		
	$\lvert\overrightarrow{OC}\rvert = \sqrt{(3)^2 + (3)^2 + (-3)^2} = \sqrt{(1)^2 + (1)^2 + (-5)^2} = \lvert\overrightarrow{AB}\rvert$	A **complete** method of proving that the diagonals are equal.	M1
	As $\lvert\overrightarrow{OC}\rvert = \lvert\overrightarrow{AB}\rvert = \sqrt{27}$	Correct result.	A1
	then the <u>diagonals are equal</u>, and OACB is a <u>rectangle</u>.	<u>diagonals are equal</u> and <u>OACB is a rectangle</u>	A1 **cso**
			[3]
	$\mathbf{a} = \overrightarrow{OA} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k} \Rightarrow \lvert\overrightarrow{OA}\rvert = 3$ $\mathbf{b} = \overrightarrow{OB} = \mathbf{i} + \mathbf{j} - 4\mathbf{k} \Rightarrow \lvert\overrightarrow{OB}\rvert = \sqrt{18}$ $\overrightarrow{BC} = \pm(2\mathbf{i} + 2\mathbf{j} + \mathbf{k}) \Rightarrow \lvert\overrightarrow{BC}\rvert = 3$ $\overrightarrow{AC} = \pm(\mathbf{i} + \mathbf{j} - 4\mathbf{k}) \Rightarrow \lvert\overrightarrow{AC}\rvert = \sqrt{18}$ $\mathbf{c} = \overrightarrow{OC} = \pm(3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}) \Rightarrow \lvert\overrightarrow{OC}\rvert = \sqrt{27}$ $\overrightarrow{AB} = \pm(-\mathbf{i} - \mathbf{j} - 5\mathbf{k}) \Rightarrow \lvert\overrightarrow{AB}\rvert = \sqrt{27}$		
Aliter 7. (b) (i) **Way 3**	$(OA)^2 + (AC)^2 = (OC)^2$ or $(BC)^2 + (OB)^2 = (OC)^2$ or $(OA)^2 + (OB)^2 = (AB)^2$ or $(BC)^2 + (AC)^2 = (AB)^2$ or equivalent		
	$\Rightarrow$ <u>$(3)^2 + (\sqrt{18})^2 = (\sqrt{27})^2$</u>	A **complete** method of proving that Pythagoras holds using their values. <u>Correct result</u>	M1 A1
	and therefore OA is <u>perpendicular</u> to OB or AC is <u>perpendicular</u> to BC and hence <u>OACB is a rectangle</u>.	<u>perpendicular</u> and <u>OACB is a rectangle</u>	A1 **cso**
			[3]
			14 marks

Question Number	Scheme	Marks
8. (a)		

x	0	1	2	3	4	5
y	e^1	e^2	$e^{\sqrt{7}}$	$e^{\sqrt{10}}$	$e^{\sqrt{13}}$	e^4
or y	2.71828…	7.38906…	14.09403…	23.62434…	36.80197…	54.59815…

Either $e^{\sqrt{7}}$, $e^{\sqrt{10}}$ and $e^{\sqrt{13}}$
or awrt 14.1, 23.6 and 36.8
or e to the power
awrt 2.65, 3.16, 3.61
(or mixture of decimals and e's)

At least two correct — B1
All three correct — B1
[2]

(b)

$$I \approx \frac{1}{2} \times 1 ; \times \left\{ e^1 + 2\left(e^2 + e^{\sqrt{7}} + e^{\sqrt{10}} + e^{\sqrt{13}}\right) + e^4 \right\}$$

Outside brackets $\frac{1}{2} \times 1$ — B1;
For structure of trapezium rule $\{............\}$; — M1 $\sqrt{}$

$$= \frac{1}{2} \times 221.1352227... = 110.5676113... = \underline{110.6} \text{ (4sf)}$$

110.6 — A1 **cao**
[3]

Beware: In part (b) candidates can add up the individual trapezia:

(b) $I \approx \frac{1}{2}.1\left(\underline{e^1 + e^2}\right) + \frac{1}{2}.1\left(\underline{e^2 + e^{\sqrt{7}}}\right) + \frac{1}{2}.1\left(\underline{e^{\sqrt{7}} + e^{\sqrt{10}}}\right) + \frac{1}{2}.1\left(\underline{e^{\sqrt{10}} + e^{\sqrt{13}}}\right) + \frac{1}{2}.1\left(\underline{e^{\sqrt{13}} + e^4}\right)$

Question Number	Scheme		Marks
(c)	$t = (3x+1)^{\frac{1}{2}} \Rightarrow \frac{dt}{dx} = \frac{1}{2}.3.(3x+1)^{-\frac{1}{2}}$	$A(3x+1)^{-\frac{1}{2}}$ or $t\frac{dt}{dx} = A$	M1
	… or $t^2 = 3x+1 \Rightarrow \underline{2t\frac{dt}{dx} = 3}$	$\underline{\frac{3}{2}(3x+1)^{-\frac{1}{2}}}$ or $\underline{2t\frac{dt}{dx} = 3}$	A1
	so $\frac{dt}{dx} = \frac{3}{2.(3x+1)^{\frac{1}{2}}} = \frac{3}{2t} \Rightarrow \frac{dx}{dt} = \frac{2t}{3}$	Candidate obtains either $\frac{dt}{dx}$ or $\frac{dx}{dt}$ in terms of t …	
	$\therefore I = \int e^{\sqrt{(3x+1)}}\ dx = \int e^t \frac{dx}{dt}.dt = \int e^t.\frac{2t}{3}.dt$	… and moves on to substitute this into I to convert an integral wrt x to an integral wrt t.	dM1
	$\therefore I = \underline{\int \frac{2}{3}te^t\ dt}$	$\underline{\int \frac{2}{3}te^t}$	A1
	change limits: when x = 0, t = 1 & when x = 5, t = 4	changes limits $x \to t$ so that $0 \to 1$ and $5 \to 4$	B1
	Hence $I = \int_1^4 \frac{2}{3}te^t\ dt$; where $a = 1, b = 4, k = \frac{2}{3}$		
			[5]
(d)	$\begin{cases} u = t & \Rightarrow \frac{du}{dt} = 1 \\ \frac{dv}{dt} = e^t & \Rightarrow v = e^t \end{cases}$	Let k be any constant for the first three marks of this part.	
	$k\int te^t\ dt = k\left(te^t - \int e^t.1\ dt\right)$	Use of 'integration by parts' formula in the correct direction.	M1
		Correct expression with a constant factor k.	A1
	$= k\left(\underline{te^t - e^t}\right) + c$	<u>Correct integration</u> with/without a constant factor k	A1
	$\therefore \int_1^4 \frac{2}{3}te^t\ dt = \frac{2}{3}\left\{\left(4e^4 - e^4\right) - \left(e^1 - e^1\right)\right\}$	Substitutes their changed limits into the integrand and subtracts oe.	dM1 **oe**
	$= \frac{2}{3}(3e^4) = \underline{2e^4} = 109.1963...$	either $2e^4$ or awrt 109.2	A1
			[5]
			15 marks

- Note: dM1 denotes a method mark which is dependent upon the award of the previous method mark
- ddM1 denotes a method mark which is dependent upon the award of the previous two method marks.