January 2007
6664 Core Mathematics C2
Mark Scheme

Question Number	Scheme	Marks
1. (a)	$f'(x) = 3x^2 + 6x$	B1
	$f''(x) = 6x + 6$	M1, A1cao
		(3)

Notes   cao = correct answer only

1(a)	
Acceptable alternatives include $3x^2 + 6x^1$; $3x^2 + 3\times 2x$; $3x^2 + 6x + 0$ Ignore LHS (e.g. use [whether correct or not] of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$) $3x^2 + 6x + c$ or $3x^2 + 6x +$ constant (i.e. the written word constant) is B0	B1
M1 Attempt to differentiate their $f'(x)$; $x^n \to x^{n-1}$. $x^n \to x^{n-1}$ seen in at least one of the terms. Coefficient of $x^{\cdots}$ ignored for the method mark. $x^2 \to x^1$ and $x \to x^0$ are acceptable.	M1
Acceptable alternatives include $6x^1 + 6x^0$; $3\times 2x + 3\times 2$ $6x + 6 + c$ or $6x + 6 +$ constant is A0	A1 cao

Examples

1(a) $f''(x) = 3x^2 + 6x$ — B1, M0 A0

1(a) $f'(x) = 3x^2 + 6x$ — B1
$f''(x) = 6x$ — M1 A0

1(a) $y = x^3 + 3x^2 + 5$
$\frac{dy}{dx} = 3x^2 + 3x$ — B0
$\frac{d^2y}{dx^2} = 6x + 3$ — M1 A0

1(a) $f'(x) = 3x^2 + 6x + c$ — B0
$f''(x) = 6x + 6$ — M1 A1

1(a) $f'(x) = x^2 + 3x$ — B0
$f''(x) = x + 3$ — M1 A0

1(a) $x^3 + 3x^2 + 5$
$= 3x^2 + 6x$ — B1
$= 6x + 6$ — M1 A1

1(a) $f'(x) = 3x^2 + 6x \quad + 5$ — B0
$f''(x) = 6x + 6$ — M1 A1

1(a) $f'(x) = 3x^2 + 6x$ — B1
$f''(x) = 6x + 6 + c$ — M1 A0

Question Number	Scheme	Marks
1. (b)	$\int (x^3 + 3x^2 + 5)\,dx = \frac{x^4}{4} + \frac{3x^3}{3} + 5x$	M1, A1
	$\left[\frac{x^4}{4} + x^3 + 5x\right]_1^2 = 4 + 8 + 10 - (\frac{1}{4} + 1 + 5)$	M1
	$= 15\frac{3}{4}$ o.e.	A1 (4) **(7)**

<u>Notes</u> o.e. = or equivalent

1(b)	
Attempt to integrate f(x); $x^n \to x^{n+1}$ Ignore incorrect notation (e.g. inclusion of integral sign)	M1
o.e. Acceptable alternatives include $\frac{x^4}{4} + x^3 + 5x$; $\frac{x^4}{4} + \frac{3x^3}{3} + 5x^1$; $\frac{x^4}{4} + \frac{3x^3}{3} + 5x + c$; $\int \frac{x^4}{4} + \frac{3x^3}{3} + 5x$ N.B. If the candidate has written the integral (either $\frac{x^4}{4} + \frac{3x^3}{3} + 5x$ or what they think is the integral) in part (a), it may not be rewritten in (b), but the marks may be awarded if the integral is used in (b).	A1
Substituting 2 and 1 into any function other than $x^3 + 3x^2 + 5$ and subtracting either way round. So using their f ′(x) or f ′′(x) or ∫ their f ′(x) dx or ∫ their f ′′(x) dx will gain the M mark (because none of these will give $x^3 + 3x^2 + 5$). Must substitute for all x s but could make a slip. $4 + 8 + 10 - \frac{1}{4} + 1 + 5$ (for example) is acceptable for evidence of subtraction ('invisible' brackets).	M1
o.e. (e.g. $15\frac{3}{4}$, 15.75, $\frac{63}{4}$) Must be a single number (so $22 - 6\frac{1}{4}$ is A0).	A1
Answer only is M0A0M0A0	

<u>Examples</u>

1(b) $\frac{x^4}{4} + x^3 + 5x + c$ M1 A1

$4 + 8 + 10 + c - (\frac{1}{4} + 1 + 5 + c)$ M1

$= 15\frac{3}{4}$ A1

1(b) $\frac{x^4}{4} + x^3 + 5x + c$ M1 A1

$x = 2$, $22 + c$

$x = 1$, $6\frac{1}{4} + c$ M0

A0

(no subtraction)

1(b) $\int_1^2 f(x)\,dx = 2^3 + 3\times2^2 + 5 - (1 + 3 + 5)$ M0 A0, M0

$= 25 - 9$

$= 16$ A0

(Substituting 2 and 1 into $x^3 + 3x^2 + 5$, so 2nd M0)

1(b) $\int_1^2 (6x+6)\,dx = \left[3x^2 + 6x\right]_1^2$ M0 A0

$= 12 + 12 - (3 + 6)$ M1 A0

1(b) $\int_1^2 (3x^2+6x)\,dx = \left[x^3 + 3x^2\right]_1^2$ M0 A0

$= 8 + 12 - (1 + 3)$ M1 A0

1(b) $\frac{x^4}{4} + x^3 + 5x$ M1 A1

$\frac{2^4}{4} + 2^3 + 5\times2 - \frac{1^4}{4} + 1^3 + 5$ M1

(one negative sign is sufficient for evidence of subtraction)

$= 22 - 6\frac{1}{4} = 15\frac{3}{4}$ A1

(allow 'recovery', implying student was using 'invisible brackets')

1(a) $f(x) = x^3 + 3x^2 + 5$

$f''(x) = \frac{x^4}{4} + x^3 + 5x$ B0 M0 A0

(b) $\frac{2^4}{4} + 2^3 + 5\times2 - \frac{1^4}{4} - 1^3 - 5$ M1 A1 M1

$= 15\frac{3}{4}$ A1

The candidate has written the integral in part (a). It is not rewritten in (b), but the marks may be awarded as the integral is used in (b).

Question Number	Scheme	Marks
2. (a)	$(1-2x)^5 = 1 + 5\times(-2x) + \frac{5\times4}{2!}(-2x)^2 + \frac{5\times4\times3}{3!}(-2x)^3 + ...$	
	$= 1 - 10x + 40x^2 - 80x^3 + ...$	B1, M1, A1, A1
		(4)
(b)	$(1 + x)(1 - 2x)^5 = (1 + x)(1 - 10x + \ldots)$	
	$= 1 + x - 10x + \ldots$	M1
	$\approx 1 - 9x$ (✱)	A1 (2)
		(6)

Notes

2(a)	
$1 - 10x$ $1 - 10x$ must be seen in this simplified form in (a).	B1
Correct structure: 'binomial coefficients' (perhaps from Pascal's triangle), increasing powers of x. Allow slips. Accept other forms: $^5C_1$, $\binom{5}{1}$, also condone $\left(\frac{5}{1}\right)$ but must be attempting to use 5. Condone use of invisible brackets and using $2x$ instead of $-2x$. Powers of x: at least 2 powers of the type $(2x)^a$ or $2x^a$ seen for $a \geq 1$.	M1
$40x^2$ (1st A1)	A1
$-80x^3$ (2nd A1)	A1
Allow commas between terms. Terms may be listed rather than added Allow 'recovery' from invisible brackets, so $1^5 + \binom{5}{1}1^4.-2x + \binom{5}{2}1^3.-2x^2 + \binom{5}{3}1^2.-2x^3$ $= 1 - 10x + 40x^2 - 80x^3 + ...$ gains full marks. $1 + 5\times(2x) + \frac{5\times4}{2!}(2x)^2 + \frac{5\times4\times3}{3!}(2x)^3 + ... = 1 + 10x + 40x^2 + 80x^3 + ...$ gains B0M1A1A0	
Misread: first 4 terms, descending terms: if correct, would score B0, M1, 1st A1: one of $40x^2$ and $-80x^3$ correct; 2nd A1: both $40x^2$ and $-80x^3$ correct.	

2(a) Long multiplication	
$(1-2x)^2 = 1 - 4x + 4x^2$, $(1-2x)^3 = 1 - 6x + 12x^2 - 8x^3$, $(1-2x)^4 = 1 - 8x + 24x^2 - 32x^3$ {+ $16x^4$} $(1-2x)^5 = 1 - 10x + 40x^2 + 80x^3 + \ldots$	
$1 - 10x$ $1 - 10x$ must be seen in this simplified form in (a).	B1
Attempt repeated multiplication up to and including $(1-2x)^5$	M1

$40x^2$ (1st A1)	A1
$-80x^3$ (2nd A1)	A1
Misread: first 4 terms, descending terms: if correct, would score B0, M1, 1st A1: one of $40x^2$ and $-80x^3$ correct; 2nd A1: both $40x^2$ and $-80x^3$ correct.	

2(b)	
Use their (a) and attempt to multiply out; terms (whether correct or incorrect) in x^2 or higher can be ignored. If their (a) is correct an attempt to multiply out can be implied from the correct answer, so $(1 + x)(1 – 10x) = 1 – 9x$ will gain M1 A1. If their (a) is correct, the 2nd bracket must contain at least $(1 – 10x)$ and an attempt to multiply out for the M mark. An attempt to multiply out is an attempt at 2 out of the 3 relevant terms (N.B. the 2 terms in x^1 may be combined – but this will still count as 2 terms). If their (a) is incorrect their 2nd bracket must contain all the terms in x^0 and x^1 from their (a) AND an attempt to multiply all terms that produce terms in x^0 and x^1. N.B. $(1 + x)(1 – 2x)^5 = (1 + x)(1 – 2x)$ [where $1 – 2x + …$ is NOT the candidate's answer to (a)] $= 1 – x$ i.e. candidate has ignored the power of 5: M0 N.B. The candidate may start again with the binomial expansion for $(1 – 2x)^5$ in (b). If correct (only needs $1 – 10x$) may gain M1 A1 even if candidate did not gain B1 in part (a).	M1
N.B. Answer given in question.	A1

<u>Example</u>

Answer in (a) is $=1+10x+40x^2-80x^3+...$

(b) $(1 + x)(1 + 10x) = 1 + 10x + x$ M1
$= 1 + 11x$ A0

Question Number	Scheme	Marks
3.	Centre $\left(\frac{-1+3}{2}, \frac{6+4}{2}\right)$, i.e. (1, 5)	M1, A1
	$r = \frac{\sqrt{(3-(-1))^2+(6-4)^2}}{2}$ or $r^2 = (1-(-1))^2 + (5-4)^2$ or $r^2 = (3-1)^2 + (6-5)^2$ o.e.	M1
	$(x-1)^2 + (y-5)^2 = 5$	M1,A1,A1 **(6)**

Notes

Some use of correct formula in x or y coordinate. Can be implied. Use of $\left(\frac{1}{2}(x_A - x_B), \frac{1}{2}(y_A - y_B)\right) \rightarrow$ (–2, –1) or (2, 1) is M0 A0 but watch out for use of $x_A + \frac{1}{2}(x_A - x_B)$ etc which is okay.	M1
(1, 5) (5, 1) gains M1 A0.	A1
Correct method to find r or r^2 using given points or f.t. from their centre. Does not need to be simplified. Attempting radius = $\sqrt{\frac{(\text{diameter})^2}{2}}$ is an incorrect method, so M0. N.B. Be careful of labelling: candidates may not use d for diameter and r for radius. Labelling should be ignored. Simplification may be incorrect – mark awarded for correct method. Use of $\sqrt{(x_1-x_2)^2-(y_1-y_2)^2}$ is M0.	M1
Write down $(x \pm a)^2 + (y \pm b)^2$ = any constant (a letter or a number). Numbers do not have to be substituted for a, b and if they are they can be wrong.	M1
LHS is $(x-1)^2 + (y-5)^2$. Ignore RHS.	A1
RHS is 5. Ignore subsequent working. Condone use of decimals that leads to exact 5.	A1
Or correct equivalents, e.g. $x^2 + y^2 - 2x - 10y + 21 = 0$.	

Alternative – note the order of the marks needed for ePEN.	
As above.	M1
As above.	A1
$x^2 + y^2$ + (constant)x + (constant)y + constant = 0. Numbers do not have to be substituted for the constants and if they are they can be wrong.	3rd M1
Attempt an appropriate substitution of the coordinates of their centre (i.e. working with coefficient of x and coefficient of y in equation of circle) and substitute (–1, 4) or (3, 6) into equation of circle.	2nd M1
$-2x - 10y$ part of the equation $x^2 + y^2 - 2x - 10y + 21 = 0$.	A1
$+21 = 0$ part of the equation $x^2 + y^2 - 2x - 10y + 21 = 0$.	A1
Or correct equivalents, e.g. $(x-1)^2 + (y-5)^2 = 5$.	

Question Number	Scheme	Marks
4.	$x\log 5 = \log 17$ or $x = \log_5 17$	M1
	$x = \dfrac{\log 17}{\log 5}$	A1
	$= 1.76$	A1 (3)

Notes N.B. It is never possible to award an A mark after giving M0. If M0 is given then the marks will be M0 A0 A0.

4	
Acceptable alternatives include $x\log 5 = \log 17$; $x\log_{10} 5 = \log_{10} 17$; $x\log_e 5 = \log_e 17$; $x\ln 5 = \ln 17$; $x = \log_5 17$ Can be implied by a correct exact expression as shown on the first A1 mark	1st M1
An exact expression for x that can be evaluated on a calculator. Acceptable alternatives include $x = \dfrac{\log 17}{\log 5}$; $x = \dfrac{\log_{10} 17}{\log_{10} 5}$; $x = \dfrac{\log_e 17}{\log_e 5}$; $x = \dfrac{\ln 17}{\ln 5}$; $x = \dfrac{\log_q 17}{\log_q 5}$ where q is a number This may not be seen (as, for example, $\log_5 17$ can be worked out directly on many calculators) so this A mark can be implied by the correct final answer or the right answer corrected to or truncated to a greater accuracy than 3 significant figures or 1.8 Alternative: $x = \dfrac{\text{a number}}{\text{a number}}$ where this fraction, when worked out as a decimal rounds to 1.76. (N.B. remember that this A mark cannot be awarded without the M mark). If the line for the M mark is missing but this line is seen (with or without the x =) and is correct the method can be assumed and M1 1st A1 given.	1st A1
1.76 cao	2nd A1
N.B. $\sqrt[5]{17} = 1.76$ and $x^5 = 17$, $\therefore x = 1.76$ are both M0 A0 A0	
Answer only 1.76: full marks (M1 A1 A1) Answer only to a greater accuracy but which rounds to 1.76: M1 A1 A0 (e.g. 1.760, 1.7603, 1.7604, 1.76037 etc) Answer only 1.8: M1 A1 A0 Trial and improvement: award marks as for "answer only".	

Examples

4. $x = \log 5^{17}$ M0 A0 $= 1.76$ A0 Working seen, so scheme applied	4. $5^{1.76} = 17$ M1 A1 A1 Answer only but clear that $x = 1.76$
4. $5^{1.8} = 17$ M1 A1 A0 Answer only but clear that $x = 1.8$	4. $5^{1.76}$ M0 A0 A0
4. $\log_5 17 = x$ M1 $x = 1.760$ A1 A0	4. $\log_5 17 = x$ M1 $x = 1.76$ A1 A1
4. $x\log 5 = \log 17$ M1 $x = \frac{1.2304...}{0.69897...}$ A1 $x = 1.76$ A1	4. $x\ln 5 = \ln 17$ M1 $x = \frac{2.833212...}{1.609437...}$ A1 $x = 1.76$ A1
4. $x\log 5 = \log 17$ M1 $x = \frac{2.57890}{1.46497}$ A1 $x = 1.83$ A0	4. $\log_{17} 5 = x$ M0 $x = \frac{\log 5}{\log 17}$ A0 $x = 0.568$ A0
4. $5^{1.8} = 18.1,\ 5^{1.75} = 16.7$ $5^{1.761} = 17$ M1 A1 A0	4. $x = 5^{1.76}$ M0 A0 A0
4. $x\log 5 = \log 17$ M1 $x = 1.8$ A1 A0	4. $x = \frac{\log 17}{\log 5}$ M1 A1 $x = 1.8$ A0

N.B.

4. $x^5 = 17$ M0 A0 $x = 1.76$ A0	4. $\sqrt[5]{17}$ M0 A0 $= 1.76$ A0

Question Number	Scheme	Marks
5. (a)	$f(-2) = (-2)^3 + 4(-2)^2 + (-2) - 6$	M1
	$\{ = -8 + 16 - 2 - 6\}$	
	$= 0, \therefore x + 2$ is a factor	A1 (2)
(b)	$x^3 + 4x^2 + x - 6 = (x + 2)(x^2 + 2x - 3)$	M1, A1
	$= (x + 2)(x + 3)(x - 1)$	M1, A1 (4)
(c)	$-3, -2, 1$	B1 (1) **(7)**

Notes Line in mark scheme in { } does not need to be seen.

5(a)	
Attempting f(±2): No x s; allow invisible brackets for M mark Long division: M0 A0.	M1
= 0 and minimal conclusion (e.g. factor, hence result, QED, ✓, □). If result is stated first [i.e. If $x + 2$ is a factor, $f(-2) = 0$] conclusion is not needed. Invisible brackets used as brackets can get M1 A1, so $f(-2) = -2^3 + 4\times-2^2 + -2 - 6$ $\{ = -8 + 16 - 2 - 6\} = 0$, $\therefore x + 2$ is a factor M1 A1, but $f(-2) = -2^3 + 4\times-2^2 + -2 - 6 = -8 - 16 - 2 - 6 = 0$, $\therefore x + 2$ is a factor M1 A0 Acceptable alternatives include: $x = -2$ is a factor, $f(-2)$ is a factor.	A1

5(b)	
1st M1 requires division by $(x + 2)$ to get $x^2 + ax + b$ where $a \neq 0$ and $b \neq 0$ or equivalent with division by $(x + 3)$ or $(x - 1)$.	M1
$(x + 2)(x^2 + 2x - 3)$ or $(x + 3)(x^2 + x - 2)$ or $(x - 1)(x^2 + 5x + 6)$ [If long division has been done in (a), minimum seen in (b) to get first M1 A1 is to make some reference to their quotient $x^2 + ax + b$.]	A1
Attempt to factorise their quadratic (usual rules).	M1
"Combining" all 3 factors is not required.	A1
Answer only: Correct M1 A1 M1 A1 Answer only with one sign slip: $(x + 2)(x + 3)(x + 1)$ scores 1st M1 1st A12nd M0 2nd A0 $(x + 2)(x - 3)(x - 1)$ scores 1st M0 1st A0 2nd M1 2nd A1	
Answer to (b) can be seen in (c).	

5(b) Alternative comparing coefficients	
$(x + 2)(x^2 + ax + b) = x^3 + (2 + a)x^2 + (2a + b)x + 2b$ Attempt to compare coefficients of two terms to find values of a and b	M1
$a = 2, b = -3$	A1
Or $(x + 2)(ax^2 + bx + c) = ax^3 + (2a + b)x^2 + (2b + c)x + 2c$ Attempt to compare coefficients of three terms to find values of a, b and c.	M1

$a = 1$, $b = 2$, $c = -3$	A1
Then apply scheme as above	

5(b) Alternative using factor theorem	
Show f(–3) = 0; allow invisible brackets	M1
$\therefore x + 3$ is a factor	A1
Show f(1) = 0	M1
$\therefore x - 1$ is a factor	A1

5(c)	
–3, –2, 1 or (–3, 0), (–2, 0), (1, 0) only. Do not ignore subsequent working. Ignore any working in previous parts of the question. Can be seen in (b)	B1

Question Number	Scheme	Marks
6.	$2(1-\sin^2 x)+1=5\sin x$	M1
	$2\sin^2 x+5\sin x-3=0$	
	$(2\sin x-1)(\sin x+3)=0$	
	$\sin x=\frac{1}{2}$	M1, A1
	$x=\frac{\pi}{6},\frac{5\pi}{6}$	M1, M1, A1cso **(6)**

Notes

Use of $\cos^2 x=1-\sin^2 x$. Condone invisible brackets in first line if $2-2\sin^2 x$ is present (or implied) in a subsequent line. Must be using $\cos^2 x=1-\sin^2 x$. Using $\cos^2 x=1+\sin^2 x$ is M0.	M1
Attempt to solve a 2 or 3 term quadratic in sin x up to sin x = … Usual rules for solving quadratics. Method may be factorising, formula or completing the square	M1
Correct factorising for correct quadratic and $\sin x=\frac{1}{2}$. So, e.g. $(\sin x+3)$ as a factor $\rightarrow$ $\sin x=3$ can be ignored.	A1
Method for finding any angle in any range consistent with (either of) their trig. equation(s) in degrees or radians (even if x not exact). [Generous M mark] Generous mark. Solving any trig. equation that comes from minimal working (however bad). So $x=\sin^{-1}/\cos^{-1}/\tan^{-1}$(number) $\rightarrow$ answer in degrees or radians correct for their equation (in any range)	M1
Method for finding second angle consistent with (either of) their trig. equation(s) in radians. Must be in range $0\le x<2\pi$. Must involve using π (e.g. $\pi \pm \ldots$, $2\pi - \ldots$) but … can be inexact. Must be using the same equation as they used to attempt the 3rd M mark. Use of π must be consistent with the trig. equation they are using (e.g. if using $\cos^{-1}$ then must be using $2\pi - \ldots$) If finding both angles in degrees: method for finding 2nd angle equivalent to method above in degrees and an attempt to change both angles to radians.	M1
$\frac{\pi}{6},\frac{5\pi}{6}$ c.s.o. Recurring decimals are okay (instead of $\frac{1}{6}$ and $\frac{5}{6}$). Correct decimal values (corrected or truncated) before the final answer of $\frac{\pi}{6},\frac{5\pi}{6}$ is acceptable.	A1 cso
Ignore extra solutions outside range; deduct final A mark for extra solutions in range.	
Special case Answer only $\frac{\pi}{6},\frac{5\pi}{6}$ M0, M0, A0, M1, M1 A1 Answer only $\frac{\pi}{6}$ M0, M0, A0, M1,	

M0 A0 Finding answers by trying different values (e.g. trying multiples of π) in $2\cos^2 x + 1 = 5\sin x$: as for answer only.	

Question Number	Scheme	Marks
7.	$y = x(x^2 - 6x + 5)$	
	$= x^3 - 6x^2 + 5x$	M1, A1
	$\int (x^3 - 6x^2 + 5x)\,dx = \frac{x^4}{4} - \frac{6x^3}{3} + \frac{5x^2}{2}$	M1, A1ft
	$\left[\frac{x^4}{4} - 2x^3 + \frac{5x^2}{2}\right]_0^1 = \left(\frac{1}{4} - 2 + \frac{5}{2}\right) - 0 = \frac{3}{4}$	M1
	$\left[\frac{x^4}{4} - 2x^3 + \frac{5x^2}{2}\right]_1^2 = (4 - 16 + 10) - \frac{3}{4} = -\frac{11}{4}$	M1, A1(both)
	$\therefore$ total area $= \frac{3}{4} + \frac{11}{4}$	M1
	$= \frac{7}{2}$ o.e.	A1cso **(9)**

Notes

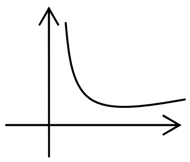
Attempt to multiply out, must be a cubic.	M1
Award A mark for their final version of expansion (but final version does not need to have like terms collected).	A1
Attempt to integrate; $x^n \to x^{n+1}$. Generous mark for some use of integration, so e.g. $\int x(x-1)(x-5)\ \mathrm{d}x = \frac{x^2}{2}\left(\frac{x^2}{2}-x\right)\left(\frac{x^2}{2}-5x\right)$ would gain method mark.	M1
Ft on their final version of expansion provided it is in the form $ax^p + bx^q + \ldots$. Integrand must have at least two terms and all terms must be integrated correctly. If they integrate twice (e.g. $\int_0^1$ and $\int_1^2$) and get different answers, take the better of the two.	A1ft
Substitutes and subtracts (either way round) for one integral. Integral must be a 'changed' function. Either 1 and 0, 2 and 1 or 2 and 0. For $[\ ]_0^1$: – 0 for bottom limit can be implied (provided that it is 0).	M1
M1 Substitutes and subtracts (either way round) for two integrals. Integral must be a 'changed' function. Must have 1 and 0 and 2 and 1 (or 1 and 2). The two integrals do not need to be the same, but they must have come from attempts to integrate the same function.	M1
$\frac{3}{4}$ and $-\frac{11}{4}$ o.e. (if using $\int_1^2 \mathrm{f}(x)$) or $\frac{3}{4}$ and $\frac{11}{4}$ o.e. (if using $\int_2^1 \mathrm{f}(x)$ or $-\int_1^2 \mathrm{f}(x)$ or $\int_1^2 -\mathrm{f}(x)$) where $\mathrm{f}(x) = \frac{x^4}{4} - 2x^3 + \frac{5x^2}{2}$. The answer must be consistent with the integral they are using (so $\int_1^2 \mathrm{f}(x) = \frac{11}{4}$ loses this A and the final A). $-\frac{11}{4}$ may not be seen explicitly. Can be implied by a subsequent line of working.	A1
5th M1 | their value for $[\ ]_0^1$ | + | their value for $[\ ]_1^2$ | Dependent on at least one of the values coming from integration (other may come from e.g. trapezium rules). This can be awarded even if both values already positive.	M1
$\frac{7}{2}$ o.e. N.B. c.s.o.	A1 cso

Question Number	Scheme	Marks
8. (a)	$\frac{dC}{dv} = -1400v^{-2} + \frac{2}{7}$	M1, A1
	$-1400v^{-2} + \frac{2}{7} = 0$	M1
	$v^2 = 4900$	**d**M1
	$v = 70$	A1cso (5)
(b)	$\frac{d^2C}{dv^2} = 2800v^{-3}$	M1
	$v = 70,\ \frac{d^2C}{dv^2} > 0$ {⇒ minimum}	A1ft
	or $v = 70,\ \frac{d^2C}{dv^2} = 2800 \times 70^{-3}$ $\{= \frac{2}{245} = 0.00816...\}$ {⇒ minimum}	(2)
(c)	$v = 70,\ C = \frac{1400}{70} + \frac{2 \times 70}{7}$	M1
	$C = 40$	A1 (2) **(9)**

Notes

8(a)	
Attempt to differentiate $v^n \to v^{n-1}$. Must be seen and marked in part (a) not part (b). Must be differentiating a function of the form $av^{-1} + bv$.	M1
o.e. ($-1400v^{-2} + \frac{2}{7} + c$ is A0)	A1
Their $\frac{dC}{dv} = 0$. Can be implied by their $\frac{dC}{dv} = P + Q \rightarrow P = \pm Q$.	M1
Dependent on both of the previous Ms. Attempt to rearrange their $\frac{dC}{dv}$ into the form v^n = number or v^n – number = 0, $n \neq 0$.	dM1
$v = 70$ cso but allow $v = \pm 70$. $v = 70$ km per h also acceptable.	A1cso
Answer only is 0 out of 5.	
Method of completing the square: send to review.	

8(a) Trial and improvement $\quad \mathrm{f}(v) = \dfrac{1400}{v} + \dfrac{2v}{7}$	
Attempts to evaluate f(v) for 3 values a, b, c where (i) $a < 70$, $b = 70$ and $c > 70$ or (ii) a, $b < 70$ and $c > 70$ or (iii) $a < 70$ and b, $c > 70$.	M1
All 3 correct and states $v = 70$ (exact)	A1
Then 2nd M0, 3rd M0, 2nd A0.	

8(a) Graph	
 Correct shape (ignore anything drawn for $v < 0$).	M1
$v = 70$ (exact)	A1
Then 2nd M0, 3rd M0, 2nd A0.	

8(b)	
Attempt to differentiate their $\dfrac{\mathrm{d}C}{\mathrm{d}v}$; $v^n \to v^{n-1}$ (including $v^0 \to 0$).	M1
$\dfrac{\mathrm{d}^2C}{\mathrm{d}v^2}$ must be correct. Ft only from their value of v and provided their value of v is +ve. Must be some (minimal) indication that their value of v is being used. Statement: "When v = their value of v, $\dfrac{\mathrm{d}^2C}{\mathrm{d}v^2} > 0$" is sufficient provided $2800v^{-3} > 0$ for their value of v. If substitution of their v seen: correct substitution of their v into $2800v^{-3}$, but, provided evaluation is +ve, ignore incorrect evaluation. N.B. Parts in mark scheme in { } do not need to be seen.	A1ft

8(c)	
Substitute their value of v that they think will give $C_{\min}$ (independent of the method of obtaining this value of v and independent of which part of the question it comes from).	M1
40 or £40 Must have part (a) completely correct (i.e. all 5 marks) to gain this A1.	A1
Answer only gains M1A1 provided part (a) is completely correct..	

Examples 8(b)

8(b) $\frac{d^2C}{dv^2} = 2800v^{-3}$ M1

$v = 70$, $\frac{d^2C}{dv^2} > 0$ A1

8(b) $\frac{d^2C}{dv^2} = 2800v^{-3}$ M1

> 0 A0 (no indication that a value of v is being used)

8(b) Answer from (a): $v = 30$

$\frac{d^2C}{dv^2} = 2800v^{-3}$ M1

$v = 30$, $\frac{d^2C}{dv^2} > 0$ A1ft

8(b) $\frac{d^2C}{dv^2} = 2800v^{-3}$ M1

$v = 70$, $\frac{d^2C}{dv^2} = 2800 \times 70^{-3}$

$= 8.16$ A1 (correct substitution of 70 seen, evaluation wrong but positive)

8(b) $\frac{d^2C}{dv^2} = 2800v^{-3}$ M1

$v = 70$, $\frac{d^2C}{dv^2} = 0.00408$ A0 (correct substitution of 70 not seen)

Question Number	Scheme	Marks
9. (a)	$\cos PQR = \dfrac{6^2 + 6^2 - (6\sqrt{3})^2}{2\times 6\times 6} \quad \left\{= -\dfrac{1}{2}\right\}$	M1, A1
	$PQR = \dfrac{2\pi}{3}$	A1 (3)
(b)	Area = $\dfrac{1}{2}\times 6^2 \times \dfrac{2\pi}{3}$ m^2	M1
	= 12π m^2 (✱)	A1cso (2)
(c)	Area of Δ = $\dfrac{1}{2}\times 6\times 6\times \sin\dfrac{2\pi}{3}$ m^2	M1
	= $9\sqrt{3}$ m^2	A1cso (2)
(d)	Area of segment = $12\pi - 9\sqrt{3}$ m^2	M1
	= 22.1 m^2	A1 (2)
(e)	Perimeter = $6 + 6 + \left[6\times\dfrac{2\pi}{3}\right]$ m	M1
	= 24.6 m	A1ft (2) **(11)**

Notes

9(a) N.B. $a^2 = b^2 + c^2 - 2bc\cos A$ is in the formulae book.	
Use of cosine rule for $\cos PQR$. Allow A, θ or other symbol for angle. (i) $(6\sqrt{3})^2 = 6^2 + 6^2 - 2.6.6\cos PQR$: Apply usual rules for formulae: (a) formula not stated, must be correct, (b) correct formula stated, allow one sign slip when substituting. or (ii) $\cos PQR = \dfrac{\pm 6^2 \pm 6^2 \pm (6\sqrt{3})^2}{\pm 2\times 6\times 6}$ Also allow invisible brackets [so allow $6\sqrt{3}^2$] in (i) or (ii)	M1
Correct expression $\dfrac{6^2 + 6^2 - (6\sqrt{3})^2}{2\times 6\times 6}$ o.e. (e.g. $-\dfrac{36}{72}$ or $-\dfrac{1}{2}$)	A1
$\dfrac{2\pi}{3}$	A1

9(a) Alternative	
$\sin\theta = \frac{a\sqrt{3}}{6}$ where θ is any symbol and $a < 6$.	M1
$\sin\theta = \frac{3\sqrt{3}}{6}$ where θ is any symbol.	A1
$\frac{2\pi}{3}$	A1

9(b)	
Use of $\frac{1}{2}r^2\theta$ with $r = 6$ and θ = their (a). For M mark θ does not have to be exact. M0 if using degrees.	M1
12π c.s.o. ($\Rightarrow$ (a) correct exact or decimal value) N.B. Answer given in question	A1
Special case: Can come from an inexact value in (a) $PQR = 2.09 \rightarrow$ Area = $\frac{1}{2}\times 6^2 \times 2.09 = 37.6$ (or 37.7) = 12π (no errors seen, assume full values used on calculator) gets M1 A1. $PQR = 2.09 \rightarrow$ Area = $\frac{1}{2}\times 6^2 \times 2.09 = 37.6$ (or 37.7) = 11.97π = 12π gets M1 A0.	

9(c)	
Use of $\frac{1}{2}r^2\sin\theta$ with $r = 6$ and their (a). $\theta = \cos^{-1}(\text{their } PQR)$ in degrees or radians Method can be implied by correct decimal provided decimal is correct (corrected or truncated to at least 3 decimal places). 15.58845727	M1
$9\sqrt{3}$ c.s.o. Must be exact, but correct approx. followed by $9\sqrt{3}$ is okay (e.g. … = 15.58845 = $9\sqrt{3}$)	A1cso

9(c) Alternative (using $\frac{1}{2}bh$)	
Attempt to find h using trig. or Pythagoras and use this h in $\frac{1}{2}bh$ form to find the area of triangle PQR	M1
$9\sqrt{3}$ c.s.o. Must be exact, but correct approx. followed by $9\sqrt{3}$ is okay (e.g. … = 15.58845 = $9\sqrt{3}$)	A1cso

9(d)	
Use of area of sector – area of Δ or use of $\frac{1}{2}r^2(\theta - \sin\theta)$.	M1
Any value to 1 decimal place or more which rounds to 22.1	A1

9(e)	
6 + 6 + [6 × their (a)].	M1
Correct for their (a) to 1 decimal place or more	A1 ft

Question Number	Scheme	Marks
10. (a)	$\{S_n = \}\ a + ar + \ldots + ar^{n-1}$	B1
	$\{rS_n = \}\ ar + ar^2 + \ldots + ar^n$	M1
	$(1 - r)S_n = a(1 - r^n)$	**d**M1
	$S_n = \dfrac{a(1-r^n)}{1-r}$ (✱)	A1cso (4)
(b)	$a = 200, r = 2, n = 10, \quad S_{10} = \dfrac{200(1-2^{10})}{1-2}$	M1, A1
	$= 204{,}600$	A1 (3)
(c)	$a = \dfrac{5}{6},\ r = \dfrac{1}{3}$	B1
	$S_\infty = \dfrac{a}{1-r}, \qquad S_\infty = \dfrac{\frac{5}{6}}{1-\frac{1}{3}}$	M1
	$= \dfrac{5}{4}$ o.e.	A1 (3)
(d)	$-1 < r < 1$ (or $\vert r \vert < 1$)	B1 (1)
		(11)

Notes

10(a)	
S_n not required. The following must be seen: at least one + sign, a, ar^{n-1} and one other intermediate term. No extra terms (usually ar^n).	B1
Multiply by r; rS_n not required. At least 2 of their terms on RHS correctly multiplied by r.	M1
Subtract both sides: LHS must be $\pm(1 - r)S_n$, RHS must be in the form $\pm a(1 - r^{pn+q})$. Only award this mark if the line for $S_n = \ldots$ or the line for $rS_n = \ldots$ contains a term of the form ar^{cn+d} Method mark, so may contain a slip but not awarded if last term of their S_n = last term of their rS_n.	dM1
Completion c.s.o. N.B. Answer given in question	A1 cso

10(a)	
S_n not required. The following must be seen: at least one + sign, a, ar^{n-1} and one other intermediate term. No extra terms (usually ar^n).	B1
On RHS, multiply by $\dfrac{1-r}{1-r}$	M1

Or Multiply LHS and RHS by $(1 - r)$	
Multiply by $(1 - r)$ convincingly (RHS) and take out factor of a. Method mark, so may contain a slip.	dM1
Completion c.s.o. N.B. Answer given in question	A1 cso

10(b)	
Substitute $r = 2$ with $a = 100$ or 200 and $n = 9$ or 10 into formula for S_n.	M1
$\dfrac{200(1-2^{10})}{1-2}$ or equivalent.	A1
204,600	A1

10(b) Alternative method: adding 10 terms	
(i) Answer only: full marks. (M1 A1 A1)	
(ii) 200 + 400 + 800 + … {+ 102,400} = 204,600 or 100(2 + 4 + 8 + … {+ 1,024)} = 204,600 M1 for two correct terms (as above o.e.) and an indication that the sum is needed (e.g. + sign or the word sum).	M1
102,400 o.e. as final term. Can be implied by a correct final answer.	A1
204,600.	A1

10(c) N.B. $S_\infty = \frac{a}{1-r}$ is in the formulae book.	
$r = \dfrac{1}{3}$ seen or implied anywhere.	B1
Substitute $a = \dfrac{5}{6}$ and their r into $\dfrac{a}{1-r}$. Usual rules about quoting formula.	M1
$\dfrac{5}{4}$ o.e.	A1

10(d) N.B. $S_\infty = \frac{a}{1-r}$ for $\lvert r \rvert < 1$ is in the formulae book.	
$-1 < r < 1$ or $\lvert r \rvert < 1$ In words or symbols. Take symbols if words and symbols are contradictory. Must be < not ≤.	B1