# Mark Scheme (Results)

# January 2007

GCE

## GCE Mathematics

## Mechanics M3 (6679)

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# 6679 Mechanics M3
# Mark Scheme

| Question Number | Scheme | Marks |
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| 1. | (a) Maximum speed when accel. = 0 (o.e.) | B1 (1) |
| | (b) $\frac{1}{12}(30-x) = v\frac{dv}{dx}$ (acceln = … + attempt to integrate) | M1 ↓ |
| | Use of $v\frac{dv}{dx}$: $\frac{v^2}{2} = \frac{1}{12}\left(30x - \frac{x^2}{2}\right)$ $(+c)$ | M1 A1 ↓ |
| | Substituting $x = 30$, $v = 10$ and finding $c$ (= 12.5), or limits | M1 |
| | $\underline{v^2 = 25 + 5x - \frac{1}{12}x^2}$ (o.e.) | A1 (5) |
| | (a) Allow "acceln > 0 for $x < 30$, acceln < 0 for $x > 30$"<br>Also "accelerating for $x < 30$, decelerating for $x > 30$"<br>But "acceln < 0 for $x > 30$" only is B0 | |
| | (b) 1st M1 will be generous for wrong form of acceln (e.g. d$v$/d$x$)!<br>3rd M1 If use limits, they must use them in correct way with correct values<br>Final A1. Have to accept any expression, but it must be for $v^2$ explicitly (not $1/2v^2$), and if in separate terms, one can expect like terms to be collected. Hence answer in form as above, or e.g. $\frac{1}{12}\left(300 + 60x - x^2\right)$; also $100 - \frac{1}{12}(30 - x)^2$ | |

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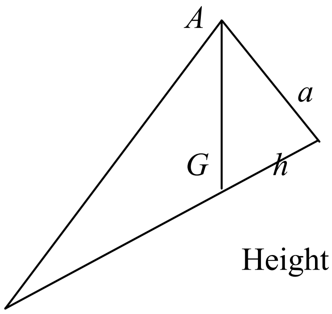
| 2. | <br>Height of cone = $\dfrac{a}{\tan\alpha} = 3a$<br><br>Hence $h = \frac{3}{4}a$<br><br>$\tan\theta = \dfrac{a}{\frac{3}{4}a} = \dfrac{4}{3} \Rightarrow \theta = 53.1°$<br><br>1st M1 (generous) allow any trig ratio to get height of cone (e.g. using sin)<br><br>3rd M1 For correct trig ratio on a suitable triangle to get $\theta$ or complement (even if they call the angle by another name – hence if they are aware or not that they are getting the required angle) | M1 A1<br>↓<br>M1<br>↓<br>M1 A1<br>(5) |

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| 3. | (a) E.P.E. $= \frac{1}{2}\frac{3.6mg}{a}x^2 = \frac{1}{2}\frac{3.6mg}{a}\left(\frac{a}{3}\right)^2$<br>$= \underline{0.2\ mga}$ | M1 A1<br><br>A1<br>(3) |
| | (b) Friction = $\mu mg \Rightarrow$ work done by friction = $\mu mg\left(\frac{4a}{3}\right)$<br>Work-energy: $\frac{1}{2}m.2ga = \mu mgd + 0.2\ mga$ (3 relevant terms)<br>Solving to find $\mu$: $\underline{\mu = 0.6}$ | M1 A1<br><br>M1 A1√<br>↓<br>M1 A1<br>(6) |
| | (b) 1st M1: allow for attempt to find work done by frictional force (i.e. not just finding friction).<br>2nd M1: "relevant" terms, i.e. energy or work terms!<br>A1 f.t. on their work done by friction | |

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| 4. | (a) Energy: $\frac{1}{2}m.3ag - \frac{1}{2}mv^2 = mga(1+\cos\theta)$ | M1 A1 |
| | $\underline{v^2 = ag(1-2\cos\theta)}$ **(o.e.)** | A1 (3) |
| | (b) $T + mg\cos\theta = m\frac{v^2}{a}$ | M1 A1 |
| | Hence $\underline{T = (1-3\cos\theta)mg}$ (*) | A1 cso (3) |
| | (c) Using $T = 0$ to find $\cos\theta$ | M1 |
| | Hence height above $A$ = $\underline{\frac{4}{3}a}$ Accept $1.33a$ (but must have 3+ s.f.) | A1 (2) |
| | (d) $v^2 = \frac{1}{3}ag$ **(o.e.)** f.t. using $\cos\theta = \frac{1}{3}$ in $v^2$ | B1√ |
| | consider vert motion: $(v\sin\theta)^2 = 2gh$ (with $v$ resolved) | M1 A1 ↓ |
| | $\sin^2\theta = \frac{8}{9}$ (or $\theta = 70.53$, $\sin\theta = 0.943$) and solve for $h$ (as $ka$) | M1 |
| | $h = \underline{\frac{4}{27}a}$ or $0.148a$ (awrt) | A1 |
| | **OR** consider energy: $\frac{1}{2}m(v\cos\theta)^2 + mgh = \frac{1}{2}mv^2$ (3 non-zero terms) | M1 A1 ↓ |
| | Sub for $v$, $\theta$ and solve for $h$ | M1 |
| | $h = \underline{\frac{4}{27}a}$ or $0.148a$ (awrt) | A1 |

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| 5. | (a) $\updownarrow \; T\cos\theta = mg$ | | B1 |
| | $\leftrightarrow \; T + T\sin\theta = mr\omega^2$ | (3 terms) | M1 A1 |
| | $r = h\tan\theta$ | | B1 |
| | $\frac{mg}{\cos\theta}(1+\sin\theta) = \frac{m\omega^2 h\sin\theta}{\cos\theta}$ | (eliminate $r$) | ↓ M1 |
| | $\underline{\omega^2 = \frac{g}{h}\left(\frac{1+\sin\theta}{\sin\theta}\right)}$ (*) | (solve for $\omega^2$ ) | ↓ M1 A1 (7) |
| | (b) $\omega^2 = \frac{g}{h}\left(\frac{1}{\sin\theta}+1\right) > \frac{2g}{h}$ $(\sin\theta < 1)$ $\Rightarrow$ $\omega > \sqrt{\frac{2g}{h}}$ (*) | | M1 A1 (2) |
| | (c) $\frac{3g}{h} = \frac{g}{h}\left(\frac{1+\sin\theta}{\sin\theta}\right)$ $\Rightarrow$ $\sin\theta = \frac{1}{2}$ | | M1 A1 |
| | $T\cos\theta = mg$ $\Rightarrow$ $\underline{T = \frac{2\sqrt{3}}{3}mg}$ or $\underline{1.15mg}$ (awrt) | | ↓ M1 A1 (4) |
| | (a) Allow first B1 M1 A1 if assume different tensions (so next M1 is effectively for eliminating *r* **and** *T*.<br><br>(b) M1 requires a *valid* attempt to derive an *in*equality for $\omega$.<br>(Hence putting $\sin\theta = 1$ immediately into expression of $\omega^2$ [assuming this is the critical value] is M0.) | | |

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| 6. | (a) Moments: $\pi \int_1^2 xy^2 \mathrm{d}x = V\bar{x}$ or $\int_1^2 xy^2 \mathrm{d}x = \bar{x}\int_1^2 y^2 \mathrm{d}x$ | M1 |
| | $\int_1^2 y^2\,\mathrm{d}x = \int_1^2 \frac{1}{4x^4}\mathrm{d}x = \left[-\frac{1}{12x^3}\right]_1^2 \quad (= \frac{7}{96})$ (either) | M1 A1 |
| | $\int_1^2 xy^2\,\mathrm{d}x = \int_1^2 \frac{1}{4x^3}\mathrm{d}x = \left[-\frac{1}{8x^2}\right]_1^2 \quad (= \frac{3}{32})$ (both) | A1 |
| | Solving to find $\bar{x}$ $(= \frac{9}{7})$ $\Rightarrow$ required dist $= \frac{9}{7} - 1 = \frac{2}{7}$ m (*) | ↓<br>M1 A1 cso<br>(6) |
| | (b) Mass: $H$: $(\rho)\frac{2}{3}\pi\left(\frac{1}{2}\right)^3$ $\left[= \frac{1}{12}(\rho)\pi\right]$, $S$: $(\rho)\frac{7\pi}{96}$, $T$: $H + S$ $\left[= \frac{5}{32}(\rho)\pi\right]$ | B1, M1 |
| | Dist of CM from base: $H$: $\frac{19}{16}$ m, $S$: $\frac{5}{7}$ m, $T$: $\bar{x}$ | B1 B1 |
| | Moments: $\left[= \frac{1}{12}(\rho)\pi\right]\left(\frac{19}{16}\right) + (\rho)\frac{7\pi}{96}\left(\frac{5}{7}\right) = \left[\frac{5}{32}(\rho)\pi\right]\bar{x}$ | M1 A1 |
| | $\bar{x} = \frac{29}{30}$ m or 0.967 m (awrt) | A1<br>(7) |
| | Allow distances to be found from different base line if necessary | |

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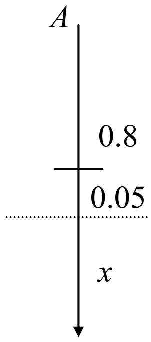
| 7. | (a)  $T = \frac{\lambda}{0.8}(0.05) = 0.25g$ | M1 |
| | $\lambda = \frac{(0.8)(0.25g)}{0.05} = 39.2$ (*) | A1 (2) |
| | (b) $T = \frac{39.2}{0.8}(x + 0.05)$ | M1 |
| | $mg - T = ma$ (3 term equn) | M1 |
| | $0.25g - \frac{39.2}{0.8}(x + 0.05) = 0.25\ddot{x}$ (or equivalent) | A1 |
| | $\ddot{x} = -196x$ | A1 ↓ |
| | SHM with period $\frac{2\pi}{\omega} = \frac{2\pi}{14} = \frac{\pi}{7}$ s (*) | M1 A1 cso (6) |
| | (c) $v = 14\sqrt{\{(0.1)^2 - (0.05)^2\}}$ | M1 A1√ |
| | $= 1.21(24\ldots) \approx$ <u>1.21 m s$^{-1}$</u> (3 s.f.) Accept $7\sqrt{3}/10$ | A1 (3) |
| | (d) Time $T$ under gravity $= \frac{1.21..}{g}$ $(= 0.1237s)$ | B1√ |
| | Complete method for time $T'$ from $B$ to slack.<br>[↑ e.g. $\frac{\pi}{28} + t$, where $0.05 = 0.1\sin 14t$<br>OR $T'$, where $-0.05 = 0.1\cos 14T'$ ] | M1 A1 |
| | $T'' = 0.1496$s | A1 |
| | Total time $= T + T' =$ <u>0.273 s</u> | A1 (5) |
| | (b) 1st M1 must have extn as $x + k$ with $k \neq 0$ (but allow M1 if e.g. $x + 0.15$), or must justify later<br><br>For last four marks, *must* be using $\ddot{x}$ (not $a$)<br><br>(c) Using $x = 0$ is M0<br>(d) M1 – must be using distance for when string goes slack. Using $x = -0.1$ (i.e. assumed end of the oscillation) is M0 | |