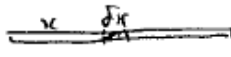
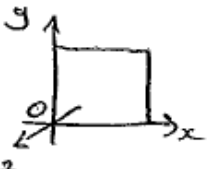
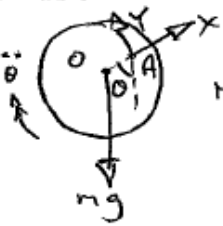
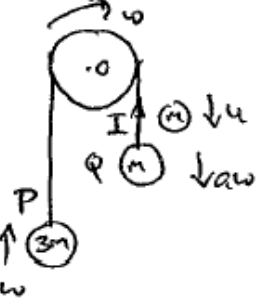


June 2006
6681 Mechanics M5
Mark Scheme

Question Number	Scheme	Marks
1. (a)	 $I = \int_0^{2a} \frac{m}{2a} x^2 dx$ $= \frac{m}{2a} [x^3]_0^{2a}$ $= \frac{4ma^2}{3} *$	M1 A1 A1 (3)
(b)	 $I_x = I_y = \frac{4}{3} ma^2 \text{ (stitching rule)}$ $I_z = I_x + I_y = \frac{8}{3} ma^2 \text{ (per axes)}$	M1 M1A1 (3) (6)
2.	$\underline{d} = \begin{pmatrix} 4 \\ 5 \\ -5 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} = 2\underline{i} + 2\underline{j} - \underline{k}$ $\underline{F} \cdot (2\underline{i} + 2\underline{j} - \underline{k}) = \frac{1}{2} \times \frac{1}{2} \times 12^2 = 36$ <p>but $\underline{F} = \lambda(2\underline{i} + 2\underline{j} - \underline{k})$ (particle starts at rest)</p> $\Rightarrow \lambda(2\underline{i} + 2\underline{j} - \underline{k}) \cdot (2\underline{i} + 2\underline{j} - \underline{k}) = 36$ $\Rightarrow \lambda^2(4 + 4 + 1) = 36$ $\Rightarrow \lambda = 4$ $\underline{F}_2 = 4 \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = \underline{7i} + \underline{6j} - \underline{3k}$	B1 M1A2 M1 M1 A1 M1A1 (9)
3. (a)	$m^2 - 2m = m(m-2) = 0$ $\Rightarrow m = 0 \text{ or } m = 2$ $\Rightarrow \underline{r} = \underline{A} + \underline{B}e^{2t}$ $t=0, \underline{r} = 3\underline{i} \Rightarrow \underline{A} + \underline{B} = 3\underline{i}$ $\underline{\dot{r}} = 2\underline{B}e^{2t}$ $t=0, \underline{\dot{r}} = \underline{j} \Rightarrow \underline{B} = \frac{1}{2}\underline{j}$ $\Rightarrow \underline{r} = (3\underline{i} - \frac{1}{2}\underline{j}) + \frac{1}{2}\underline{j}e^{2t} = 3\underline{i} + \frac{1}{2}\underline{j}(e^{2t} - 1)$	M1 A1 A1 M1A1 M1 A1 A1 (8)
(b)	<p>Particle moves in a straight line Equation of line is $x=3$</p>	B1 B1 (2) (10)

Question Number	Scheme	Marks
4. (a)	$\underline{R} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix} = \underline{(4\mathbf{i} + 2\mathbf{k})}$	M1 A1 (2)
(b)	$\begin{pmatrix} -1 \\ 1 \\ 6 \end{pmatrix} \times \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \times \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$ $= \begin{pmatrix} 0 \\ -6 \\ -6 \end{pmatrix} + \begin{pmatrix} 2 \\ -2 \\ -2 \end{pmatrix} + \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$ $= \begin{pmatrix} 3 \\ 3 \\ -6 \end{pmatrix}$ $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \times \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ -6 \end{pmatrix}$ $\begin{pmatrix} 2y \\ 4z - 2x \\ -4y \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ -6 \end{pmatrix}$ <p>e.g. $x = -3/2, y = 3/2, z = 0$</p> $\underline{r} = \begin{pmatrix} -3/2 \\ 3/2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$	M1 A1 A1 A1 A1 M1 A1 ft. B1 M1 A1 (10) (12)
5. (a)	$m\mathbf{v} \equiv (m + \delta m)(\mathbf{v} + \delta \mathbf{v}) + (-\delta m)(\mathbf{K} + \mathbf{v} + \delta \mathbf{v})$ $m\mathbf{v} \equiv m\mathbf{v} + m\delta\mathbf{v} + v\delta m - K\delta m - v\delta m$ $K\delta m \approx m\delta v$ <p>In the limit, as $\delta t \rightarrow 0$,</p> $\frac{dm}{dv} = \frac{m}{K} *$	M1 A3 M1 A1 (6)
(b)	$m_1 \int \frac{dm}{m} = \int \frac{dv}{K}$ $\ln m_1 - \ln M = \frac{1}{K} (V - U)$ $\ln \frac{m_1}{M} = \frac{1}{K} (V - U)$ $m_1 = M e^{\frac{V-U}{K}}$ $\text{Amount of fuel} = M - m_1 = M \left(1 - e^{\frac{V-U}{K}} \right)$	M1 A1 M1 A1 M1 A1 (6) (12)

Question Number	Scheme	Marks
6(c)	 $I_A = \frac{1}{2}ma^2 + m\left(\frac{1}{2}a\right)^2 = \frac{3}{4}ma^2$ $M(A), -mg\frac{a}{2}\sin\theta = \frac{3}{4}ma^2\ddot{\theta}$ $-\frac{2g}{3a}\sin\theta = \ddot{\theta}$	M1 A1 M1 A2 (5)
(b)	$F\sin\theta = 0, \quad -\frac{2g}{3a}\theta = \ddot{\theta}$ $T = 2\pi\sqrt{\frac{3a}{2g}}$	M1 A1 (2)
(c)	$R(F), \quad Y - mg\sin\theta = m\frac{a}{2}\ddot{\theta}$ $\Rightarrow Y = mg\sin\theta + \frac{ma}{2}\left(-\frac{2g}{3a}\sin\theta\right)$ $= \frac{2mg\sin\theta}{3}$	M1 A2 M1 A1 (5) (12)
7.(a)	 $u = \sqrt{2ag}$ <p style="text-align: center;"><u>CAM about O:</u></p> $m\sqrt{2ag}a = 2ma^2\omega + 3ma^2\omega + \frac{1}{2}2ma^2\omega$ $\frac{\sqrt{2ag}}{6a} = \omega$ $\frac{1}{3}\sqrt{\frac{g}{2a}} = \omega \quad *$	B1 M1 A2 A1 (5)
(b)	<p><u>For Q:</u></p> $-I = 2ma\omega - m\omega$ $\Rightarrow I = 6ma\omega - 2ma\omega = 4ma\omega$ $= \frac{4ma}{3}\sqrt{\frac{g}{2a}} = \frac{m}{3}\sqrt{8ag}$	M1 A1 A1 (5)
(c)	$PE_{Gain} \text{ of } P = KE_{Loss} \text{ of } P + KE_{Loss} \text{ of } Q + KE_{Loss} \text{ of pulley} + PE_{Loss} \text{ of } Q$ $3mgd = \frac{1}{2}3ma^2\omega^2 + \frac{1}{2}2ma^2\omega^2 + \frac{1}{2}ma^2\omega^2 + 2ngd$ $\cancel{gd} = 3ma^2\omega^2$ $gd = 3a^2 \cdot \frac{1}{9} \frac{g}{2a} = \frac{a}{6}$	M1 A3 M1 A1 (6) (14)

