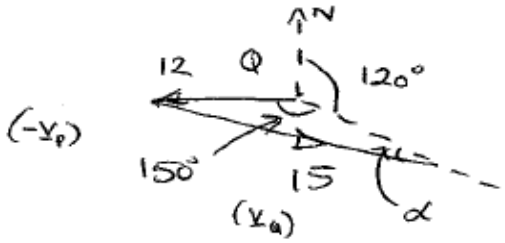
June 2006
6680 Mechanics M4
Mark Scheme

Question Number	Scheme	Marks
1.	 $\frac{\sin\alpha}{12} = \frac{\sin 150°}{15}$ $\Rightarrow \sin\alpha = \frac{6}{15}$ $\Rightarrow \alpha = 23.6°$ $\therefore$ Course is 096 (.4°)	M1 M1 A1 A1 A1 (5)
2.	 Plan view $(\rightarrow)$ $u\cos\alpha = v\cos\theta$ $(\uparrow)$ $eu\sin\alpha = v\sin\theta$ $\Rightarrow v^2 = u^2(\cos^2\alpha + e^2\sin^2\alpha)$ $\Rightarrow KE = \frac{1}{2}mu^2(\cos^2\alpha + e^2\sin^2\alpha)$	M1 A1 M1 A1 M1 A1 (6)

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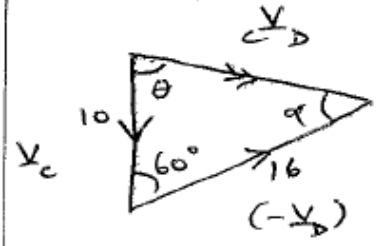
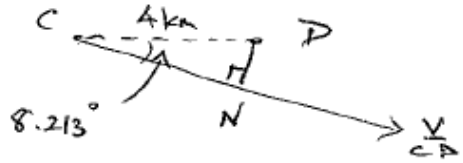
3.(a)	 $\lvert \underset{\sim}{V}_{CD} \rvert^2 = 10^2 + 16^2 - 2\times10\times16\cos 60°$ $= 196$ $\lvert \underset{\sim}{V}_{CD} \rvert = 14 \text{ ms}^{-1}$ *	M1 A1 A1 (3)
(b)	α is acute (opposite shortest side) $\frac{\sin\alpha}{10} = \frac{\sin 60°}{14}$ $\Rightarrow \alpha = 38.213..$	M1 A1
	 (i) $DN = 4000 \sin 8.213$ $\approx$ 571 m $(\frac{4000}{7})$	M1 A1
	(ii) $t = \frac{4000\cos 8.213°}{14}$ secs. $\approx 282.78...$ secs. Time is 2.05 pm (nearest minute)	M1 A1 A1 (7)
		(10)

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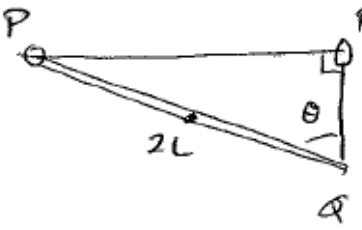
4.(a)	 PE of rod $= -mgL\cos\theta$	B1
	EPE of string $= \frac{kmg}{2L}(2L\cos\theta - L)^2$	M1 A1
	Total PE of system, $V = -mgL\cos\theta + \frac{kmgL}{2}(2\cos\theta - 1)^2 + C$	M1
	$= -mgL\cos\theta + \frac{kmgL}{2}(4\cos^2\theta - 4\cos\theta + 1) + C$	M1 A1
	$= mgL(-\cos\theta + 2k\cos^2\theta - 2k\cos\theta) + c'$	
	$= \underline{mgL[2k\cos^2\theta - (2k+1)\cos\theta] + c'}$ *	A1 (7)
(b)	$\frac{dV}{d\theta} = mgL(-4k\cos\theta\sin\theta + (2k+1)\sin\theta)$	M1 A1
	At equil^m, $mgL\sin\theta(-4k\cos\theta + (2k+1)) = 0$	M1
	$\Rightarrow \sin\theta = 0$ or $\cos\theta = \frac{2k+1}{4k}$	
	$\Rightarrow \theta = 0$ $(\theta > 0)$ $\frac{2k+1}{4k} < 1$	M1
	$2k+1 < 4k$	
	$1 < 2k$	
	$\underline{\frac{1}{2} < k}$ *	A1 (5)
		(12)

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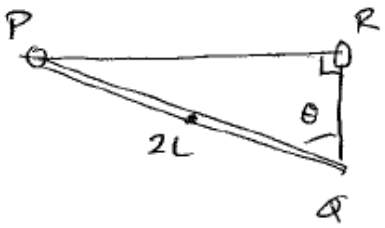
4.(a)	 PE of rod $= -mgL\cos\theta$	B1
	EPE of string $= \frac{kmg}{2L}(2L\cos\theta - L)^2$	M1 A1
	Total PE of system, $V = -mgL\cos\theta + \frac{kmgL}{2}(2\cos\theta - 1)^2 + C$	M1
	$= -mgL\cos\theta + \frac{kmgL}{2}(4\cos^2\theta - 4\cos\theta + 1) + C$	M1 A1
	$= mgL(-\cos\theta + 2k\cos^2\theta - 2k\cos\theta) + c'$	
	$= \underline{mgL[2k\cos^2\theta - (2k+1)\cos\theta] + c'}$ *	A1 (7)
(b)	$\frac{dV}{d\theta} = mgL(-4k\cos\theta\sin\theta + (2k+1)\sin\theta)$	M1 A1
	At equil^{m}, $mgL\sin\theta(-4k\cos\theta + (2k+1)) = 0$	M1
	$\Rightarrow \sin\theta = 0$ or $\cos\theta = \frac{2k+1}{4k}$	
	$\Rightarrow \theta = 0$ $(\theta > 0)$ $\frac{2k+1}{4k} < 1$	M1
	$2k+1 < 4k$	
	$1 < 2k$	
	$\underline{\frac{1}{2} < k}$ *	A1 (5)
		(12)

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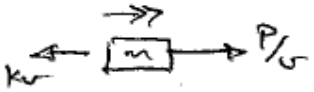
5.(a)	 $\frac{P}{v}$	B1
	$(\rightarrow)$: $\frac{P}{v} - kv = m\frac{dv}{dt}$	M1
	$\Rightarrow \underline{P = mv\frac{dv}{dt} + kv^2}$ *	A1 (3)
(b)	$\int_0^T dt = \int_u^{2u} \frac{mv\,dv}{P-kv^2}$ $\left(u=\frac{1}{3}\sqrt{\frac{P}{k}}\right)$	M1 A1
	$\Rightarrow T = \frac{-m}{2k}\left[\ln(P-kv^2)\right]_u^{2u}$	A2
	$= \frac{m}{2k}\left\{\ln\left(P-\frac{k}{9}\cdot\frac{P}{k}\right) - \ln\left(P-\frac{4k}{9}\cdot\frac{P}{k}\right)\right\}$	M1 A1
	$= \frac{m}{2k}\left\{\ln\frac{8P}{9} - \ln\frac{5P}{9}\right\}$	
	$= \frac{m}{2k}\ln\left(\frac{8P}{9}\times\frac{9}{5P}\right)$	M1
	$= \frac{m}{2k}\ln\frac{8}{5}$	A1 (8)
		(11)

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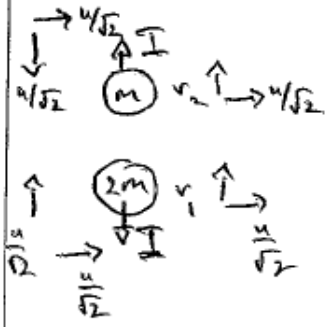
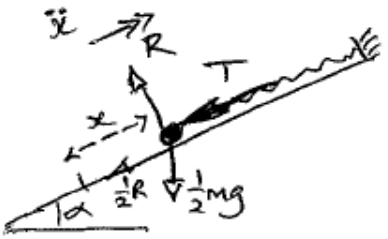
6.(a)	 Form: $I = m\left(v_2 + \frac{u}{\sqrt{2}}\right)$	M1 A1
	CLM(↑): $2m\frac{u}{\sqrt{2}} - m\frac{u}{\sqrt{2}} = 2mv_1 + mv_2$	
	$\frac{u}{\sqrt{2}} = 2v_1 + v_2$ —(1)	M1 A1
	NIL: $e\frac{2u}{\sqrt{2}} = \frac{u}{\sqrt{2}} = -v_1 + v_2$ —(2)	M1 A1
	$\Rightarrow \frac{2u}{\sqrt{2}} = 2v_2$	M1 A1
	$\Rightarrow I = m\left(\frac{u}{\sqrt{2}} + \frac{u}{\sqrt{2}}\right)$	
	$= \underline{\underline{mu\sqrt{2}}}$	A1 (9)
(b)	$v_2 - v_1 = \frac{u}{\sqrt{2}}$ (Separation speed)	M1
	time to wall $= \frac{d}{u/\sqrt{2}} = \frac{d\sqrt{2}}{u}$	M1 A1
	$\therefore$ Separation $= \frac{d\sqrt{2}}{u} \times \frac{u}{\sqrt{2}} = \underline{\underline{d}}$	M1 A1
		(5)
		(14)

Question number	Scheme	Marks
7.(a)	 $F=\frac{1}{2}R$	M1
	$R=mg\cos\alpha$	B1
	$T=\frac{4mgx}{L}$	B1
	$(\rightarrow): \quad -F-mg\sin\alpha-T=m\ddot{x}$	M1 A1
	$-\frac{1}{2}\cdot\frac{4}{5}mg-\frac{3}{5}mg-\frac{4mgx}{L}=m\ddot{x}$	
	$\Rightarrow \frac{d^2x}{dt^2}+4\omega^2x=-g$ * $(\omega=\sqrt{\frac{g}{L}})$	A1 (6)
(b)	$m^2+4\omega^2=0 \Rightarrow m=\pm 2\omega i$	
	C.F. : $x=A\sin 2\omega t+B\cos 2\omega t$	M1
	P.I. : $x=\frac{-g}{4\omega^2}=-\frac{L}{4}$	B1
	G.S. : $x=A\sin 2\omega t+B\cos 2\omega t-\frac{L}{4}$	B1
	$t=0, x=0$: $B=\frac{L}{4}$	
	$\dot{x}=2\omega A\cos 2\omega t-2\omega B\sin 2\omega t$	M1 A1
	$t=0, \dot{x}=\frac{1}{2}\sqrt{gL}$: $\frac{\sqrt{gL}}{2}=2\sqrt{\frac{g}{L}}A \Rightarrow A=\frac{L}{4}$	M1
	$\Rightarrow x=\frac{L}{4}(\sin 2\omega t+\cos 2\omega t-1)$	A1 (7)
(c)	$\dot{x}=0 \Rightarrow 2\omega A\cos 2\omega t-2\omega B\sin 2\omega t=0$	M1
	$\Rightarrow \tan 2\omega t=\frac{A}{B}=1$	
	$\Rightarrow 2\omega t=\pi/4$ (first value)	A1
	$\Rightarrow x=\frac{L}{4}\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}-1\right)$	M1
	$=\frac{L}{4}(\sqrt{2}-1)$	A1 (4)
		(17)