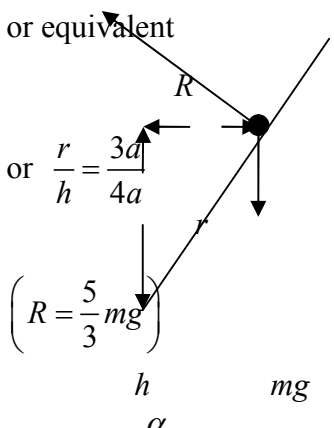
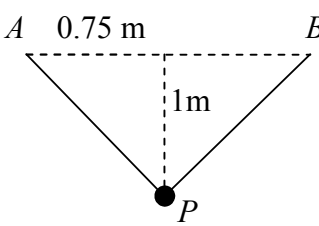
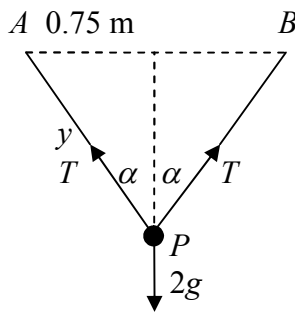


**June 2006**  
**6679 Mechanics M3**  
**Mark Scheme**

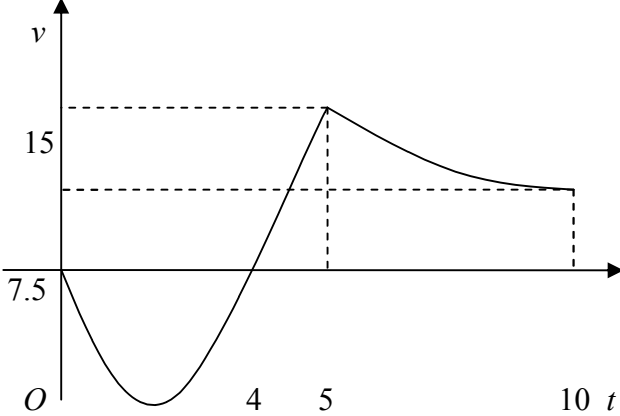
Question Number	Scheme	Marks
1.	<p style="text-align: center;">Use of <math>(\pi) \int y^2 dx \times \bar{x} = (\pi) \int xy^2 dx</math></p> <p style="text-align: center;"><math>\int x dx \times \bar{x} = \int x^2 dx</math></p> <p style="text-align: center;"><math>\left[ \frac{1}{2} x^2 \right]_{\dots}^{\dots} \times \bar{x} = \left[ \frac{1}{3} x^3 \right]_{\dots}^{\dots}</math></p> <p>Using limits 0 and 4</p> <p style="text-align: center;"><math>\frac{16}{2} \times \bar{x} = \frac{64}{3}</math></p> <p style="text-align: center;"><math>\bar{x} = \frac{8}{3}</math></p>	<p>M1</p> <p>A1 = A1</p> <p>M1</p> <p>A1     <b>(5)</b></p> <p style="text-align: right;"><b>[5]</b></p>
2.	<p>(a)                      Small Hemisphere    Bowl    Large Hemisphere</p> <p>Mass ratios            <math>\frac{2}{3} \pi \left( \frac{a}{2} \right)^3</math>    <math>\frac{2}{3} \pi \frac{7a^3}{8}</math>    <math>\frac{2}{3} \pi a^3</math></p> <p style="text-align: right;">Anything in the ratio 1 : 7 : 8</p> <p><math>\bar{x}</math>                      <math>\frac{3}{16} a</math>            <math>\bar{x}</math>                      <math>\frac{3}{8} a</math></p> <p style="text-align: center;"><math>1 \times \frac{3}{16} a + 7 \times \bar{x} = 8 \times \frac{3}{8} a</math></p> <p>Leading to            <math>\bar{x} = \frac{45}{112} a</math> *                                      cso</p> <p>(b)                      Bowl    Liquid    Bowl and Liquid</p> <p>Mass Ratios            <math>M</math>            <math>kM</math>            <math>(k+1)M</math></p> <p><math>\bar{x}</math>                      <math>\frac{45}{112} a</math>            <math>\frac{3}{16} a</math>            <math>\frac{17}{48} a</math></p> <p style="text-align: center;"><math>M \times \frac{45}{112} a + kM \times \frac{3}{16} a = (k+1)M \times \frac{17}{48} a</math></p> <p>Leading to            <math>k = \frac{2}{7}</math></p>	<p>B1</p> <p>B1</p> <p>M1 A1</p> <p>A1     <b>(5)</b></p> <p>B1</p> <p>B1</p> <p>M1 A1</p> <p>A1     <b>(5)</b></p> <p style="text-align: right;"><b>[10]</b></p>

Question Number	Scheme	Marks
3.	<p>(a)</p> $a = 0.1$ $\frac{2\pi}{\omega} = \frac{1}{5} \Rightarrow \omega = 10\pi$ $F_{\max} = ma\omega^2$ $= 0.2 \times 0.1 \times (10\pi)^2$ $\approx 19.7 \text{ (N)}$ <p>cao</p> <p>(b)</p> $a' = 0.2, \quad \omega' = 10\pi$ $v^2 = \omega'^2 (a^2 - x^2) = 100\pi^2 (0.2^2 - 0.1^2) \quad (= 3\pi^2 \approx 29.6 \dots)$ $v \approx 5.44 \text{ (ms}^{-1}\text{)}$ <p>cao</p> <p><i>If answers are given to more than 3 significant figures a maximum of one A mark is lost in the question.</i></p>	<p>B1</p> <p>M1 A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p><b>(6)</b></p> <p>B1ft, B1ft</p> <p>M1 A1</p> <p>A1</p> <p><b>(5)</b></p> <p><b>[11]</b></p>
4.	<p>or equivalent</p>  <p>or <math>\frac{r}{h} = \frac{3a}{4a}</math></p> <p><math>\left( R = \frac{5}{3}mg \right)</math></p> <p><math>h</math>      <math>mg</math></p> <p><math>\alpha</math></p> <p><math>= mr \times \frac{8g}{9a} \quad \left( R = \frac{10mrg}{9a} \right)</math></p> <p><math>\tan \alpha = \frac{9a}{8r} \quad \left( \frac{5}{3}mg = \frac{10mrg}{9a} \right)</math></p> <p>Eliminating R</p> <p><math>\left( \frac{3}{4} = \frac{9a}{8r} \Rightarrow r = \frac{3}{2}a \right)</math></p> <p><math>h = \frac{r}{\tan \alpha} = \frac{3a}{2} \times \frac{4}{3} = 2a</math></p>	<p><math>\tan \alpha = \frac{3}{4}</math> B1</p> <p><math>\tan \alpha = \frac{r}{h}</math> B1</p> <p><math>R(\uparrow) \quad R \sin \alpha = mg</math> M1 A1</p> <p><math>R(\leftarrow) \quad R \cos \alpha = mr\omega^2</math> M1 A1</p> <p>A1</p> <p>M1 A1</p> <p>M1 A1</p> <p><b>(11)</b></p>

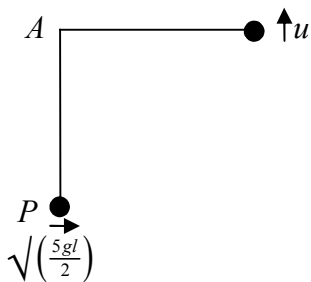
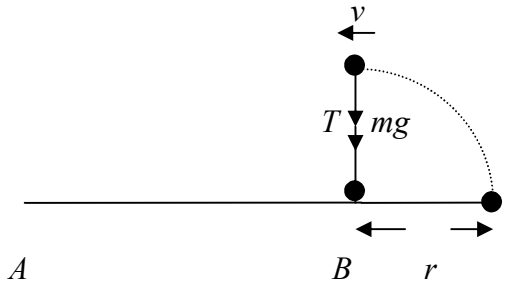


Question Number	Scheme	Marks
5.	<p>(a)</p>  $AP = \sqrt{(0.75^2 + 1^2)} = 1.25$ <p>Conservation of energy</p> $\frac{1}{2} \times 2 \times v^2 + 2 \times \frac{49 \times 0.5^2}{2 \times 0.75} = 2g \times 1 \quad -1$ <p>for each incorrect term</p> <p>Leading to <math>v \approx 1.8 \text{ (ms}^{-1}\text{)}</math></p> <p>accept 1.81</p> <p>(b)</p>  <p><math>R(\uparrow) \quad 2T \cos \alpha = 2g</math></p> $y = \frac{0.75}{\sin \alpha}$ <p>Hooke's Law</p> $T = \frac{49}{0.75} \left( \frac{0.75}{\sin \alpha} - 0.75 \right)$ $= 49 \left( \frac{1}{\sin \alpha} - 1 \right)$ $\frac{9.8}{\cos \alpha} = 49 \left( \frac{1}{\sin \alpha} - 1 \right)$ <p>Eliminating <math>T</math></p> $\tan \alpha = 5(1 - \sin \alpha)$ $5 = \tan \alpha + 5 \sin \alpha \quad *$ <p>cs0</p>	<p>M1 A1</p> <p>M1 A2 (1, 0)</p> <p>A1 (6)</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1</p> <p>A1 (6)</p> <p>[12]</p>



Question Number	Scheme	Marks
6.	<p>(a)</p>  <p>Parabola Hyperbola Points</p> <p>(b) Identifying the minimum point of the parabola and 5 as the end points.</p> $2 < t < 5$ <p>(c) Splitting the integral into two part, with limits 0 and 4, and 4 and 5, and evaluating both integrals.</p> $\int_0^4 3t(t-4) dt = [t^3 - 6t^2]_0^4 = -32 \quad \text{and} \quad \int_4^5 3t(t-4) dt = [t^3 - 6t^2]_4^5 = 7$ <p>Both</p> <p style="text-align: center;">Total distance = 39 (m) *</p> <p>cs0</p> <p>(d)</p> $\int_5^{t_1} \frac{75}{t} dt = 32 - 7$ $75[\ln t]_5^{t_1} = 25$ $\ln \frac{t_1}{5} = \frac{1}{3} \Rightarrow t_1 = 5e^{\frac{1}{3}}$ $\approx 6.98$ <p>cao</p>	<p>B1 B1 B1 <b>(3)</b></p> <p>M1 A1 <b>(2)</b></p> <p>M1</p> <p>A1</p> <p>A1 <b>(3)</b></p> <p>M1 A1</p> <p>A1</p> <p>M1</p> <p>A1 <b>(5)</b></p> <p><b>[13]</b></p>

--	--	--	--

Question Number	Scheme	Marks
7.	<p>(a)</p>  <p>Conservation of Energy</p> $\frac{1}{2}m\left(\frac{5gl}{2} - u^2\right) = mgl$ <p>Leading to <math>u = \sqrt{\left(\frac{gl}{2}\right)}</math></p> <p>(b)</p>  <p>Conservation of Energy</p> $\frac{1}{2}m(u^2 - v^2) = mgr$ $v^2 = u^2 - 2gr$ <p><math>R(\downarrow) \quad T + mg = \frac{mv^2}{r}</math></p> $T = \frac{m}{r}(u^2 - 2gr) - mg$ $= \frac{mu^2}{r} - 3mg$ $= \frac{mgl}{2r} - 3mg$ <p><math>T \geq 0 \Rightarrow \frac{mgl}{2r} \geq 3mg</math></p> $\Rightarrow \frac{1}{6} \geq r$ $AB_{\text{MIN}} = \frac{5l}{6}$	<p>M1 A1= A1</p> <p>A1 (4)</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1 (9)</p> <p>[13]</p>







