Paper Reference(s)

6676/01

Edexcel GCE

Pure Mathematics P6
Further Pure Mathematics FP3

Advanced/Advanced Subsidiary

Monday 16 January 2006 – Morning
Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Lilac)	Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Pure Mathematics P6/Further Pure Mathematics FP3), the paper reference (6676), your surname, initials and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions on this paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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1. Solve the equation

$$z^5 = \mathrm{i},$$

giving your answers in the form $\cos\theta + \mathrm{i}\sin\theta$.

(5)

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2. The variable y satisfies the differential equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 2 + 3y^2\frac{\mathrm{d}y}{\mathrm{d}x}.$$

It is given that $y = 1$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 2$ at $x = 0.5$.

Use the approximations

$$\left(\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}\right)_0 \approx \frac{y_1 - 2y_0 + y_{-1}}{h^2} \text{ and } \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 \approx \frac{y_1 - y_{-1}}{2h}, \quad \text{with } h = 0.1,$$

to find an estimate of y at $x = 0.4$.

(6)

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3. A transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ is represented by the matrix

$$\mathbf{A} = \begin{pmatrix} k & 2 \\ 2 & -1 \end{pmatrix}, \text{ where } k \text{ is a constant.}$$

For the case $k = -4$,

(*a*) find the image under T of the line with equation $y = 2x + 1$.

(2)

For the case $k = 2$, find

(*b*) the two eigenvalues of $\mathbf{A}$,

(4)

(*c*) a cartesian equation for each of the two lines passing through the origin which are invariant under T.

(3)

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4. $$\mathbf{A} = \begin{pmatrix} k & 1 & -2 \\ 0 & -1 & k \\ 9 & 1 & 0 \end{pmatrix}, \text{ where } k \text{ is a real constant.}$$

(*a*) Find values of k for which **A** is singular.

(4)

Given that **A** is non-singular,

(*b*) find, in terms of k, $\mathbf{A}^{-1}$.

(5)

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5. Prove by induction that, for $n \in \mathbb{Z}^+$,

(i) $\sum_{r=1}^{n} r2^r = 2\{1 + (n-1)2^n\}$,

(5)

(ii) for $y = \ln(2 + 3x)$, where $x > -\frac{2}{3}$,

$$\frac{d^n y}{dx^n} = (-1)^{n+1}\frac{(n-1)!3^n}{(2+3x)^n}.$$

(5)

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6. $$(1 + 2x)\frac{dy}{dx} = x + 4y^2.$$

(*a*) Show that

$$(1 + 2x)\frac{d^2 y}{dx^2} = 1 + 2(4y - 1)\frac{dy}{dx}. \qquad (1)$$

(2)

(*b*) Differentiate equation (1) with respect to x to obtain an equation involving

$\frac{d^3 y}{dx^3}$, $\frac{d^2 y}{dx^2}$, $\frac{dy}{dx}$, x and y.

(3)

Given that $y = \frac{1}{2}$ at $x = 0$,

(*c*) find a series solution for y, in ascending powers of x, up to and including the term in x^3.

(6)

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7. The plane Π passes through the points

$$P(-1, 3, -2),\ \ Q(4, -1, -1) \text{ and } R(3, 0, c), \text{ where } c \text{ is a constant.}$$

(*a*) Find, in terms of c, $\overrightarrow{RP} \times \overrightarrow{RQ}$.

(3)

Given that $\overrightarrow{RP} \times \overrightarrow{RQ} = 3\mathbf{i} + d\mathbf{j} + \mathbf{k}$, where d is a constant,

(*b*) find the value of c and show that $d = 4$,

(2)

(*c*) find an equation of Π in the form $\mathbf{r.n} = p$, where p is a constant.

(3)

The point S has position vector $\mathbf{i} + 5\mathbf{j} + 10\mathbf{k}$. The point S' is the image of S under reflection in Π.

(*d*) Find the position vector of S'.

(5)

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8. In the Argand diagram the point P represents the complex number z.

Given that $\arg\left(\dfrac{z - 2\text{i}}{z + 2}\right) = \dfrac{\pi}{2}$,

(*a*) sketch the locus of P,

(4)

(*b*) deduce the value of $|z + 1 - \text{i}|$.

(2)

The transformation T from the z-plane to the w-plane is defined by

$$w = \frac{2(1 + \text{i})}{z + 2}, \quad z \neq -2.$$

(*c*) Show that the locus of P in the z-plane is mapped to part of a straight line in the w-plane, and show this in an Argand diagram.

(6)

TOTAL FOR PAPER: 75 MARKS

END