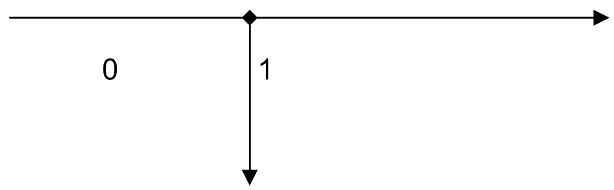
Question Number	Scheme	Marks
1.	$\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$	B1
	$\cos\frac{\pi}{10} + i\sin\frac{\pi}{10}$	B1
	$\cos\left(\frac{(4k+1)\pi}{10}\right) + i\sin\left(\frac{(4k+1)\pi}{10}\right)$, $k =$ 2, 3, 4(or equiv.)	M1A2,1,0
		(5)
	$[\cos\left(\frac{9\pi}{10}\right) + i\sin\left(\frac{9\pi}{10}\right), \cos\left(\frac{13\pi}{10}\right) + i\sin\left(\frac{13\pi}{10}\right), \cos\left(\frac{17\pi}{10}\right) + i\sin\left(\frac{17\pi}{10}\right)]$ [Degrees : 18, 90, 162, 234, 306]	
		Total 5 marks
2.	$\left(\frac{dy}{dx}\right)_0 \approx \frac{y_1 - y_{-1}}{2h} \Rightarrow 2 \approx \frac{y_1 - y_{-1}}{0.2} \Rightarrow y_1 - y_{-1} \approx 0.4$	M1A1
	$\left(\frac{d^2y}{dx^2}\right)_0 \approx \frac{y_1 - 2y_0 + y_{-1}}{h^2} \Rightarrow 8 \approx \frac{y_1 - 2y_0 + y_{-1}}{0.01}$	M1
	[For M1, an attempt at evaluating $\left(\frac{d^2y}{dx^2}\right)_0$ is required.]	
	$\Rightarrow y_1 + y_{-1} \approx 2.08$	A1
	Subtracting to give $y_{-1} \approx 0.84$	M1A1 (6)
		Total 6 marks
3.	(a) Complete method for finding image: e.g. $\begin{pmatrix} -4 & 2 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ 2x+1 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$	M1
	The image is the point (2, –1)	A1
		(2)
	(b) $\begin{vmatrix} k-\lambda & 2 \\ 2 & -1-\lambda \end{vmatrix} = 0$	M1
	Characteristic equation: $\lambda^2 - \lambda - 6 = 0$	A1
	Solving : $(\lambda - 3)(\lambda + 2) = 0 \Rightarrow \lambda = ...$	M1
	$\lambda = -2,\ \lambda = 3$ (both)	A1 (4)
	(c) Method for finding an eigenvector $\begin{pmatrix} 2 & 2 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 3\begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and	M1

	$\begin{pmatrix} 2 & 2 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = -2\begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$	A1√
	Equations are: $y = \frac{1}{2}x$ and $y = -2x$.	A1 (3)
	Alt: $\begin{pmatrix} 2 & 2 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ mx \end{pmatrix} = \begin{pmatrix} y \\ my \end{pmatrix} \Rightarrow 2m^2 + 3m - 2 = 0$ M1A1 [m = ½, –2] Correct equations A1	**Total 9 marks**
4.	$\mathbf{A} = \begin{pmatrix} k & 1 & -2 \\ 0 & -1 & k \\ 9 & 1 & 0 \end{pmatrix}$	
	(a) Det. **A** = $-k^2 + 9k - 18$	M1A1
	Setting to zero and solving for k [$(k-6)(k-3) = 0$] $\Rightarrow k = 3,\ k = 6$	M1 A1 (4)
	(b) Cofactors $\begin{pmatrix} -k & 9k & 9 \\ -2 & 18 & 9-k \\ k-2 & -k^2 & -k \end{pmatrix}$ [B1 for each row (or column)]	B3
	$\mathbf{A}^{-1} = \frac{1}{\det}\begin{pmatrix} -k & -2 & k-2 \\ 9k & 18 & -k^2 \\ 9 & 9-k & -k \end{pmatrix}$ [A1 f.t. is on determinant or cofactors]	M1A1√ (5) **Total 9 marks**

5.	(i) When $n = 1$, LHS $= 1(2)^1 = 2$; RHS $= 2\{1 + 0\} = 2 \Rightarrow$ true for $n = 1$	B1
	Suppose true for $n = k$, then $\sum_1^{k+1} r\,2^r = 2\{1 + (k-1)2^k + (k+1)2^{k+1}\}$	M1
	$= 2 + k\,2^{k+1} + k\,2^{k+1}$	A1
	$= 2(1 + k\,2^{k+1})$	
	$= 2[1 + \{(k+1) - 1\}2^{k+1}]$	M1
	So, if true for $n = k$ then true for $n = k + 1$, but true for $n = 1$, $\therefore$ true, by induction, for all values of n $\in Z^+$.	A1(cso) (5)
	(ii) Showing true for $n = 1$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{2+3x}$	B1
	$\frac{\mathrm{d}^{n+1}y}{\mathrm{d}x^{n+1}} = (-1)^{n+1}\frac{3^n(n-1)!(-3n)}{(2+3x)^{n+1}}$	M1A1
	$= (-1)^{n+2}\frac{3^{n+1}(n)!}{(2+3x)^{n+1}}$	A1
	Conclusion	A1(cso) (5)
		Total 10 marks
6.	(a) Correct method for producing 2nd order differential equation	
	e.g. $\frac{\mathrm{d}}{\mathrm{d}x}\left\{(1+2x)\frac{\mathrm{d}y}{\mathrm{d}x}\right\} = \frac{\mathrm{d}}{\mathrm{d}x}\{x + 4y^2\}$ attempted	M1
	$(1+2x)\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2\frac{\mathrm{d}y}{\mathrm{d}x} = 1 + 8y\frac{\mathrm{d}y}{\mathrm{d}x}$ seen + conclusion AG	A1* (2)
	(b) Differentiating again w.r.t. x: $(1+2x)\frac{\mathrm{d}^3y}{\mathrm{d}x^3} + 2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 8y\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 8\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 - 2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ or equiv.	
	[e.g. $(1+2x)\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = 8\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 + 4(2y-1)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$	M1A2,1,0 (3)
	(c) $\frac{\mathrm{d}y}{\mathrm{d}x}$ (at $x = 0$) = 1	

	Finding $\frac{d^2y}{dx^2}$ (at $x = 0$) (= 3) Finding $\frac{d^3y}{dx^3}$, at $x = 0$; = 8 [A1 f.t. is on part (c) values only] $y = \frac{1}{2} + x + \frac{3}{2}x^2 + \frac{4}{3}x^3 + ...$ [Alternative (c): Polynomial for y: $y = ½ + ax + bx^2 + cx^3 + ...$ M1 In given d.e.: $(1 + 2x)(a + 2bx + 3cx^2 + \dots) \equiv x + 4(½ + ax + bx^2 + cx^3 + ...)^2$ M1A1 a = 1 B1, Complete method for other coefficients M1, answer A1	B1 M1 M1A1√ M1A1 (6) **Total 11 marks**
7.	(a) $\vec{RQ} = \begin{pmatrix} 1 \\ -1 \\ -1-c \end{pmatrix}$, $RP = \begin{pmatrix} -4 \\ 3 \\ -2-c \end{pmatrix}$ (both) $\vec{RP} \times \vec{RQ} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & 3 & -2-c \\ 1 & -1 & -1-c \end{vmatrix}$ $= (-5-4c)\,\mathbf{i} - (6+5c)\,\mathbf{j} + \mathbf{k}$ (b) $c = -2$ $d = -6 - 5c = 4$ AG (c) $\mathbf{r}.\begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} = p$ Substituting point in plane to give p, $\mathbf{r}.\begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} = 7.$ (d) Equation of normal to plane through S : $\mathbf{r} = \begin{pmatrix} 1 \\ 5 \\ 10 \end{pmatrix} + t\begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}$ Meets plane where $\begin{pmatrix} 1+3t \\ 5+4t \\ 10+t \end{pmatrix}.\begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} = 7 \Rightarrow t = -1$ S' has position vector $\begin{pmatrix} 1 \\ 5 \\ 10 \end{pmatrix} + 2t\begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} = \begin{pmatrix} -5 \\ -3 \\ 8 \end{pmatrix}$	B1 M1A1√ (3) A1√ A1*(cso) (2) M1 M1A1 (3) B1 M1A1√

	(a) Relating lines and angle (generous) [*angle between* $\pm 2\mathrm{i}$ *to P and* ± 2 *to P*]	M1A1 (5) **Total 13 marks**
8.	Angle between correct lines is $\frac{\pi}{2}$ Circle Selecting correct ("top half") semi-circle . [If algebraic approach: Method for finding Cartesian equation M1 Correct equation, any form, $\Rightarrow x(x+2)+y(y-2)=0$ A1 Sketch: showing circle M1 Correct circle { centre (–1, 1)}, choosing only "top half" A1]	M1 A1 M1 A1 (4)
	(b) $\lvert z+1-\mathrm{i} \rvert$ is radius; $=\sqrt{2}$	M1A1 (2)
	(c) $z=\frac{2(1+\mathrm{i})-2\omega}{\omega}$ $\left(=\frac{2(1+\mathrm{i})}{\omega}-2\right)$ $\frac{z-2\mathrm{i}}{z+2}=\frac{2(1+\mathrm{i})-2(1+\mathrm{i})\omega}{2(1+\mathrm{i})}$ $(=1-\omega)$ Arg $(1-\omega)=\frac{\pi}{2}$ is line segment, passing through (1,0)  Alt ©: $u+iv=\frac{2+2i}{(x+2)+iy}=\frac{(2x+2y+4)+i(x+2-y)}{(x+2)^2+y^2}$ M1 $x=-1+\sqrt{2}\cos\theta,\ y=1+\sqrt{2}\sin\theta$ M1 $\Rightarrow w=\frac{(2\sqrt{2}\cos\theta+2\sqrt{2}\sin\theta+4)+i.....}{(2\sqrt{2}\cos\theta+2\sqrt{2}\sin\theta+4)}$ {= $1+i\,\mathrm{f}(\theta)$ } A1, $\Rightarrow$ part of line $u=1$, show lower "half" of line A1,A1	M1 M1A1 A1,A1 A1 (6) **Total 12 marks**